

Flavor changing fermion-graviton vertices

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Introduction

- Gravity is universally coupled to all matter fields
- Universality is provided by the tree-level coupling of graviton field with the (conserved) matter energy-momentum tensor
- **quantum corrections modify this picture**
 - how virtual photons alter graviton-electron vertex,
 - how virtual electrons alter graviton-photon vertex, etc.
- If virtual gravitons are not included in loops, **quantum corrections to on-shell matrix elements are finite:** due to Ward Identities from matter energy-momentum conservation

- once the result is finite, one can ask for the magnitude of the finite radiative corrections
 - QED corrections are typical of order of a percent
 - should we expect that Newton law is only 99% accurate ?
 - same question applies to the light deflection experiment
- Berends and Gastmans, Ann. Phys. 98, 225 ('76)
- **Quantum corrections due to Weak interactions**
 - a complete study of these effects is still missing
 - **charged Weak currents generate at one-loop flavor-changing neutral currents (FCNC) in the fermion sector**
 - analogous FC contribution in fermion-graviton couplings
 - **Could induce a flavor-changing term in Newton potential ?**

To analyze this issue we computed the flavor-changing fermion-graviton couplings at one-loop level in the standard model

Theoretical framework

Invariant action under general coordinate transformation

$$\mathcal{S} = \int d^4x \sqrt{-g} \mathcal{L}_{\text{SM}}[g_{\mu\nu}] \quad g = \det g_{\mu\nu}$$

 **SM Lagrangian**

➤ Expand the metric $g_{\mu\nu}$ around Minkowski metric $\eta_{\mu\nu}$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \quad \kappa = \sqrt{32\pi G_N}$$

 **graviton field**

Newton constant

➤ At the first order in κ we obtain

$$\mathcal{L}_{int} = -\frac{1}{\bar{M}_P} T^{\mu\nu} h_{\mu\nu} \quad \bar{M}_P \equiv 2/\kappa$$

 **energy-momentum tensor**

QED corrections to electron-graviton vertex

Starting point: **evaluation of the on-shell matrix element**

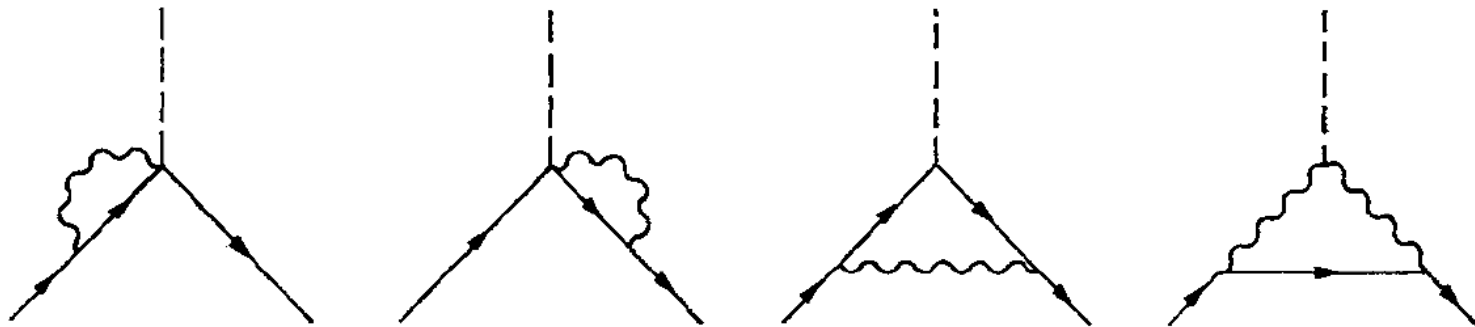
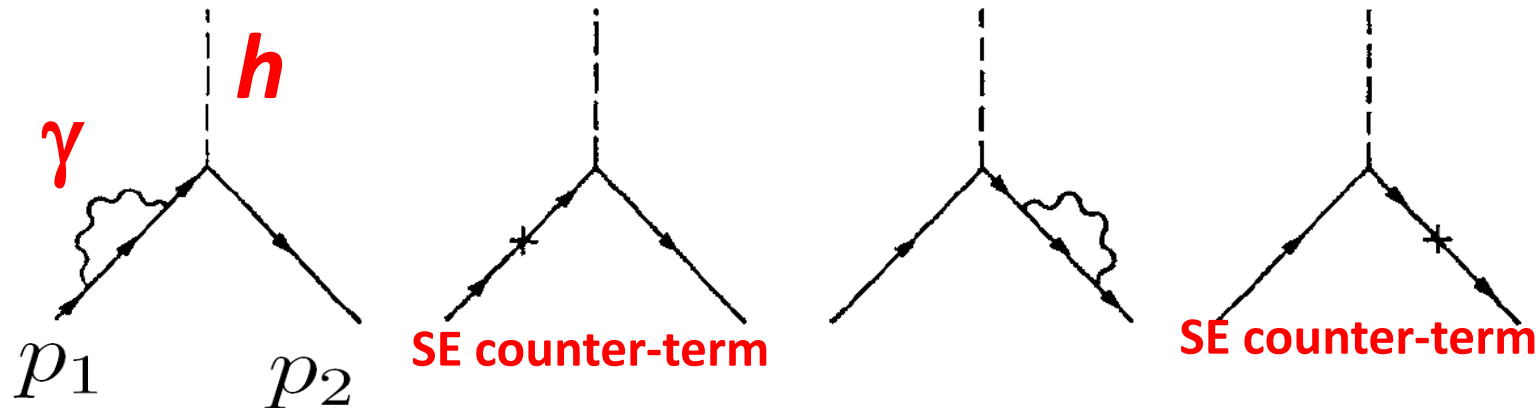
$$\hat{T}^{\mu\nu} \equiv -i \langle p_2 | T^{\mu\nu}(0) | p_1 \rangle$$

$$T_{\mu\nu} = \left(-\eta_{\mu\nu} \mathcal{L} + 2 \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} \right) |_{g=\eta}$$

$T_{\mu\nu}$ expression in QED:

$$\begin{aligned} T_{\mu\nu} = & \frac{i}{4} \bar{\psi} (\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu) \psi - \frac{i}{4} (\partial_\mu \bar{\psi} \gamma_\nu + \partial_\nu \bar{\psi} \gamma_\mu) \psi \\ & + F_{\sigma\mu} F_\nu{}^\sigma + \frac{1}{4} \eta_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} - \frac{1}{2} e \bar{\psi} (\gamma_\mu A_\nu + \gamma_\nu A_\mu) \psi \end{aligned}$$

One-loop Feynman diagrams in QED (Feynman gauge) graviton-electron vertex (in/out fermions are on-shell)



result is finite after renormalization, due to WI:

$$\partial_\mu T^{\mu\nu}(x) = q_\mu \langle p_2 | T^{\mu\nu}(0) | p_1 \rangle = 0$$

$$q = p_1 - p_2$$

Matrix element at one-loop

$$p = p_1 + p_2$$

$$q = p_1 - p_2$$

$$\begin{aligned}\hat{T}^{\mu\nu} = & -\frac{i}{4\bar{M}_P} \bar{u}(p_2) \left[(\gamma^\mu p^\nu + \gamma^\nu p^\mu) F_1(q^2) \right. \\ & \left. + 2p^\mu p^\nu \frac{F_2(q^2)}{m_e} + (q^2 \eta^{\mu\nu} - q^\mu q^\nu) \frac{F_3(q^2)}{m_e} \right] u(p_1)\end{aligned}$$

only three form factors: $q^2 < 0$

$$\begin{aligned}F_1(q^2) &= 1 + \frac{\alpha}{\pi} \left\{ \mathbf{f}_1(\mathbf{x}) - \log \frac{\lambda}{m_e} \left(1 + 2 \frac{1+x^4}{1-x^4} \log x \right) \right\} \\ F_2(q^2) &= \frac{\alpha}{\pi} \mathbf{f}_2(\mathbf{x}), \quad F_3(q^2) = \frac{\alpha}{\pi} \frac{4m_e^2}{(-q^2)} \mathbf{f}_3(\mathbf{x})\end{aligned}$$

$$x = (-b)^{1/2}, \quad b = (1-a)/(1+a), \quad a = \left(1 - 4m_e^2/q^2 \right)^{1/2}$$

λ is a fictitious photon mass (needed to regularize infrared divergencies)

- matrix element is finite and gauge invariant
- **Newton constant is not renormalized by QED corrections**
- infrared divergence is canceled in the usual way by considering soft photon bremsstrahlung

Interesting to examine the $q^2 \rightarrow 0$ limit

$$F_1(0) \simeq 1 - \frac{\alpha\pi}{8} \left(\frac{-q^2}{m_e^2} \right)^{1/2}$$

$$F_2(0) \simeq -\frac{\alpha\pi}{16} \left(\frac{-q^2}{m_e^2} \right)^{1/2}$$

$$F_3(0) \simeq -\frac{\alpha\pi}{8} \left(\frac{m_e^2}{-q^2} \right)^{1/2}$$

Electron scattering by a heavy stationary scalar mass M

Spin-independent term in the scattering amplitude $\mathcal{M}$

$$\mathcal{M} \simeq i \frac{4\pi G_N M m_e}{|\mathbf{q}|^2} u^\dagger \left[1 - \frac{\alpha\pi}{4} \left(\frac{|\mathbf{q}|^2}{m_e^2} \right)^{1/2} + \mathcal{O} \left(\frac{q^2}{m_e^2} \right) \right] u$$

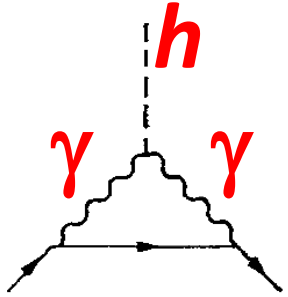
In coordinate space this leads to a **small modification of Newton potential at large distances** ($r \gg m_e^{-1}$)

$$V(r) \sim -\frac{G_N M m_e}{r} \left(1 - \frac{\alpha}{2m_e r} \right)$$

correction of order **$O(10^{-22})$** on earth surface

repulsive force

➤ $1/r^2$ term has an infrared origin due to h - γ - γ coupling



→ loop integral induces a term proportional to
 $\propto \sqrt{-q^2 / m_e^2}$ instead of q^2 .

graviton pole is not fully canceled in scattering amplitude

➤ after Fourier transforming ➡ $1/(r^2 m_e)$ generated

➤ no counterpart of such effect in QED corrections to Coulomb potential

➤ Remarkably, $1/r^2$ term has a classical origin:

Derived by C.M. De Witt and B.S. De Witt, Physics 1, 3 (1964) on the basis of pure classical considerations in General Relativity.

“total mass of a particle is not concentrated at a point, but partly distributed as field energy in the space around the particle”

Gravity coupled to weak interactions

- one-loop radiative corrections to graviton-fermion vertex include also the exchanges of virtual W and Z
- a complete study of this effect is still missing
- Weak radiative corrections are expected to give a small contribution **at low energies $E \ll m_W$** , of order **$O(G_F E^2)$**
- no infrared divergencies, being W and Z massive
- therefore, **no modification of Newton potential as $1/r^2$ in QED is expected**

- **Peculiar aspects of Weak interactions:** at one-loop, W -exchanges can induce FCNC in the fermion sector
- FCNC is a **pure quantum effect**
- Flavor-changing graviton couplings are not present at tree-level
- One-loop Weak corrections can induce **an off-diagonal (flavor) $\Delta F=1$ contribution to matter energy-momentum tensor**
- **spin-2 counter part of FCNC $\rightarrow$ Tensorial FCNC (TFCNC)**

Flavor changing quark-graviton vertex

One needs first to evaluate the (flavor) **off-diagonal matrix element** of energy-momentum tensor at one-loop: (**$\Delta F=1$ transition**)

$$\hat{T}^{\mu\nu} \equiv -i \langle p_2 | T^{\mu\nu}(0) | p_1 \rangle$$

in/out are on-shell quarks with different flavor

- we performed the calculation in **R_ξ gauge ($\xi=1$)** and cross-checked the results in unitary gauge
- if standard t'Hooft gauge-fixing term is taken,
→ graviton- W -Goldstone vertex appears
- in order to avoid that, we used a modified version of the standard t'Hooft gauge-fixing term

a convenient gauge-fixing term

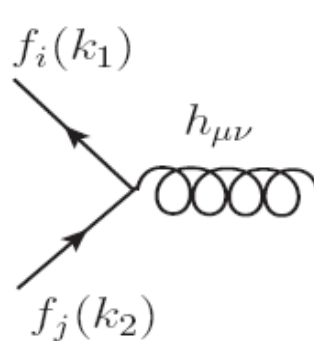
$$\mathcal{L}_{g.f.} = -\frac{1}{2\xi} \left[g_{\mu\nu} \partial^\mu W_a^\nu + g^{\nu\rho} \Gamma_{\nu\rho}^\mu W_{a\mu} + i \frac{g\xi}{2} (\phi'^\dagger \sigma^a \langle \phi \rangle - \langle \phi^\dagger \rangle \sigma^a \phi') \right]^2 \\ - \frac{1}{2\xi} \left[g_{\mu\nu} \partial^\mu B^\nu + g^{\nu\rho} \Gamma_{\nu\rho}^\mu B_\mu + i \frac{g'\xi}{2} (\phi'^\dagger \langle \phi \rangle - \langle \phi^\dagger \rangle \phi') \right]^2$$

$$\Gamma_{\nu\rho}^\mu = \frac{1}{2} g^{\mu\alpha} (\partial_\rho g_{\alpha\nu} + \partial_\nu g_{\alpha\rho} - \partial_\alpha g_{\nu\rho})$$

$$\phi' = \begin{pmatrix} \phi_+ \\ \frac{\phi_1 + i\phi_2}{\sqrt{2}} \end{pmatrix} \quad \langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

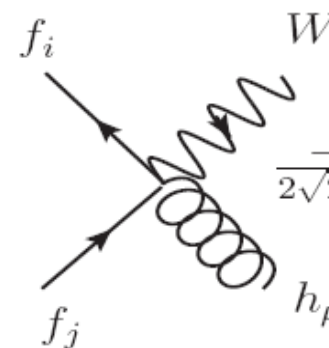
eliminates the graviton- W -Goldstone vertices

relevant Feynman rules for FC fermion-graviton vertex



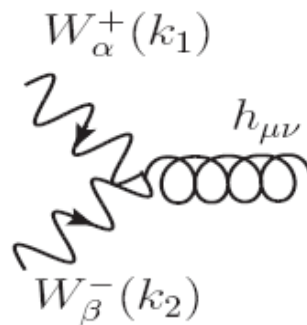
Feynman diagram showing two fermion lines, $f_i(k_1)$ and $f_j(k_2)$, meeting at a vertex connected by a graviton line $h_{\mu\nu}$ (represented by a coiled line).

$$\frac{-i}{4M_P} \delta_{ij} (X_{\mu\nu}^{(f)} + X_{\nu\mu}^{(f)})$$



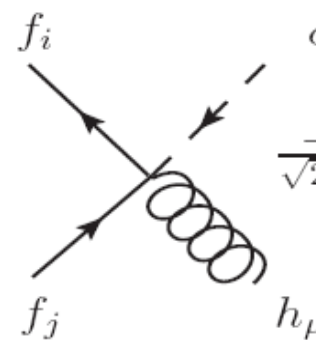
Feynman diagram showing two fermion lines, f_i and f_j , meeting at a vertex connected by a $W_\alpha^+(k_1)$ boson line (wavy) and a graviton line $h_{\mu\nu}$ (coiled).

$$\frac{-ig}{2\sqrt{2}M_P} X_{\mu\nu\alpha}^{(f,V)}$$



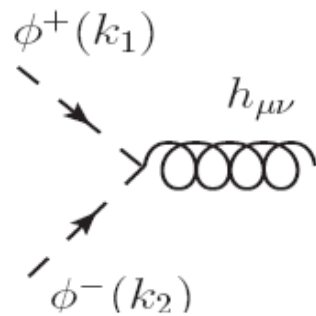
Feynman diagram showing two W boson lines, $W_\alpha^+(k_1)$ and $W_\beta^-(k_2)$, meeting at a vertex connected by a graviton line $h_{\mu\nu}$ (coiled).

$$\frac{-i}{M_P} (X_{\mu\nu\alpha\beta}^{(V)} + X_{\nu\mu\alpha\beta}^{(V)})$$



Feynman diagram showing two fermion lines, f_i and f_j , meeting at a vertex connected by a scalar line ϕ^+ (dashed) and a graviton line $h_{\mu\nu}$ (coiled).

$$\frac{-ig}{\sqrt{2}M_P} X_{\mu\nu}^{(f,S)}$$



Feynman diagram showing two scalar lines, $\phi^+(k_1)$ and $\phi^-(k_2)$, meeting at a vertex connected by a graviton line $h_{\mu\nu}$ (coiled).

$$\frac{-i}{M_P} X_{\mu\nu}^{(S)}$$

symbols entering in the previous Feynman rules

$$\begin{aligned}
 X_{\mu\nu}^{(f)} &= \gamma_\mu (k_{1\nu} + k_{2\nu}) - \eta_{\mu\nu} (\not{k}_1 + \not{k}_2 - 2m_i) \\
 X_{\mu\nu\alpha\beta}^{(V)} &= \frac{1}{2} \eta_{\mu\nu} k_{2\alpha} k_{1\beta} + \eta_{\alpha\beta} k_{1\mu} k_{2\nu} - \eta_{\mu\beta} k_{1\nu} k_{2\alpha} - \eta_{\mu\alpha} k_{2\nu} k_{1\beta} \\
 &\quad + (k_1 \cdot k_2 + m_W^2) \left(\eta_{\mu\alpha} \eta_{\nu\beta} - \frac{1}{2} \eta_{\mu\nu} \eta_{\alpha\beta} \right) \\
 &\quad + \frac{1}{\xi} \left[\frac{1}{2} \eta_{\mu\nu} (k_{1\alpha} k_{1\beta} + k_{2\alpha} k_{2\beta} + k_{1\alpha} k_{2\beta}) - \eta_{\nu\beta} k_{1\alpha} k_{1\mu} - \eta_{\nu\alpha} k_{2\beta} k_{2\mu} \right] \\
 X_{\mu\nu}^{(S)} &= (k_1 \cdot k_2 + \xi m_W^2) \eta_{\mu\nu} - k_{1\nu} k_{2\mu} - k_{1\mu} k_{2\nu} \\
 X_{\mu\nu\alpha}^{(f,V)} &= K_{ij} (2\gamma_\alpha \eta_{\mu\nu} - \gamma_\mu \eta_{\nu\alpha} - \gamma_\nu \eta_{\mu\alpha}) P_L \\
 X_{\mu\nu}^{(f,S)} &= K_{ij} \left(\frac{m_j}{m_W} P_R - \frac{m_i}{m_W} P_L \right) \eta_{\mu\nu} .
 \end{aligned}$$

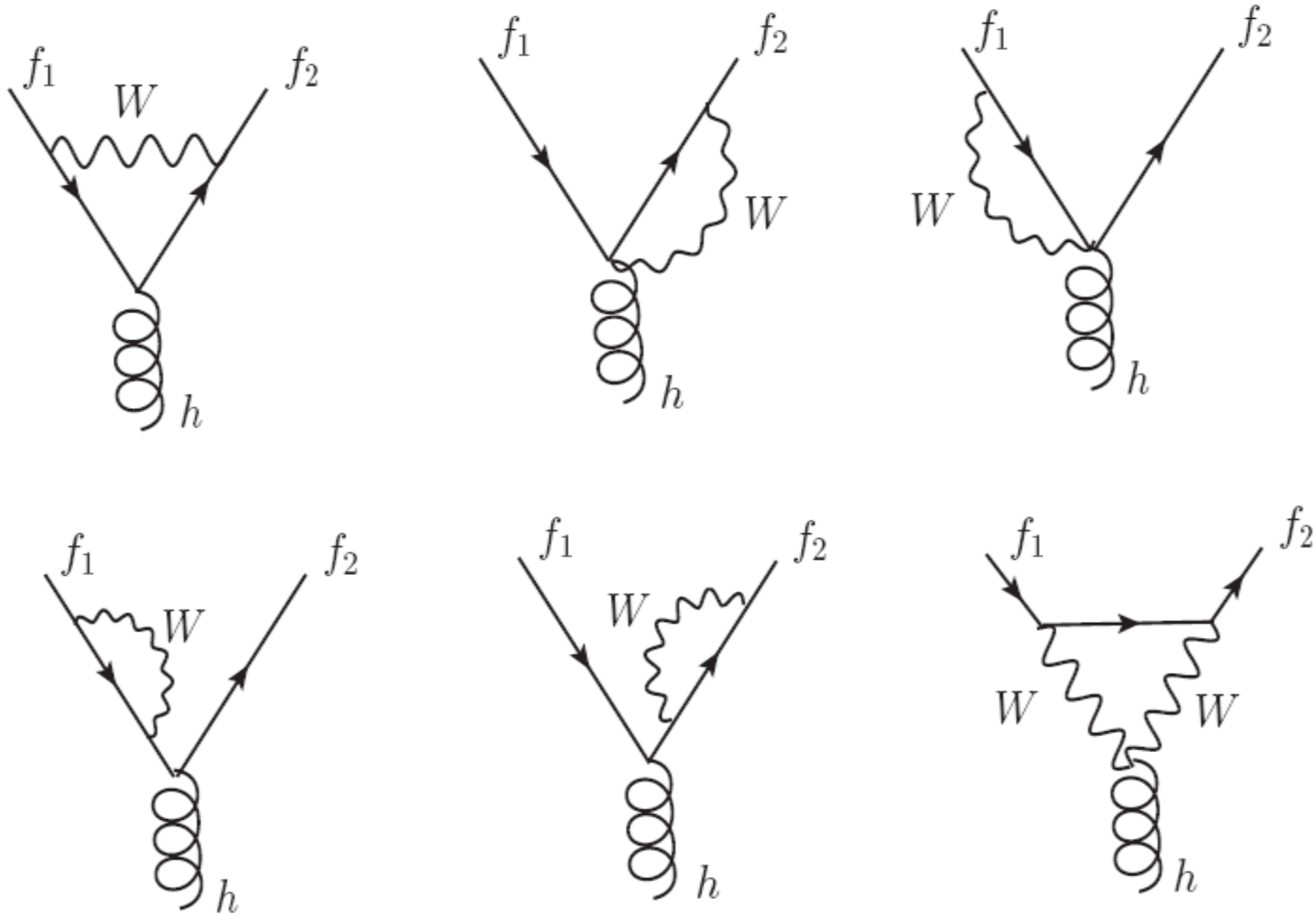
$$P_{L,R} = (1 \mp \gamma_5)/2$$

K_{ij}

CKM matrix

Feynman diagrams for FC quark-graviton vertex

unitary gauge (FC graviton-penguin diagrams)



off-diagonal (flavor) matrix element $\Delta F=1$

$p=p_1+p_2$ $q=p_1-p_2$ in SM 12 scalar form factors $f_i(p,q)$

$$\hat{T}^{\mu\nu} = \frac{iG_F}{16\pi^2\sqrt{2}} \sum_{i=1}^{12} f_i(p, q) \bar{u}_2(p_2) O_i^{\mu\nu} u_1(p_1)$$

$$O_1^{\mu\nu} = (\gamma^\mu p^\nu + \gamma^\nu p^\mu) P_L$$

$$O_2^{\mu\nu} = (\gamma^\mu q^\nu + \gamma^\nu q^\mu) P_L$$

$$O_3^{\mu\nu} = \eta^{\mu\nu} M_+$$

$$O_4^{\mu\nu} = p^\mu p^\nu M_+$$

$$O_5^{\mu\nu} = q^\mu q^\nu M_+$$

$$O_6^{\mu\nu} = (p^\mu q^\nu + q^\mu p^\nu) M_+$$

$$O_7^{\mu\nu} = \eta^{\mu\nu} M_-$$

$$O_8^{\mu\nu} = p^\mu p^\nu M_-$$

$$O_9^{\mu\nu} = q^\mu q^\nu M_-$$

$$O_{10}^{\mu\nu} = (p^\mu q^\nu + q^\mu p^\nu) M_-$$

$$O_{11}^{\mu\nu} = \frac{m_1 m_2}{m_W^2} (\gamma^\mu p^\nu + \gamma^\nu p^\mu) P_R$$

$$O_{12}^{\mu\nu} = \frac{m_1 m_2}{m_W^2} (\gamma^\mu q^\nu + \gamma^\nu q^\mu) P_R$$

$$M_\pm \equiv m_1 P_R \pm m_2 P_L$$

only six independent scalar form factors

due to Ward Identities

$$q_\mu \langle p_2 | T^{\mu\nu}(0) | p_1 \rangle = 0$$

$$(p \cdot q) f_1(p, q) + q^2 f_2(p, q) = 0$$

$$f_3(p, q) + q^2 f_5(p, q) + (p \cdot q) f_6(p, q) + \frac{(p \cdot q)}{2m_W^2} f_{12}(p, q) = 0$$

$$(p \cdot q) f_4(p, q) + q^2 f_6(p, q) + \frac{(p \cdot q)}{2m_W^2} f_{11}(p, q) = 0$$

$$f_2(p, q) + f_7(p, q) + q^2 f_9(p, q) + (p \cdot q) f_{10}(p, q) - \frac{p^2 + q^2}{4m_W^2} f_{12}(p, q) = 0$$

$$f_1(p, q) + (p \cdot q) f_8(p, q) + q^2 f_{10}(p, q) - \frac{p^2 + q^2}{4m_W^2} f_{11}(p, q) = 0$$

$$(p \cdot q) f_{11}(p, q) + q^2 f_{12}(p, q) = 0$$

- we performed an exact calculation of the form factors, by retaining all masses and momenta
 - all WIs analytically verified in $\xi = 1$ gauge
 - results cross checked in unitary gauge
 - $\xi = 1$ gauge: UV div. cancel without GIM
 - unitary gauge: GIM needed to cancel UV div.

- when only the leading terms in q^2/m_W^2 and $(p \cdot q)/m_W^2$ are retained, by keeping dependence on internal masses exact, **matrix element depends only by two functions of**

$$x_f = m_f^2/m_W^2 \quad (m_f = \text{internal fermion mass})$$

- we also provided exact analytical expressions in q^2 and internal masses, but at zero order in (external masses)²

off-diagonal matrix element at first order in q^2

CKM factor λ_f , $g_a(x), g_b(x)$ analogous of Inami-Lim functions

$$\hat{T}^{\mu\nu} = \frac{iG_F}{16\pi^2\sqrt{2}} \sum_f \lambda_f \bar{u}(p_2) \left\{ (\gamma^\mu p^\nu + \gamma^\nu p^\mu) P_L(-q^2) g_a(\mathbf{x}_f) + (\gamma^\mu q^\nu + \gamma^\nu q^\mu) P_L(p \cdot q) g_a(\mathbf{x}_f) \right. \\ \left. - q^\mu q^\nu M_+ g_b(\mathbf{x}_f) + \eta^{\mu\nu} \left[-2(p \cdot q) M_- g_a(\mathbf{x}_f) + q^2 M_+ g_b(\mathbf{x}_f) \right] \right. \\ \left. + (p^\mu q^\nu + p^\nu q^\mu) M_- g_a(\mathbf{x}_f) \right\} u(p_1),$$

$$\mathbf{x}_f = m_f^2 / m_W^2$$

CKM factor $\lambda_f \equiv K_{f1} K_{f2}^*$

m_f = internal fermion mass

GIM mechanism $\Rightarrow \sum_f \lambda_f = 0$

gauge invariance $q_\mu T^{\mu\nu} = 0$, easily verified

$$q_\mu \bar{u}(p_2) \gamma^\mu P_L u(p_1) = \bar{u}(p_2) M_- u(p_1)$$

➤ FC graviton-penguin diagrams:

- turn out to be **strongly GIM suppressed** at small x .
 $g_a(x)$ and $g_b(x)$ both of order **$O(x)$ for $x \rightarrow 0$**

$$g_a(x) = \frac{1}{36(x-1)^4} \left[44 - 194x + 243x^2 - 98x^3 + 5x^4 + 6x(2 - 15x + 10x^2) \log(x) \right]$$
$$g_b(x) = \frac{1}{6(x-1)^4} \left[8 - 14x + 21x^2 - 14x^3 - x^4 + 2x(4 + 3x + 2x^3) \log(x) \right]$$

➤ FC photon-penguin diagrams:

- FC matrix-element of EM current J_μ is involved
- turn out to be **mild GIM suppressed**
form factors of order **$O(\text{Log}(x))$ for $x \rightarrow 0$**

- difference due to different infrared behaviour for $m_f \rightarrow 0$
 $T_{\mu\nu} \sim D_\mu(J_\nu)$, so that $\dim[T_{\mu\nu}] > \dim[J_\mu]$

Special limit $(p \cdot q) = p^2 + q^2 = 0$ (terms $m_{1,2}^2$ neglected)

$$\hat{T}^{\mu\nu} = \frac{iG_F}{16\pi^2\sqrt{2}} \sum_{i=1}^5 F_i(p, q) \bar{u}_2(p_2) \hat{O}_i^{\mu\nu} u_1(p_1)$$

$$\hat{O}_1^{\mu\nu} = (\gamma^\mu p^\nu + \gamma^\nu p^\mu) P_L - \frac{p^\mu q^\nu + q^\mu p^\nu}{q^2} M_-$$

$$\hat{O}_2^{\mu\nu} = (-\eta^{\mu\nu} q^2 + q^\mu q^\nu) M_+,$$

$$\hat{O}_3^{\mu\nu} = (-\eta^{\mu\nu} q^2 + q^\mu q^\nu) M_-,$$

$$\hat{O}_4^{\mu\nu} = (p^\mu p^\nu) M_+,$$

$$\hat{O}_5^{\mu\nu} = (p^\mu p^\nu) M_-.$$

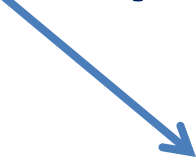
each term satisfies WI

$$q_\mu \hat{O}_i^{\mu\nu} = 0$$

$$F_1(p, q) = f_1(p, q), F_2(p, q) = f_5(p, q), F_3(p, q) = f_9(p, q), F_4(p, q) = f_4(p, q) \\ F_5(p, q) = f_8(p, q)$$

Flavor-changing gravitational couplings

- Quark scattering on external gravitational source
- tree-level coupling with TFCNC


$$f_1(p_1) + T^{\text{ext}} \rightarrow f_2(p_2) + T^{\text{ext}}$$

- one graviton-exchange amplitude

$$\mathcal{M} = \frac{1}{\bar{M}_P^2} \hat{T}_{\mu\nu} P_h^{\mu\nu\alpha\beta}(q^2) \hat{T}_{\alpha\beta}^{\text{ext}}$$

graviton propagator


$$q = p_1 - p_2 \quad \partial^\mu T_{\mu\nu}^{\text{ext}} = 0$$

massless graviton propagator (covariant gauge)

$$P_h^{\mu\nu\alpha\beta}(q^2) = \frac{i}{q^2 - i\varepsilon} \frac{1}{2} \left(\eta^{\mu\alpha} \eta^{\nu\beta} + \eta^{\nu\alpha} \eta^{\mu\beta} - \eta^{\mu\nu} \eta^{\alpha\beta} + Q^{\mu\nu\alpha\beta}(q) \right)$$

gauge dependent term, vanishes
when contracted with conserved $T_{\mu\nu}$

in general one gets

$$\mathcal{M} = \frac{i}{(q^2 - i\varepsilon) 2\bar{M}_P^2} \left[2\hat{T}^{\mu\nu} \hat{T}_{\mu\nu}^{\text{ext}} - C \hat{T}^\mu_\mu \hat{T}^{\text{ext}}_\nu{}^\nu \right]$$

C=1 → massless graviton (General Relativity)
C=2/3 → massive graviton (see following slides)

$$\mathcal{M} = \frac{G_F}{16\pi^2\sqrt{2}\bar{M}_P^2} \sum_f \lambda_f \bar{u}(p_2) \left\{ \left(\gamma^\mu p^\nu + \gamma^\nu p^\mu \right) P_L \mathbf{g}_a(\mathbf{x}_f) - \eta^{\mu\nu} M_+ \left(C g_a(x_f) - \left(\frac{3}{2} C - 1 \right) \mathbf{g}_b(\mathbf{x}_f) \right) \right. \\ \left. + \eta^{\mu\nu} M_- \frac{(p \cdot q)}{(q^2 - i\varepsilon)} \mathbf{g}_a(\mathbf{x}_f) [2 + C(-2)] \right\} u(p_1) \hat{T}_{\mu\nu}^{\text{ext}}$$

$1/q^2$ pole cancels if $C=1 \rightarrow$ massless graviton

➤ If graviton is massless, only FC short-distance interactions generated. Scale of effective theory:

$$\Lambda = \sqrt{M_P m_W} \sim 10^{10} \text{ GeV}$$

➤ consequence of angular-momentum conservation
i.e. on-shell decay $f_1 \rightarrow f_2$ G forbidden in vacuum

massive graviton scenario

- infrared modification of gravity (large distances)
- if graviton has small mass $m_G \rightarrow$ 5 polarizations states

massive graviton propagator

$$P_G^{\mu\nu\alpha\beta}(q^2) = \frac{i}{q^2 - m_G^2 - i\varepsilon} \frac{1}{2} \left(\tilde{\eta}^{\mu\alpha} \tilde{\eta}^{\nu\beta} + \tilde{\eta}^{\nu\alpha} \tilde{\eta}^{\mu\beta} - \frac{2}{3} \tilde{\eta}^{\mu\nu} \tilde{\eta}^{\alpha\beta} \right)$$
$$\tilde{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} - q^\mu q^\nu / m_G^2$$

- scalar (spin-0) polarization does not decouple in limit $m_G \rightarrow 0$

Van-dam Veltman Zakharov (vDVZ) discontinuity

- different deflection angle from light bending from the sun
predicted $\rightarrow$ 25% difference from the measured one (ruled out)
- criticisms on the validity of one-graviton exchange amplitude
approximation: Defayet, Dvali, Gabadadze, Vainshtein, PRD 65 044026 (2002)

a new aspect of vDVZ discontinuity

➤ $f_1 \rightarrow f_2$ G allowed by spin-0 polarization

➤ $1/q^2$ pole is not canceled: $C=2/3$

long-distance FC contributions induced at 1-loop !

One-(massive)graviton-exchange amplitude

$$\mathcal{M} = \frac{G_F}{16\pi^2 \sqrt{2} \bar{M}_P^2} \sum_f \lambda_f \bar{u}(p_2) \left\{ \frac{q^2}{(q^2 - m_G^2 - i\varepsilon)} \left((\gamma^\mu p^\nu + \gamma^\nu p^\mu) P_L - \frac{2}{3} \eta^{\mu\nu} M_+ \right) \mathbf{g}_a(\mathbf{x}_f) \right. \\ \left. + \frac{2}{3} \eta^{\mu\nu} M_- \frac{(p \cdot q)}{(q^2 - m_G^2 - i\varepsilon)} \mathbf{g}_a(\mathbf{x}_f) \right\} u(p_1) \hat{T}_{\mu\nu}^{\text{ext}}$$

long-distance FC contribution

vanishes for equal in/out masses, $(p \cdot q) = m_1^2 - m_2^2$

modification of Newton potential $V(r)$

from non-relativistic limit of scattering amplitude:

$$\langle j | V(r) | i \rangle = -\frac{G_N M}{r} e^{-r m_G} (m_i \delta_{ij} + (m_i - m_j) \Delta_{ij})$$

$$\Delta_{ij} = \frac{G_F (m_i^2 - m_j^2)}{32\pi^2 \sqrt{2}} \sum_f K_{fi} K_{fj}^* \mathbf{g}_a(\mathbf{x}_f),$$

 off-diagonal contribution

- Δ vanishes for equal in/out fermion masses
- but NOT in the $m_G \rightarrow 0$ limit, vDVZ discontinuity
- attractive/repulsive nature depends on $\text{sign} \sum_f K_{fi} K_{fj}^* \mathbf{g}_a(\mathbf{x}_f)$

FC processes with massive spin-2 particles

I. Scenario containing a massive spin-2 particle

$$\mathcal{L}_{\text{eff}} = -\frac{1}{\Lambda} T_{\mu\nu}(x) G^{\mu\nu}(x)$$


- assume G coupled to $T_{\mu\nu}$ of SM fields
- Λ is free scale, not necessarily M_P
- consider the fermion decay induced by TFCNC

$$f_1(p_1) \rightarrow f_2(p_2) G(q)$$

spin-2 counterpart of radiative decay

$$f_1 \rightarrow f_2 \gamma$$

- **approximation:** external masses lighter than internal loop masses, final fermion mass neglected

Decay amplitude

(spin-2 polarization tensor, $i=1-5$)

$$M(f_1 \rightarrow f_2 G) = -\frac{im_G^2 G_F}{16\pi^2 \sqrt{2} \Lambda} \sum_f \lambda_f \mathbf{g}_a(\mathbf{x}_f) \bar{u}_2(p_2) [\gamma^\mu p^\nu + \gamma^\nu p^\mu] P_L u_1(p_1) \epsilon_{\mu\nu}^i(q)$$


- square amplitude and sum over polarizations:

Decay width

$$\Gamma(f_1 \rightarrow f_2 G) = \frac{G_F^2 m_1^7 f(x_G)}{192 (2\pi)^5 \Lambda^2} \left| \sum_f \lambda_f \mathbf{g}_a(\mathbf{x}_f) \right|^2$$

$$f(x) = (1-x)(1 - \frac{3}{2}(x+x^2+x^4) + \frac{7}{2}x^3)$$

applications to QG in large extra-dimensions

$$x_G = m_G^2 / m_1^2$$

II. Quantum gravity in large extra-dimensions

If (large) compact extra-dimensions $D=4+\delta$ exist with only gravity propagating inside: **ADD scenario**

- new solution to the hierarchy problem
- **fundamental scale of quantum gravity $M_D \ll M_P$**

$$G_N^{-1} = 8\pi R^\delta M_D^{2+\delta}$$

- If $M_D \sim \text{TeV}$, tests on gravity law implies $\delta > 1$
- effective theory in $D=4$ contains:
- **(almost) continuous spectrum of massive spin2 particles: Kaluza-Klein excitations $\rightarrow$ stable particles**

Inclusive KK decay width $\sum_G \Gamma(f_1 \rightarrow f_2 G)$ **is large**

N^0 density of KK states $dN = S_{\delta-1} \frac{\bar{M}_P^2}{M_D^{2+\delta}} m_G^{\delta-1} dm_G$

Integrate Γ ($\Lambda \rightarrow M_P$) in dN

$$\sum_G \Gamma(f_1 \rightarrow f_2 G) = \frac{G_F^2 m_1^{7+\delta} S_{\delta-1} I(\delta)}{192 (2\pi)^5 M_D^{2+\delta}} \left| \sum_f \lambda_f \mathbf{g}_a(\mathbf{x}_f) \right|^2$$

Geometrical factor

$$I(\delta) = 960 / (\delta(2+\delta)(6+\delta)(8+\delta)(10+\delta))$$

$1/M_P^2$ suppression canceled by integrating over KK modes

$$\sum_G BR(B_d \rightarrow X_s G) \simeq 10^{-13} \left(\frac{\text{TeV}}{M_D} \right)^4 \left(\frac{m_b}{4.3 \text{ GeV}} \right)^4$$

$m(t)=171.2 \text{ GeV}$

FC decay of a heavy spin-2 particle

- **approximation:** massless fermion masses
- **full q^2 dependence included** $\mathbf{g}_{a,b}(\mathbf{x}) \rightarrow \mathbf{G}_{a,b}(\mathbf{x}, q^2)$

$$\Gamma(G \rightarrow f_2 \bar{f}_1) = \Gamma_f \frac{G_F^2 m_G^4}{64\pi^4} \left| \sum_f \lambda_f \mathbf{G}_a(\mathbf{x}_f, \mathbf{x}_G) \right|^2$$

$$x_G = m_G^2/m_W^2 \quad \nearrow \quad \Gamma_f = \frac{m_G^3 N_c}{160\pi \Lambda^2}$$

tree-level decay width of G in 2 massless fermions

asymptotic mass limit $m_G \gg m_W$

$$\Gamma(G \rightarrow f_2 \bar{f}_1) \Big|_{m_G \gg m_W} = \Gamma_f \frac{G_F^2 m_W^4}{64\pi^4} \left| \sum_f \lambda_f \left(\mathbf{F}(\mathbf{x}_f) - \frac{x_f}{2} \left(\log\left(\frac{m_G^2}{m_W^2}\right) - i\pi \right) \right) \right|^2$$

$$F(x) = \left(\frac{3x(3-x)}{4(1-x)} + \frac{x(8-6x+x^2)\log(x)}{2(x-1)^2} \right) - 4 \left(\text{Li}_2(1-x) - \frac{\pi^2}{6} \right)$$

consistency checks

- $G_F^2 m_G^4 \rightarrow G_F^2 m_W^4$ unitarity recovered
- absence of pure $\text{Log}(x)$ terms in $F(x)$
- GIM mechanism strongly suppressed in all range of q^2

high energy FC gravitational scattering

$$f_i(p_1)X(q_1) \rightarrow f_j(p_2)X(q_2)$$

(one-graviton exchange diagram)

cross-section factorization

$$\left. \frac{d\sigma^{f_i \rightarrow f_j}}{dt} \right|_{|t| \gg m_W^2} \simeq \frac{d\sigma^{f_i \rightarrow f_i}}{dt} \times \frac{G_F^2 m_W^4}{64\pi^4} \left| \sum_f \lambda_f \left[\mathbf{F}(\mathbf{x}_f) - \frac{x_f}{2} \log\left(\frac{-t}{m_W^2}\right) \right] \right|^2$$

flavor-conserving cross-section

holds regardless if graviton has mass or not

Conclusions

- We computed the one-loop FC quark-graviton vertex
- results easily generalizable to leptonic sector with Dirac neutrinos
- form factors turn out to be **strong GIM suppressed** (in all range of q^2) for small internal fermion masses
- gravity + weak-interaction induce **local $\Delta F=1$ interactions** if graviton is massless
- if graviton has a small mass, **$\Delta F=1$ contribution to Newton potential appears**
- we also analyzed the FC decay $f_1 \rightarrow f_2 G$
- in the framework of large-extra dimension scenarios (ADD) **$BR(B \rightarrow X_s G) \sim 10^{-13}$** for $M_D \sim 1$ TeV and $\delta = 2$

➤ results easily generalizable to a **graviscalar** ϕ_S

$$\mathcal{L}_{\text{int}} = -\frac{1}{\Lambda_S} T_{\mu\nu}(x) \phi_S(x) \eta^{\mu\nu}$$

➤ **graviscalar contribution** to the Newton potential

$$\langle j | V(r) | i \rangle = -\frac{1}{\Lambda_S^2} \frac{M}{4\pi r} e^{-r m_{\phi_S}} \left(m_i \delta_{ij} + (m_i - m_j) \Delta_{ij}^S \right)$$

$$\Delta_{ij}^S = 6\Delta_{ij}$$

Although FC gravitational interactions are too small to be detected in laboratory experiments, they can have potential astrophysical implications in the framework of **massive gravity** or **ultralight graviscalar** scenarios.