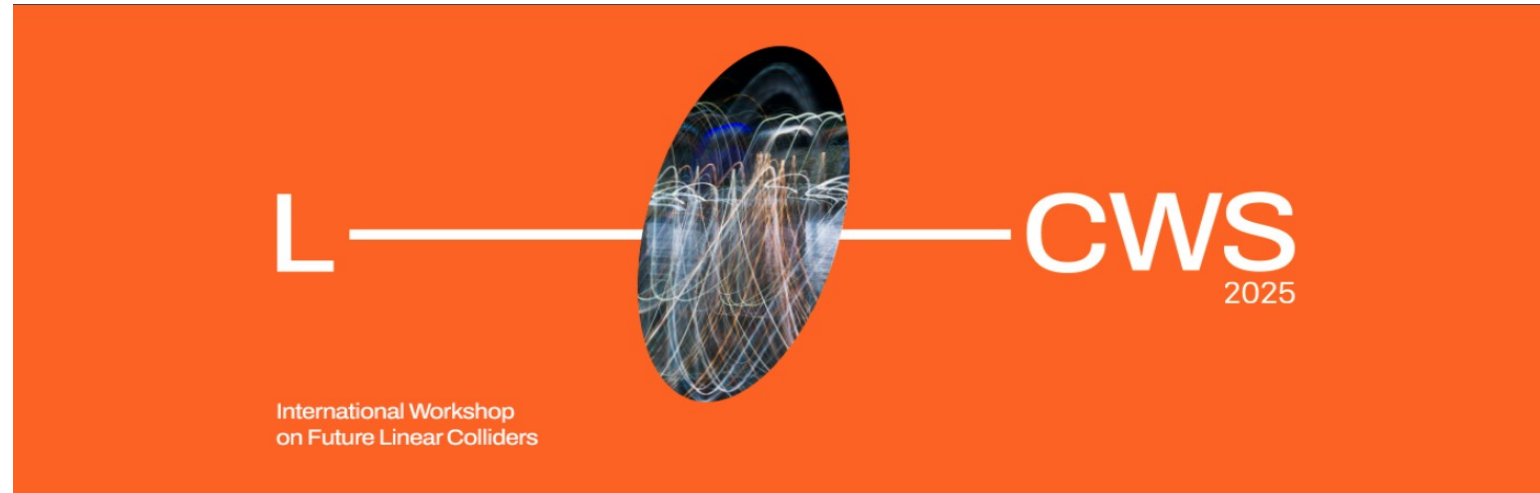
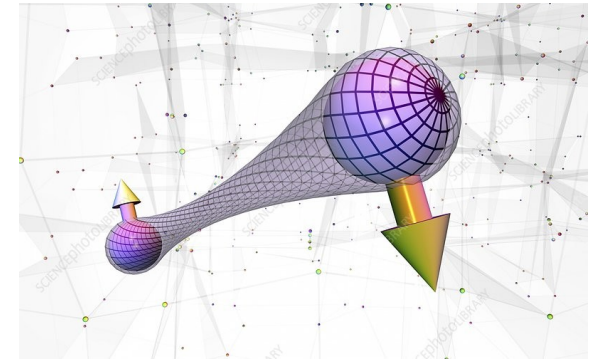


Quantum Entanglement and Bell Inequality Violation @ Future Linear Colliders

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Works in collaboration with: M. Fabbrichesi, R. Floreanini, L. Marzola

- “Entanglement” between two systems is a pure quantum phenomena
- Expected to violates **Bell inequalities** → set of (classical) probability correlations
- Violations incompatible with classical physics based on causality and local realism (locality) (**EPR paradox, hidden variables theories**)

● Violation of Bell inequality observed at **low energy**

pair of photons Freedman-Clauser PRL 28 (1972), Aspect-Dalibard-Roger PRL 49 (1982),
Zelinger et al. , Nature 433, 230 (2005)

● first observation of **Bell violation** at high energy in B meson decays and in the presence of **WEAK and STRONG** interactions ! (large significance $\gg 5\sigma$)

Fabbrichesi, Floreanini, EG, Marzola, PRD 109 (2024) 3, L031104 ; 2305.04982

● observation of Bell violation also in **Charmonium decays in Barions, Mesons**

Fabbrichesi, Floreanini, EG, Marzola, PRD 110 (2024) 5 053008; 2408.05010

● observation of **Entanglement** in top quark pairs (ATLAS [arXiv:2311.07288] and CMS [CMS-PAS-TOP-23001])

● Future Linear Colliders suitable to test QE & Bell violation in **VV** and **tau-pairs**

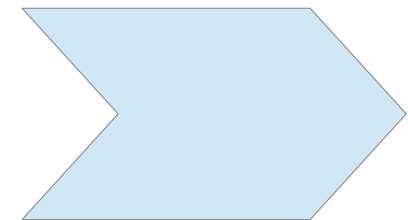
Quantifying entanglement and Bell inequality violation

- Requires the knowledge of the **Polarization Density Matrix** (PDM) of two-particles **A,B** production
- PDM can be fully reconstructed from the angular distributions of the single **A,B decay products** from data or a Monte Carlo simulation in some specific basis → **quantum tomography**
- or analogously by measuring the complete set of **helicity amplitudes** from A,B decay products
- **but PDM can also be computed analytically** (using the SM theory) from the A,B polarizations
- knowledge of **PDM** allows to quantify (where possible) entanglement and Bell inequality violations
- use tools of **quantum information theory**



A. J. Barr, M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, Quantum entanglement and Bell inequality violation at colliders, *Prog. Part. Nucl. Phys.* **139**, 104134 (2024).

the toolbox



Qubits

spin-1/2 particles, photon

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \frac{1}{4} \left[\mathbb{1} \otimes \mathbb{1} + \sum_i B_i^+ (\sigma_i \otimes \mathbb{1}) + \sum_j B_j^- (\mathbb{1} \otimes \sigma_j) + \sum_{ij} C_{ij} (\sigma_i \otimes \sigma_j) \right]$$

σ_i = 2 x 2 Pauli matrices

4 x 4 matrix

Polarization coefficients

spin-correlations coefficients

• Entanglement

(for pure states)

Concurrence $\mathcal{C}[|\psi\rangle] \equiv \sqrt{2 (1 - \text{Tr}[(\rho_A)^2])} = \sqrt{2 (1 - \text{Tr}[(\rho_B)^2])}$

vanishes for separable states,
max value = 1

$$R = \rho (\sigma_2 \otimes \sigma_2) \rho^* (\sigma_2 \otimes \sigma_2)$$

$$\mathcal{C}[\rho] = \max(0, r_1 - r_2 - r_3 - r_4)$$

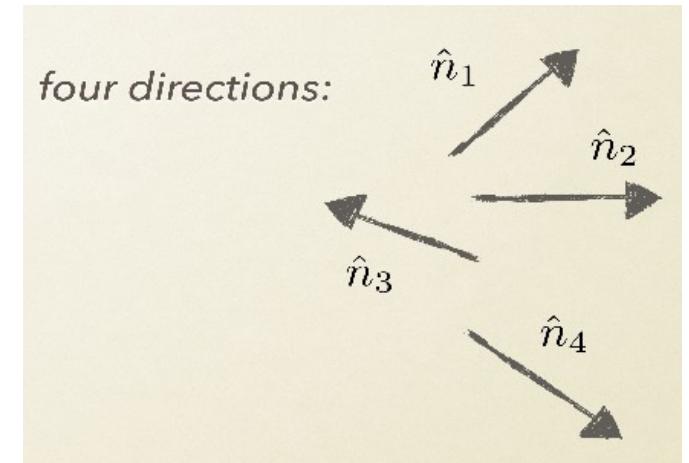
If > 0 it signals the presence of entanglement

Eigenvalues of R, with 1 the largest one

Qubits

• Bell inequality violation

$\vec{n}_1, \vec{n}_3 \longrightarrow$ for Alice $\Rightarrow (\hat{A}_1, \hat{A}_2)$
 $\vec{n}_2, \vec{n}_4 \longrightarrow$ for Bob $\Rightarrow (\hat{B}_1, \hat{B}_2)$ 2 outcomes (qubits)



CHSH inequality

$$\mathcal{I}_2 = \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle \leq 2$$

Clauser-Horne-Schimony-Holt
Phys. Rev. Lett. 24 (1970) 549

$$\left| \hat{n}_1 \cdot C \cdot (\hat{n}_2 - \hat{n}_4) + \hat{n}_3 \cdot C \cdot (\hat{n}_2 + \hat{n}_4) \right| \leq 2$$

$C_{ij} \longrightarrow$ Correlation matrix

$M = C^T C \Rightarrow [m_1, m_2, m_3]$ eigenvalues $\Rightarrow m_1$ and m_2 the largest ones

Horodecki condition

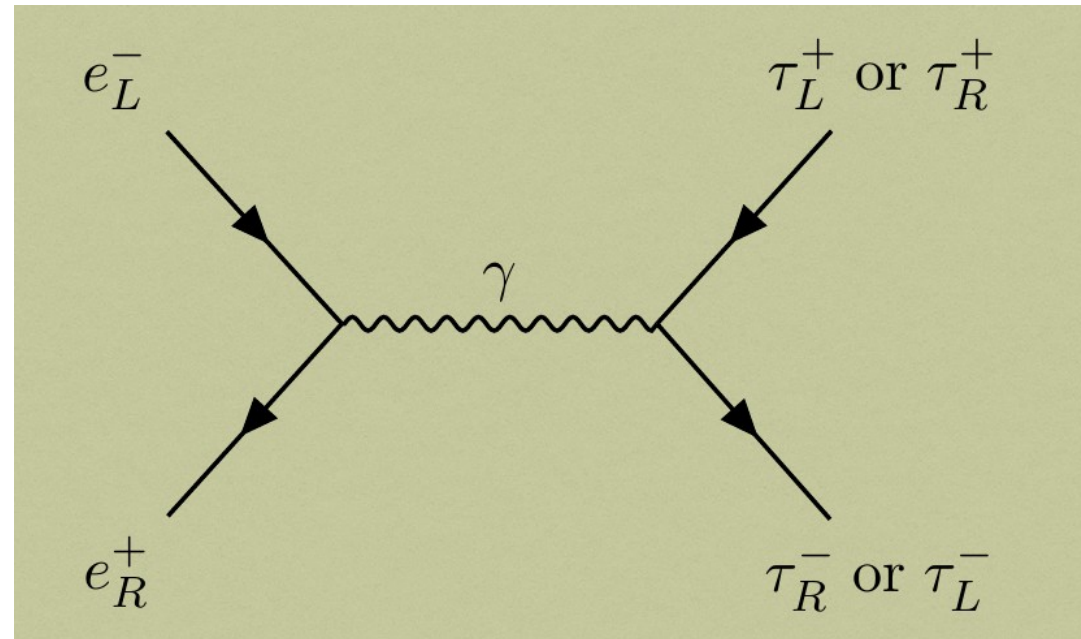
$$\mathbf{m}_{12} \equiv m_1 + m_2 > 1$$

Violation of Bell inequality

R. Horodecki et al, Phys. Lett. A200 5 (1995) 340

Tau pair production @ lepton colliders

in CM frame
massless limit



QM $\rightarrow$ **non separable entangled** states

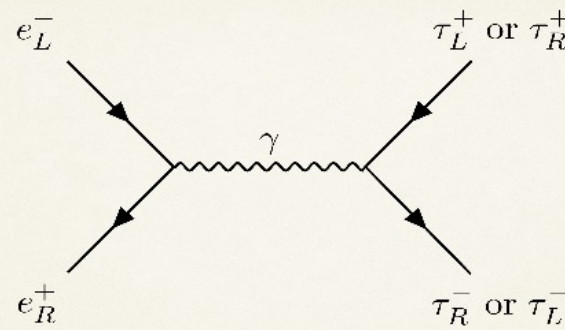
$$|\Psi\rangle = \zeta_1 |\tau_L^-\rangle |\tau_L^+\rangle + \zeta_2 |\tau_R^-\rangle |\tau_L^+\rangle + \zeta_3 |\tau_L^-\rangle |\tau_R^+\rangle + \zeta_4 |\tau_R^-\rangle |\tau_R^+\rangle$$

Deterministic theories $\rightarrow$ **separable states** (example)

$$\left(\sum_i |\xi_i|^2 = 1 \right)$$

$$|\Psi\rangle_{\text{cl}} = |\tau_R^-\rangle |\tau_L^+\rangle \quad \text{OR} \quad |\tau_L^-\rangle |\tau_R^+\rangle$$

Scattering angle in CM frame

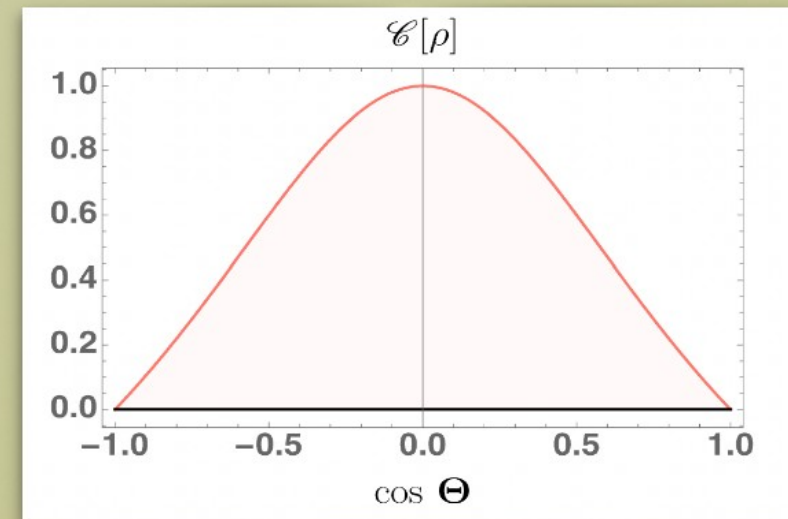


massless limit

$$(1 + \cos \Theta) |\tau_R^-\rangle |\tau_L^+\rangle + (1 - \cos \Theta) |\tau_L^-\rangle |\tau_R^+\rangle$$

Concurrence

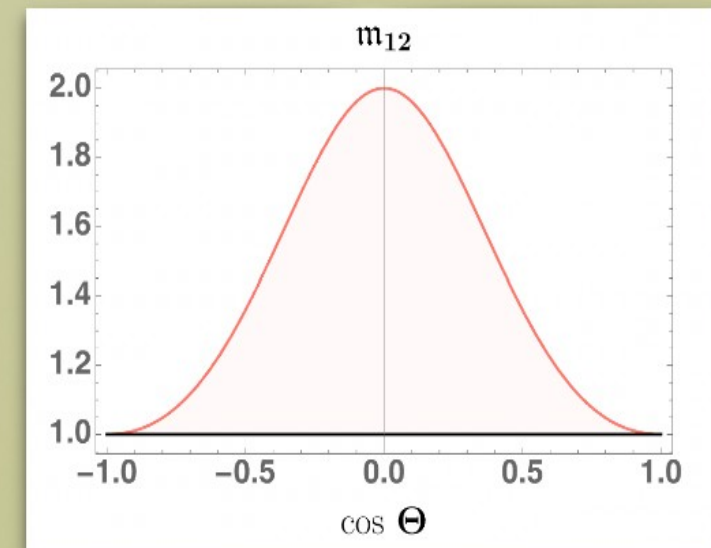
$$\mathcal{C}[\rho] = 2|\zeta_1\zeta_4 - \zeta_2\zeta_3| = \frac{\sin^2 \Theta}{1 + \cos^2 \Theta}$$

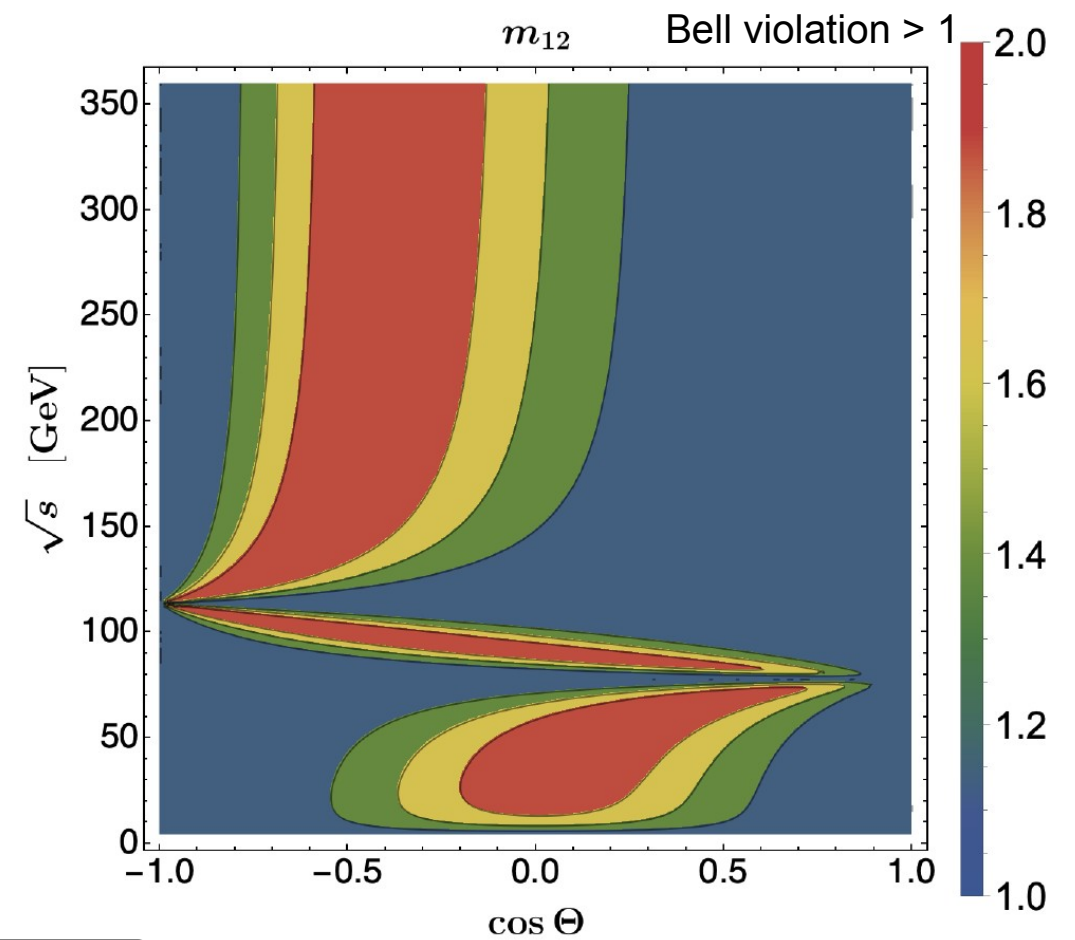
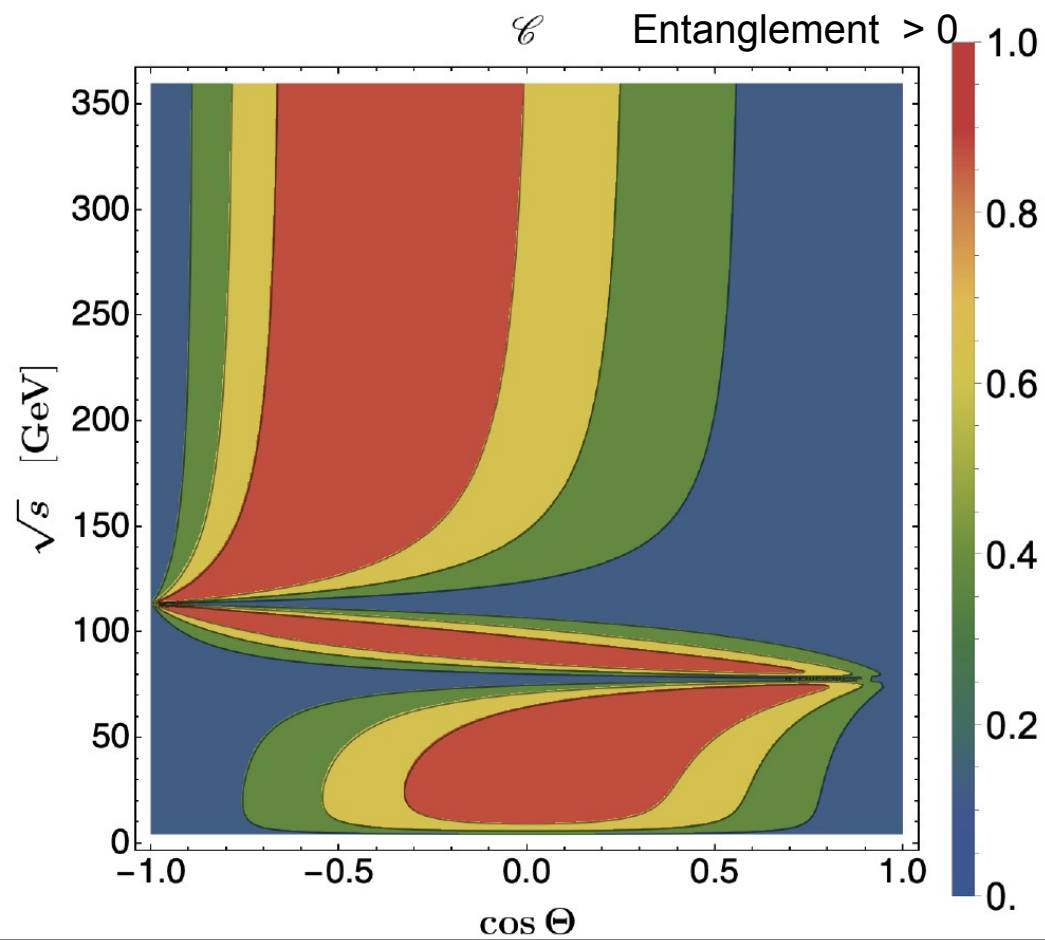


$$(1 + \cos \Theta) |\tau_R^-\rangle |\tau_L^+\rangle + (1 - \cos \Theta) |\tau_L^-\rangle |\tau_R^+\rangle$$

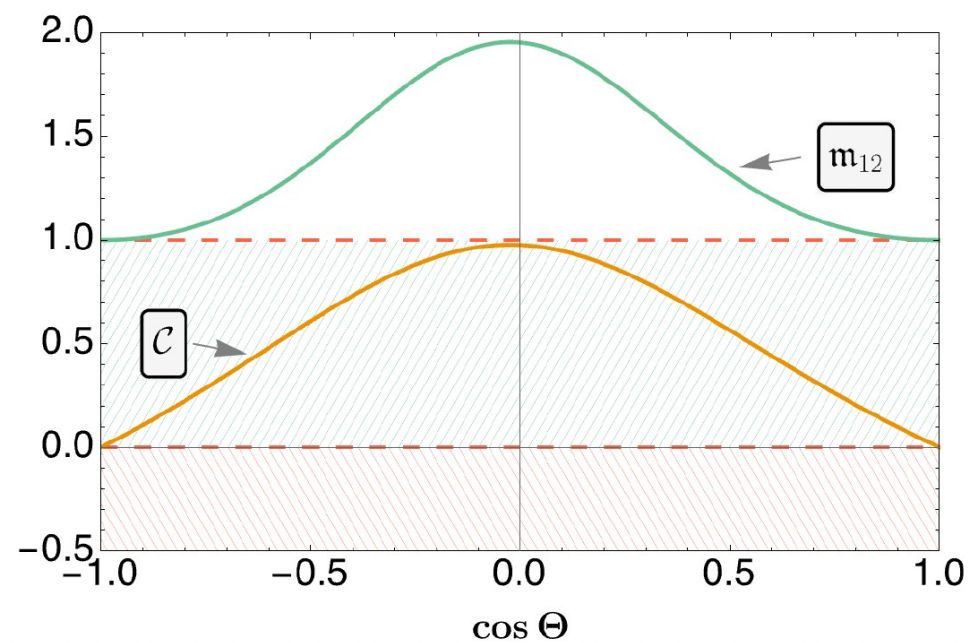
Horodecki condition $\mathfrak{m}_{12} \equiv m_1 + m_2 > 1$

$$\mathfrak{m}_{12} = 1 + \frac{\sin^4 \Theta}{(1 + \cos^2 \Theta)^2}$$





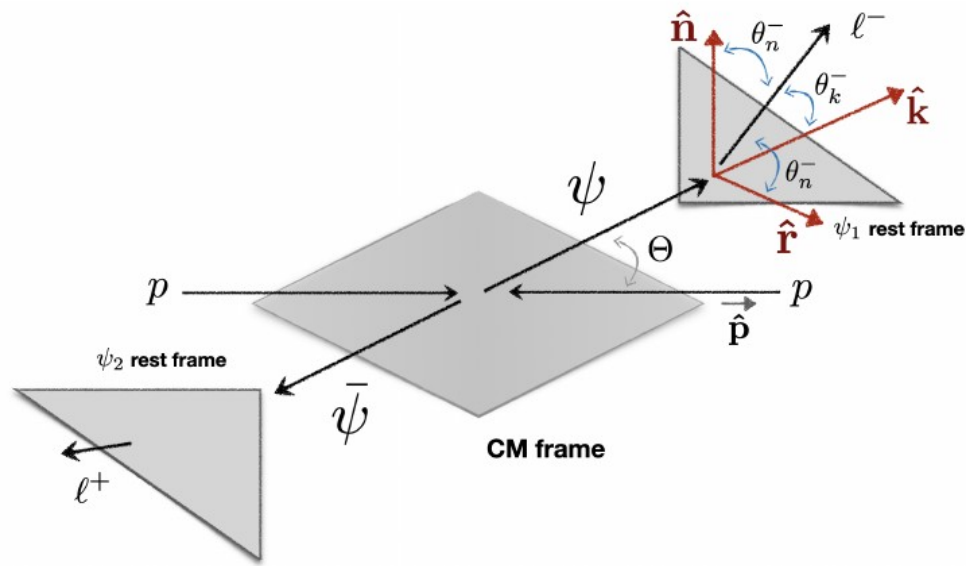
M. Fabbrichesi, L. Marzola PRD 110 (2024) 7, 076004; 2405.09201



$$\sqrt{s} = 91.19 \text{ GeV}$$

Tomography of tau-pair production

$$e^+e^- \rightarrow Z, \gamma \rightarrow \tau^+\tau^- \rightarrow \pi^+\pi^-\nu_\tau\bar{\nu}_\tau$$



Right-handed system

$$\hat{\mathbf{n}} = \frac{1}{\sin \Theta} (\hat{\mathbf{p}} \times \hat{\mathbf{k}})$$

$$\hat{\mathbf{r}} = \frac{1}{\sin \Theta} (\hat{\mathbf{p}} - \cos \Theta \hat{\mathbf{k}})$$

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_i^\pm} = \frac{1}{2} (1 \mp B_i^\pm \cos \theta_i^\pm),$$

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_i^+ d \cos \theta_j^-} = \frac{1}{4} \left(1 + C_{ij} \cos \theta_i^+ \cos \theta_j^- \right)$$

in which $\cos \theta_i^\pm$ are the projections of the $\pi^\pm$ momentum direction on the $\{\mathbf{n}, \mathbf{r}, \mathbf{k}\}$ basis, as computed in the rest frame of the decaying $\tau^\pm$.

Results of Monte Carlo full simulation at FCC (with detector and ISR effects included)

M. Fabbrichesi, L. Marzola PRD 110 (2024) 7, 076004; 2405.09201

Luminosity = 17.6 fb^{-1} corresponding to 10^7 events at Z resonance

entanglement requires > 0

$$\mathcal{C} = 0.4805 \pm 0.0063|_{\text{stat}} \pm 0.0012|_{\text{syst}} ,$$

Bell violation requires > 1

$$\mathfrak{m}_{12} = 1.239 \pm 0.017|_{\text{stat}} \pm 0.008|_{\text{syst}} ,$$

Bell inequality is violated @ 13σ level of significance

● with $L = 150 \text{ ab}^{-1}$ is violated @ 30σ level of significance

Di-boson production @ Lepton Colliders

$$\ell^+ \ell^- \rightarrow W^+ W^-$$

$$\ell^+ \ell^- \rightarrow ZZ$$

Massive spin-1 particles are qutrits

PDM ρ can be decomposed on the basis of tensor products of **Gell-Mann matrices**

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \left(\frac{1}{9} [\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [\mathbb{1} \otimes T^a] + \sum_a g_a [T^a \otimes \mathbb{1}] + \sum_{ab} h_{ab} [T^a \otimes T^b] \right)_{\lambda_1 \lambda'_1, \lambda_2 \lambda'_2}$$

9x9 matrix

- extract f, g, h coefficients from data using the angular distributions of their decay products
- can be also computed analytically from the SM
- f, g, h coefficients depends on the scattering angle in the c.m. frame and invariant mass of VV system

Bell inequality for Two-QUTRITS

- take a bi-partite system with components A, B
- perform two measurements $\longrightarrow (A_1, A_2) \quad (B_1, B_2)$ each can take values $\rightarrow \{0, 1, 2\}$
- consider the following correlator $\mathcal{I}_3$ for probability measurements

$$\begin{aligned} \mathcal{I}_3 = & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ & - P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1) \end{aligned}$$

can be expressed as

$$\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}]$$

with $\mathcal{B}$ a suitable Bell operator

Collins, Gisin, Linden,
Massar, Popescu, PRL 88 (2002)

Bell inequalities for two-qutrits

For deterministic local models

$$\mathcal{I}_3 \leq 2$$

QM for qutrits can violate this inequality with upper bound = 4

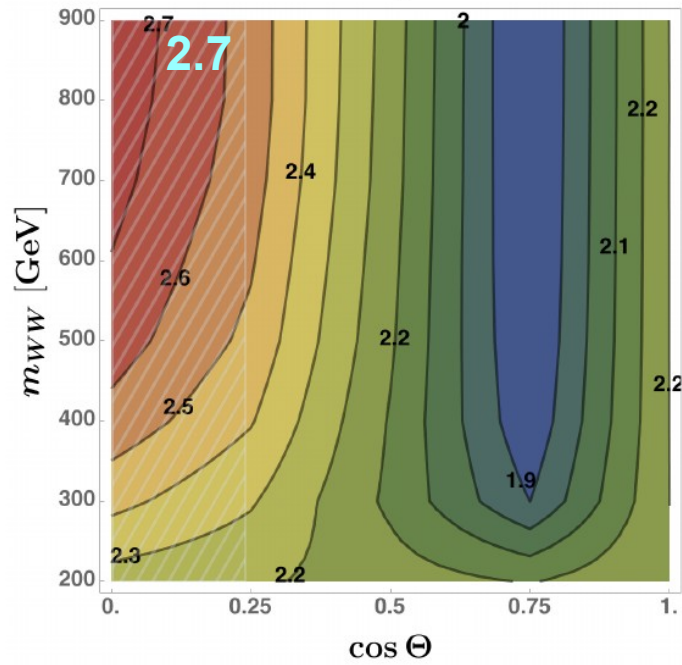
in order to maximize Bell's inequality

$$\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

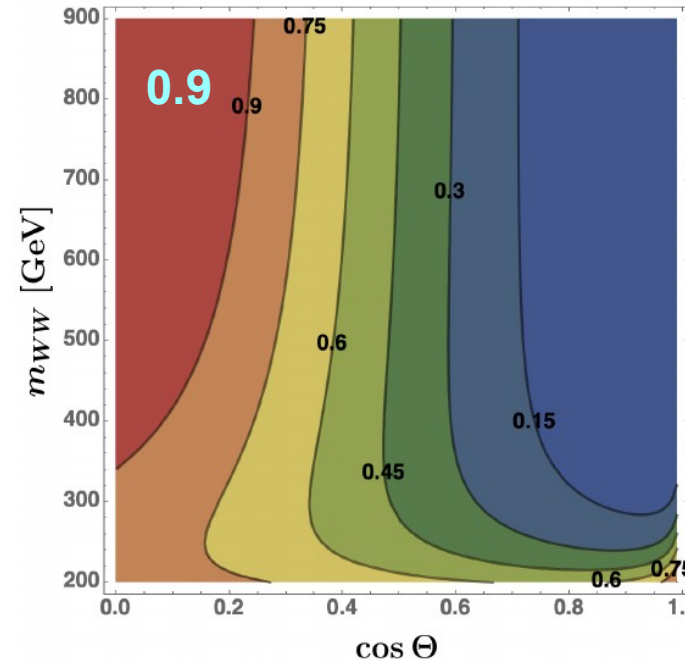
$\swarrow$
U, V unitary 3x3 matrices

Lepton colliders

$\mathcal{I}_3$ Bell violation > 2



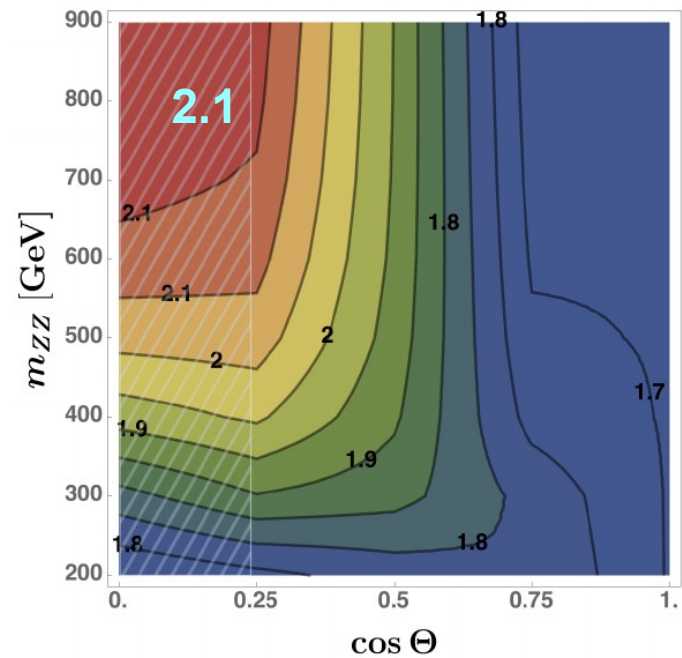
$\mathcal{C}_2$ Entanglement > 0



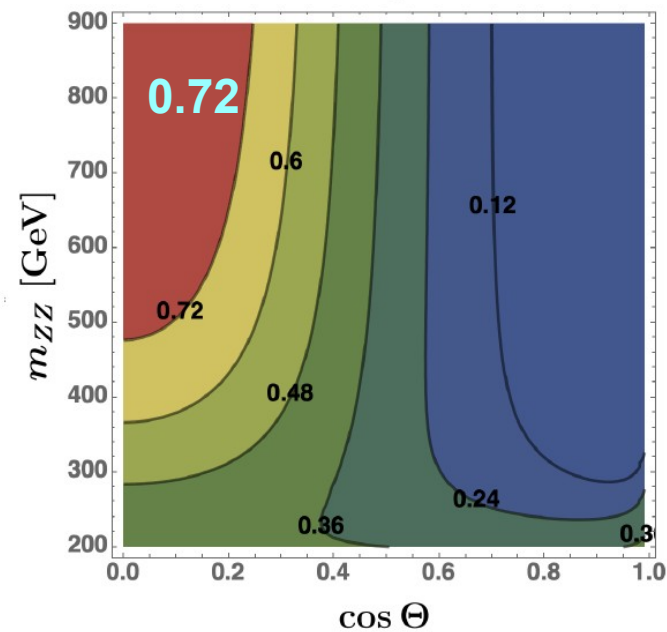
$$e^+ e^- \rightarrow W^+ W^-$$

With respect to LHC absence of PDF increases entanglement and Bell violations in WW

$\mathcal{I}_3$



$\mathcal{C}_2$



$$e^+ e^- \rightarrow Z Z$$

ZZ channel not much different than LHC $\rightarrow$ due to the fact that PDF almost factorize (t,u channels)

Lepton colliders : events and sensitivity

$$e^+ e^- \rightarrow Z Z$$

promising channel

$$\sqrt{s} = 1 \text{ TeV}$$

$$\sqrt{s} = 368 \text{ GeV}$$

$$(\text{muon}) \mathcal{L} = 1 \text{ ab}^{-1}$$

$$(\text{FCC}) \mathcal{L} = 1.5 \text{ ab}^{-1}$$

events

44

748

$$m_{WW} > 200 \text{ GeV and } \cos \Theta < 0.25$$

Significance to reject null hypothesis:

2 @ muon collider and > 4 @ FCC

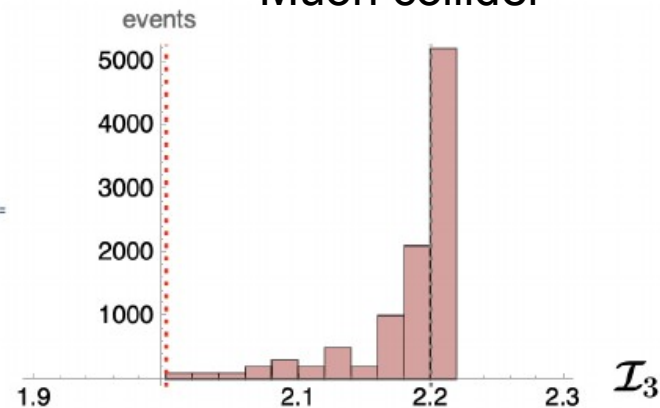
- Events computed with MADGRAPH@LO
- Efficiency 70% in each lepton identification
→ n. of events reduced accordingly
- only statistic error assumed
- no background considered

FCC significance is larger than muon collider due to larger n. of events at FCC (statistical error reduced)

10⁴ pseudo experiments

$$\mathcal{I}_3 = 2.2$$

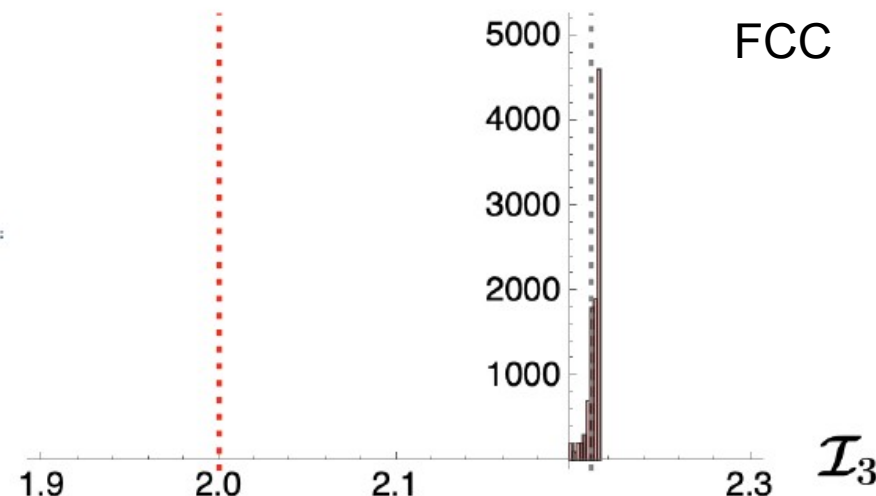
Muon collider



$$\mathcal{I}_3 = 2.2$$

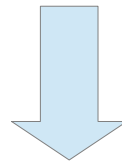
events

FCC



Conclusions

- Quantum tomography (QT) is a powerful tool to reconstruct the quantum spin state of a bipartite spin-system via polarization density matrix
- this allows to test quantum entanglement and Bell inequality violations
- Linear colliders are ideal laboratories to test these QM properties at high energy
- suitable for tau-pair production and vector boson VV production (difficult at LHC)
- Monte Carlo full simulation has been done for tau-pair production at FCC
- Expected observation level of Bell inequality violation at FCC (max luminosity)



$$e^+ e^- \rightarrow \tau^+ \tau^-$$

30 σ significance

$$e^+ e^- \rightarrow Z Z$$

> 4 σ significance

Backup slides

W W

$$p p \rightarrow V_1 + V_2 + X \rightarrow \ell^+ \ell^- + \text{jets} + E_T^{\text{miss}}$$

Differential cross section

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega^+ d\Omega^-} = \left(\frac{3}{4\pi} \right)^2 \text{Tr} \left[\rho_{V_1 V_2} (\Gamma_+ \otimes \Gamma_-) \right]$$

depend on the invariant mass m_{VV} (or velocity β) and **scattering angle** Θ in the $V_1 V_2$ cm frame

R. Rahaman, R.K. Singh, NPB 984 (2022) 115984, [arXiv:2109.09345]

$$d\Omega^\pm = \sin \theta^\pm d\theta^\pm d\phi^\pm \longrightarrow$$

$\downarrow$ polar angle $\ell^\pm$ $\downarrow$ azimuthal angle $\ell^\pm$

phase space written in terms of the spherical coordinates (with **independent polar axis**) for the momenta of the **final charged leptons** in the respective **rest frames** of the decaying spin-1 particles

$$\rho_{V_1 V_2} = \text{density matrix of } V_1 V_2$$

$$\Gamma_\pm \longrightarrow \text{Density matrices that describe the polarization of the two decaying W into final leptons (the charged ones assumed to be massless)}$$

these are projectors in the case of the W-bosons because of their chiral couplings to leptons

can be computed by **rotating** to an arbitrary polar axis the spin states of gauge bosons from the ones given in the **k-direction** quantization axis

$$\Gamma_{\pm} = \frac{1}{3} \mathbb{1} + \sum_{i=1}^8 q_{\pm}^a T^a$$

→ Density matrices for W-bosons

$q_{\pm}^a$ (Wigner q-symbols) are functions of the corresponding spherical coordinates

↓
set of polynomials
of spherical
coordinates
(see backup slide)

$$h_{ab} = \frac{1}{\sigma} \int \int \frac{d\sigma}{d\Omega^+ d\Omega^-} p_+^a p_-^b d\Omega^+ d\Omega^-$$

$$f_a = \frac{1}{\sigma} \int \frac{d\sigma}{d\Omega^+} p_+^a d\Omega^+$$

$$g_a = \frac{1}{\sigma} \int \frac{d\sigma}{d\Omega^-} p_-^a d\Omega^-$$

$p_{\pm}^n$ a particular set of orthogonal functions $\Rightarrow \left(\frac{3}{4\pi}\right) \int p_{\pm}^n q_{\pm}^m d\Omega^{\pm} = \delta^{nm}$
(see next slide)

For **ZZ** case, the set of functions are linear combinations of $q_{\pm}^a \rightarrow$ see backup slides

Wigner's Q symbols

$$\begin{aligned}
 q_{\pm}^1 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(\cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 q_{\pm}^2 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(\cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 q_{\pm}^3 &= \frac{1}{8} \left(1 \pm 4 \cos \theta^{\pm} + 3 \cos 2\theta^{\pm} \right), \\
 q_{\pm}^4 &= \frac{1}{2} \sin^2 \theta^{\pm} \cos 2\phi^{\pm}, \\
 q_{\pm}^5 &= \frac{1}{2} \sin^2 \theta^{\pm} \sin 2\phi^{\pm}, \\
 q_{\pm}^6 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(-\cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 q_{\pm}^7 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(-\cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 q_{\pm}^8 &= \frac{1}{8\sqrt{3}} \left(-1 \pm 12 \cos \theta^{\pm} - 3 \cos 2\theta^{\pm} \right),
 \end{aligned}$$

$$\begin{aligned}
 p_{\pm}^1 &= \sqrt{2} \sin \theta^{\pm} \left(5 \cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 p_{\pm}^2 &= \sqrt{2} \sin \theta^{\pm} \left(5 \cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 p_{\pm}^3 &= \frac{1}{4} \left(5 \pm 4 \cos \theta^{\pm} + 15 \cos 2\theta^{\pm} \right), \\
 p_{\pm}^4 &= 5 \sin^2 \theta^{\pm} \cos 2\phi^{\pm}, \\
 p_{\pm}^5 &= 5 \sin^2 \theta^{\pm} \sin 2\phi^{\pm}, \\
 p_{\pm}^6 &= \sqrt{2} \sin \theta^{\pm} \left(-5 \cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 p_{\pm}^7 &= \sqrt{2} \sin \theta^{\pm} \left(-5 \cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 p_{\pm}^8 &= \frac{1}{4\sqrt{3}} \left(-5 \pm 12 \cos \theta^{\pm} - 15 \cos 2\theta^{\pm} \right).
 \end{aligned}$$