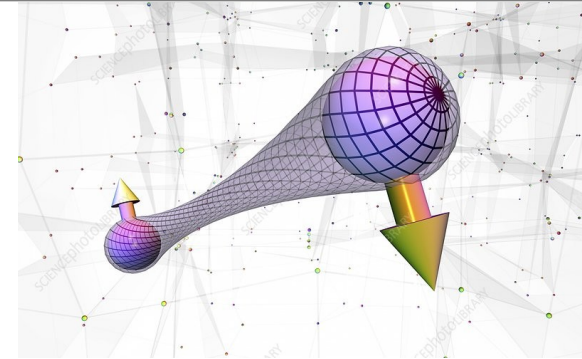


Quantum Entanglement and Violation of Bell Inequality at High Energies

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JHEP 09 (2023) 195, (arXiv:2304.02403 [hep-ph])
EPJC 83 (2023) 9, 823, (arXiv:2302.00683 [hep-ph])
EPJC 83 (2023) 2, 162, (arXiv:2208.11723 [hep-ph])

Outline

- EPR paradox and Bell inequality
- Entanglement and violation of Bell inequality in spin-correlation systems
- Main observables for entanglement and Bell inequalities: **qubit** and **qutrit** systems
- Quantum tomography to reconstruct polarization density matrix
- Bell inequality violation in B meson decays : **$B \rightarrow J/\Psi K^*$**
- Other tests of Bell inequality at the LHC in qutrit and qubit systems
- Conclusions

- “Entanglement” between two systems is a pure quantum phenomena
- Expected to violates **Bell inequalities** (set of correlation measurements)
- Violations incompatible with classical physics based on causality and local realism (locality) (**EPR paradox, hidden variables theories**)
- **pairs of two outcome measurements is at least required**
- Violation of Bell inequality verified at **low energy** (**Nobel price 2022**) using:
 - pair of photons Freedman-Clauser PRL 28 (1972), Aspect-Dalibard-Roger PRL 49 (1982), Zeilinger et al. , Nature 433, 230 (2005)
 - photons emanating from ions ($^9\text{Be}^+$) M.A. Rowe *et al.* Nature 409 (2001)
 - superconductive systems M. Ansmann *et al.* Nature 461 (2009)
 - nitrogen vacancy centers W. Pfaff et al. Nature Physics 9 (2013)
 - pairs of three-outcome measurements with photons A.Vaziri *et al* PRL 89 (2002)
- Entanglement expected at high energy as well as Bell inequality violation
- We will show that for the first time Bell inequality is violated at high energy ($\sim 5\text{ GeV}$) and in the presence of **WEAK and STRONG** interactions

Local realism

- Based on the (classical physics) idea that objects have definite properties whether or not they are measured
- and that measurements of these properties are not affected by events taking place sufficiently far away
- Einstein, Podolsky, Rosen (EPR) used these reasonable assumptions to conclude that quantum mechanics is incomplete (hidden variables..)

EPR Paradox

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in

quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

deal with P and X operators

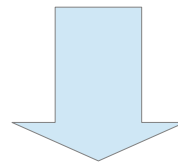
Einstein Locality Principle

Consider two systems **A** and **B** that have interacted in the past and are separated (space-like) far away

The results of a measurement on A is unaffected by operations on the distance system B

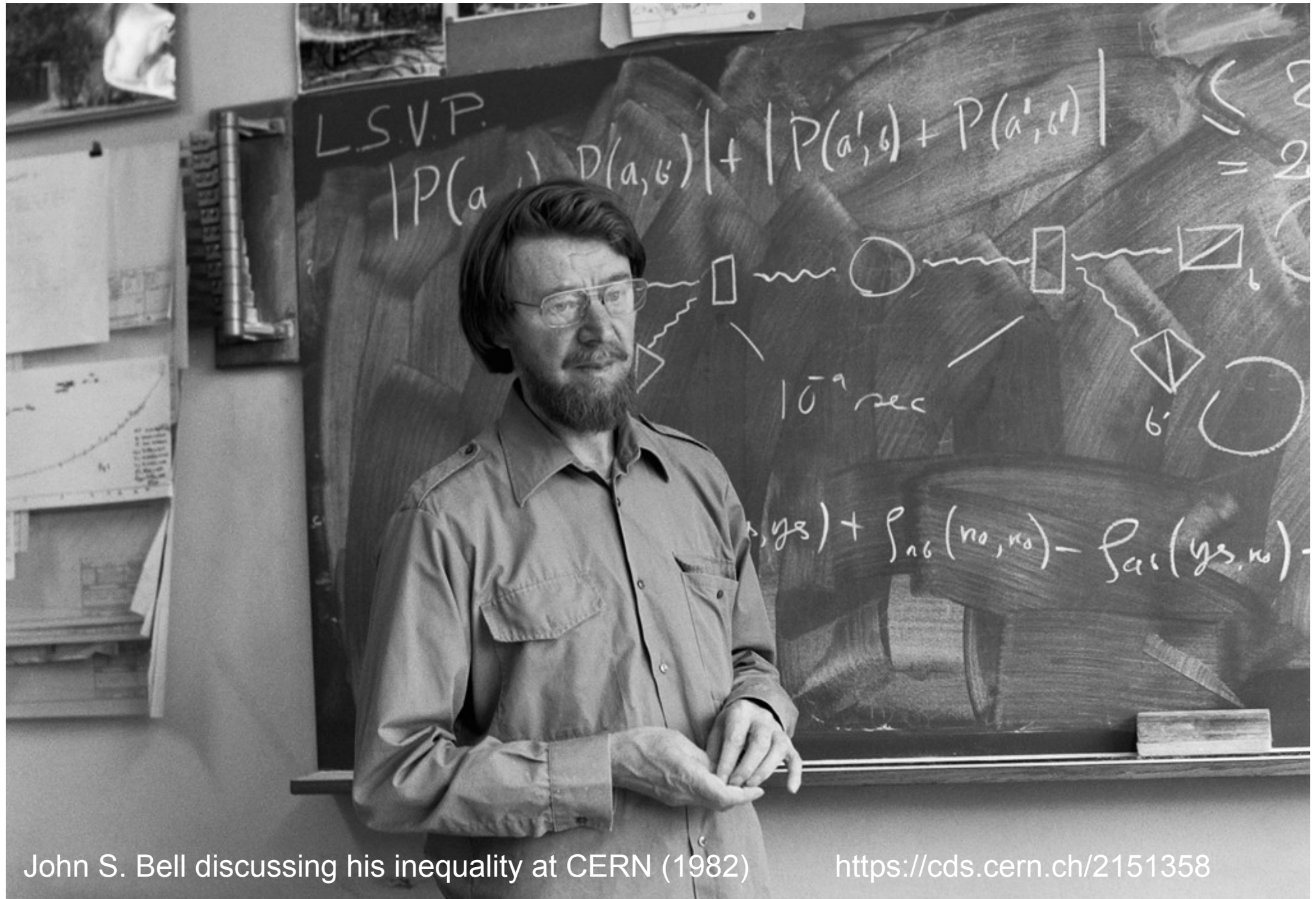
- One may argue that the incompleteness of QM followed from EPR paradox is inherent in the probabilistic interpretation of Quantum Mechanics
- Dynamic behavior at microscopic level appears probabilistic only because some yet unknown parameters (hidden variables) have not been specified
- **Untill 1964** it could be thought that these alternative theories could be cooked up such a way that they would give no predictions, other than the usual QM predictions that could be experimentally verified
- This debate would have belonged to metaphysics rather than physics until...

1964, when J. S. Bell took a different approach to the problem



Do such theories make predictions different from those of Quantum Mechanics and that could be observed ?

Bell inequality



This work laid to the foundation of modern field of quantum-information science

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL[†]

Department of Physics, University of Wisconsin, Madison, Wisconsin

(Received 4 November 1964)

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is

Proof based on locality and non-commuting spin operators S_x S_y S_z instead of X, P as in EPR

What is entanglement ?

In Quantum Mechanics

classical concept of phase space

replaced by

by abstract Hilbert space

makes a gap in the description of composite systems

Consider multipartite system of n subsystems

Classical description → Cartesian product of n subsystems → product of the n separate systems

Quantum description → Hilbert space H → tensorial product of subsystem spaces

$$H = H_1 \otimes H_2 \otimes H_3 \otimes \cdots \otimes H_n$$

superposition principle

$$|\psi\rangle = \sum_{\mathbf{i}_n} c_{\mathbf{i}_n} |\mathbf{i}_n\rangle$$

$$|\mathbf{i}_n\rangle = |i_1\rangle \otimes |i_2\rangle \cdots |i_n\rangle$$

$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \cdots |\psi_n\rangle$$

In general not possible to assign a single state vector to any of n subsystems

appearing as result of physical interactions

giving rise to phenomenon of entanglement

Entanglement in **bi-partite systems**

spin - $\frac{1}{2}$ systems

(qubit)

$$H = H_1 \otimes H_2 \quad \dim H_1 = \dim H_2 = 2$$

example H is in a $J=0$ state

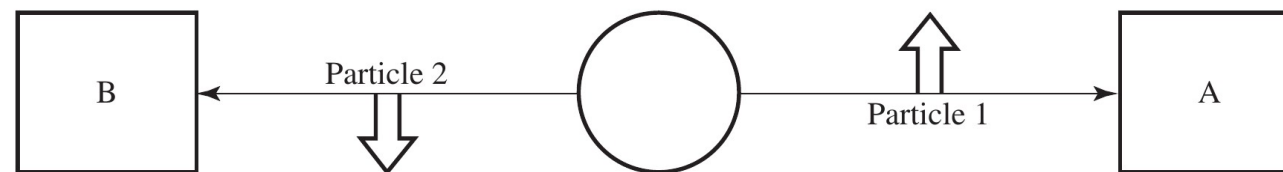
$$H \rightarrow \mu^+ + \mu^-$$

total spin of $\mu^+ \mu^-$ is zero

Total angular momentum is conserved

● measurement of spin in particle 1 induces correlation on spin measurement of particle 2

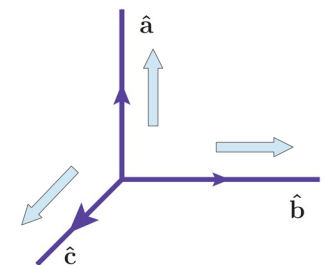
● measuring spin correlations of **particles 1 and 2** along same directions just test **property of angular momentum conservation** → QM gives same expectations as in **classical theory**



● to check departure from **deterministic nature of reality** → require to perform correlated measurements of spin-projection **in different** directions (where spin operators **S** **do not commute**)

Not necessarily to be orthogonal

$$\{\hat{a}, \hat{b}, \hat{c}\} \Rightarrow [\hat{S}_{\hat{a}}, \hat{S}_{\hat{b}}] \neq 0 \quad [\hat{S}_{\hat{a}}, \hat{S}_{\hat{c}}] \neq 0 \quad [\hat{S}_{\hat{b}}, \hat{S}_{\hat{c}}] \neq 0$$



- ▶ Consider two particles produced entangled and separated far away each other and a measurement of spin along **Z** and **X** direction in observer **A** and **B**:
- ▶ two outcome of spin measurement are possible = **+** or **-** (in unit of spin)
- ▶ Suppose to measure spin along Z and X directions (in QM **S_Z** and **S_X** do not commute)

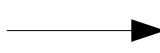
when we measure S_X we do not measure S_Z and viceversa

notation $(\hat{\mathbf{Z}}+, \hat{\mathbf{X}}-)$ 

If S_Z is measured we obtain a + sign with certainty
If S_X is measured we obtain a - sign with certainty

Consider now measurement of spin along 3 arbitrary directions $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$

notation

$\hat{\mathbf{a}}+$  measurement of $\mathbf{S}_1 \cdot \hat{\mathbf{a}}$ gives a **+** sign,..etc

suppose that one of the particle belongs to $(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$

means that if we measure spin on **a** direction is **-**, along **b** is **+** and along **c** is **+**

- separate the two particles and measure spin correlations

Spin-correlations as predicted by deterministic (causal) local theories

Population	Particle 1	Particle 2	8 possible combinations
N_1	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$	
N_2	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$	
N_3	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$	
N_4	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$	
N_5	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$	
N_6	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$	
N_7	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}+)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}-)$	
N_8	$(\hat{\mathbf{a}}-, \hat{\mathbf{b}}-, \hat{\mathbf{c}}-)$	$(\hat{\mathbf{a}}+, \hat{\mathbf{b}}+, \hat{\mathbf{c}}+)$	

Since each N_{1-8} is semipositive defined

$$N_3 + N_4 \leq (N_2 + N_4) + (N_3 + N_7)$$

Probability

$$P(\hat{\mathbf{a}}+; \hat{\mathbf{b}}+) = \frac{(N_3 + N_4)}{\sum_i^8 N_i}$$

↓
spin+ along **a** for particle 1

↓
spin+ along **b** for particle 2

$$P(\hat{\mathbf{c}}+; \hat{\mathbf{b}}+) = \frac{(N_3 + N_7)}{\sum_i^8 N_i} \qquad P(\hat{\mathbf{a}}+; \hat{\mathbf{c}}+) = \frac{(N_2 + N_4)}{\sum_i^8 N_i}$$

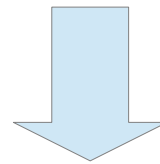
Deterministic causal local theory predictions

Bell inequality

for correlated probabilities

$$P(\hat{\mathbf{a}}+; \hat{\mathbf{b}}+) \leq P(\hat{\mathbf{a}}+; \hat{\mathbf{c}}+) + P(\hat{\mathbf{c}}+; \hat{\mathbf{b}}+)$$

spin+ along **a** for particle 1 spin+ along **b** for particle 2 ..etc



Entangled states can (but not necessarily) violate Bell inequality !

- Bell inequality allows to test if Nature has a causal and local (hidden variable) description or a QM (non-local) one
- demonstration was more general, including hidden variable description (see original paper)

We need to apply the rules of QM to compute these probabilities and check if they satisfy the inequality above

in the $\mathbf{S}_z$ eigenstate basis

$$|\text{spin singlet}\rangle = \left(\frac{1}{\sqrt{2}}\right) (|\hat{\mathbf{z}}+; \hat{\mathbf{z}}-\rangle - |\hat{\mathbf{z}}-; \hat{\mathbf{z}}+\rangle)$$

in the $\mathbf{S}_x$ eigenstate basis

$$|\text{spin singlet}\rangle = \left(\frac{1}{\sqrt{2}}\right) (|\hat{\mathbf{x}}-; \hat{\mathbf{x}}+\rangle - |\hat{\mathbf{x}}+; \hat{\mathbf{x}}-\rangle)$$

Relations of eigenstates in the $\mathbf{S}_x$ and $\mathbf{S}_z$ spin eigenstates basis

$$|\hat{\mathbf{x}}\pm\rangle = \left(\frac{1}{\sqrt{2}}\right) (|\hat{\mathbf{z}}+\rangle \pm |\hat{\mathbf{z}}-\rangle), \quad |\hat{\mathbf{z}}\pm\rangle = \left(\frac{1}{\sqrt{2}}\right) (|\hat{\mathbf{x}}+\rangle \pm |\hat{\mathbf{x}}-\rangle)$$

We can generalize these relations to arbitrary directions by using the spin-1/2 rotation operators of QM

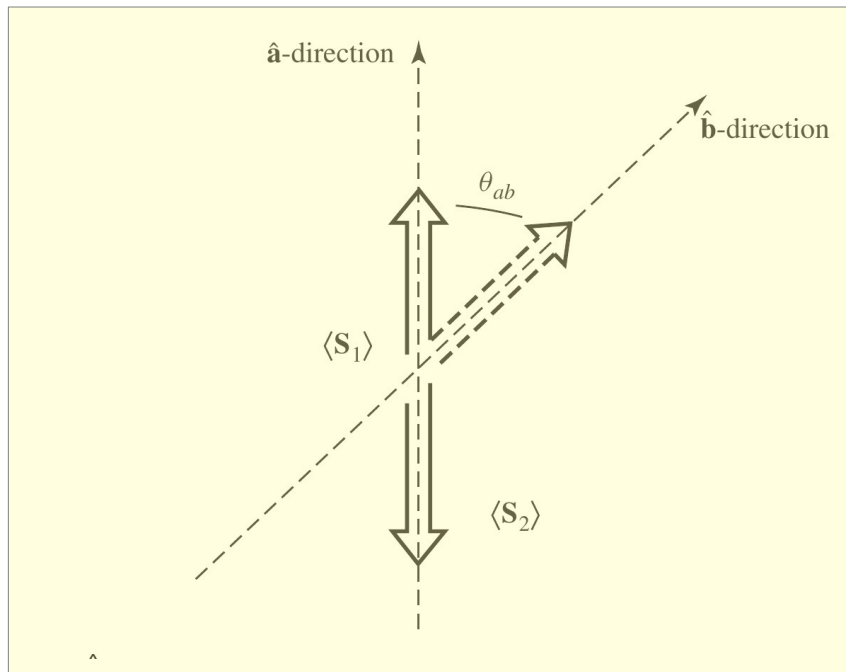
QM predictions

suppose observer A finds $\mathbf{S}_1 \cdot \hat{\mathbf{a}}$ to be positive (+) with certainty

observer B's measurement of $\mathbf{S}_2 \cdot \hat{\mathbf{a}}$ will find it negative (-) with certainty

In order to compute $P(\hat{\mathbf{a}}+; \hat{\mathbf{b}}+)$ we must consider a new quantization axis

$\hat{\mathbf{b}}$ that makes an angle θ_{ab} with $\hat{\mathbf{a}}$



The probability that $\mathbf{S}_2 \cdot \hat{\mathbf{b}}$ measurement yields + when particle 2 is known

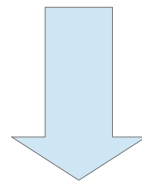
to be in a eigenstate of $\mathbf{S}_2 \cdot \hat{\mathbf{a}}$ with + is =

$$P(\hat{\mathbf{a}}+; \hat{\mathbf{b}}+) = \left(\frac{1}{2}\right) \sin^2 \left(\frac{\theta_{ab}}{2}\right)$$

→
plug in into the Bell
inequality and we get...

QM prediction of Bell inequality

$$\sin^2 \left(\frac{\theta_{ab}}{2} \right) \leq \sin^2 \left(\frac{\theta_{ac}}{2} \right) + \sin^2 \left(\frac{\theta_{cb}}{2} \right)$$



NOT always possible for all directions (simple geometry)

For example, choose all three direction to lie on a plane

$$\theta_{ab} = 2\theta, \quad \theta_{ac} = \theta_{cb} = \theta$$

In this case Bell inequality is **violated** for $0 < \theta < \frac{\pi}{2}$

For example for $\theta = \pi/4$ $\longrightarrow 0.500 \leq 0.292$???

optimization problem: goal is to find directions where the Bell inequality is maximally violated

Conclusion: QM is a non-local theory

measurements in A affects what will be measured in B, even if A and B are space-like separated apart, and no causal exchange of information between them is possible

The Nobel Prize in Physics 2022



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Alain Aspect

Prize share: 1/3



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John F. Clauser

Prize share: 1/3



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Anton Zeilinger

Prize share: 1/3

- [1] A. Zeilinger *et al.*, Nature **433**, 230 (2005)
- [2] S.J. Freedman and J.F. Clauser, Phys. Rev. Lett. **28**, 938 (1972).
<https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.28.938>
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- [4] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, A. Zeilinger, Phys. Rev. Lett. **81**, 5039 (1998). D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger, Nature **390**, 575 (1997)
- [5] A. Aspect, Physics **8**, 123. (2015)

The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

new challenge is to prove Bell inequality violation at **high energies**

**We explored entanglement and Bell inequality
at the Large Hadron Collider (LHC)**

Testing entanglement and Bell inequality at colliders

(recent activity → starting from 2021)

(qubit)

+ top-quark pair production

SM →

New Physics →

- Afik, de Nova, Euro Phys. J Plus 136 (2021) 2003.02280 [quant-ph]
Fabbrichesi, Floreanini, Panizzo, PRL 127 (2021), 2102.11883 [hep-ph]
Severi, Boschi, Maltoni, Sioli, EPJC 82 (2022), 2110.10112 [hep-ph]
Afik, de Nova, Quantum 6 (2022), 2203.05582 [quant-ph]
Aguilar-Saavedra, Casas, EPJC 82 (2022), 2205.00542 [hep-ph]

(qubit)

+ tau-pair production (Drell-Yan + Higgs decays)

- Fabbrichesi, EG, Floreanini, EPJC 83 (2023), 2302.00683 [hep-ph]
Aoude, Madge, Maltoni, Mantani, PRD 106 (2022), 2203.05619 [hep-ph]
Severi, Vryonidou, JHEP 01 (2023), 2210.09339 [hep-ph]

(qutrit)

+ Higgs boson → weak-boson pairs

- Fabbrichesi, EG, Floreanini, EPJC 83 (2023), 2302.00683 [hep-ph]
Altakach, Lambda, Maltoni, Mawatari, Sakurai, 2211.10513 [hep-ph]

(qutrit)

+ WW, ZZ, WZ (Drell-Yan)

- Alan Barr, PLB 285 (2022), 2106.01377 [hep-ph]
Barr, Caban, Rembielinski, 2204.11063 [hep-ph]
Aguilar-Saavedra, Bernal, Casas, Moreno, 2209.13441 [hep-ph]
Aguilar-Saavedra, 2209.14033 [hep-ph]
Fabbrichesi, Floreanini, EG, Marzola, 2302.00683 [hep-ph]

(qubit)

+ Λ hyperons

- Ashby-Pickering, Barr, Wierzchucka, 2209.13990 [quant-ph]
Fabbrichesi, Floreanini, EG, Marzola, 2302.00683 [hep-ph]

- Gong, Parida, Tu, Venugopalan, 2107.13007 [hep-ph]

(new results)

(qutrit)

+ B-meson decays J/Ψ K^* (Bell inequality is violated!)

- Fabbrichesi, Floreanini, EG, Luca Marzola, 2305.04982 [hep-ph]

Polarization density matrix ρ

- Requires the knowledge of the **polarization density matrix** of two-particles **A,B** production
- it can be fully reconstructed from the angular distributions of the single **A,B decay products**
- **but it can also be computed analytically**
- knowledge of the full **polarization density matrix** allows to quantify (where possible) entanglement and Bell inequality violations

Quantifying entanglement

difficult task → simplifies in the case of pure states

✓ Pure states (or pure ensembles): the system is composed only by **one** quantum state $|\psi\rangle$

$$\text{Density matrix } \rho = |\psi\rangle\langle\psi| \quad \text{for pure states}$$

✓ in real life we do not have pure states → but **mixture of states** $|\psi_n\rangle_{n=1,2,\dots}$

$$\rho = \sum_n p_n |\psi_n\rangle\langle\psi_n|, \quad \sum_n p_n = 1 \quad \Rightarrow \quad \text{Tr}[\rho] = 1$$

$\langle A \rangle = \text{Tr}[\rho A]$ → expectation value of operators (A) → can be computed in any base since Trace is base independent

✓ For pure states

$$\rho \doteq \begin{pmatrix} 0 & & & & & & 0 \\ & 0 & & & & & \\ & & \ddots & & & & \\ & & & 0 & & & \\ & & & & 1 & & \\ & & & & & 0 & \\ & & & & & & 0 \\ & & & & & & & 0 \\ & & & & & & & \ddots & \\ & & & & & & & & 0 \end{pmatrix} \quad \text{(diagonal form)}$$

$$\rho^2 = \rho$$

$$\text{Tr}[\rho^2] = 1$$

- A suitable observable (**concurrence**) to quantify entanglement in a bi-partite (A,B) system for a pure state $|\Psi\rangle$ (with matrix density $\rho = |\Psi\rangle\langle\Psi|$) is defined as

concurrence

$$\mathcal{C}[|\Psi\rangle] = \sqrt{1 - \text{Tr}[(\rho_r)^2]} \quad r = A \text{ or } B$$

pure states

Rungta, Buzek, Caves, Hillery, Milburn, PRA 64 (2001)

vanishes for separable states $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$

$$\rho_A = \text{Tr}_B[|\Psi\rangle\langle\Psi|] \text{ and similar for } \rho_B$$

Trace performed in the subsystem B

- For mixed states with matrix density $\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$, $p_i \geq 0$, $\sum_i p_i = 1$

$$\mathcal{C}[\rho] = \inf_{\{|\Psi_i\rangle\}} \sum_i p_i \mathcal{C}[|\Psi_i\rangle]$$

infimum taken over all possible decompositions in pure states

$\mathcal{C}[\rho]$ vanishes for separable states

Two-Qubit systems

System of two spin-1/2 particles

Useful to decompose polarization density matrix along the Pauli-matrices σ_i basis

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \frac{1}{4} \left[\mathbb{1} \otimes \mathbb{1} + \sum_i B_i^+ (\sigma_i \otimes \mathbb{1}) + \sum_j B_j^- (\mathbb{1} \otimes \sigma_j) + \sum_{ij} C_{ij} (\sigma_i \otimes \sigma_j) \right]_{\lambda_1 \lambda'_1, \lambda_2 \lambda'_2}$$

Tensorial product $[A \otimes B]_{ii' jj'} = A_{ii'} B_{jj'}$

$$B_i^+ = \text{Tr}[\rho (\sigma_i \otimes \mathbb{1})] \quad B_j^- = \text{Tr}[\rho (\mathbb{1} \otimes \sigma_j)] \quad \Rightarrow \quad \text{polarization coefficients of the two-qubits}$$

$$C_{ij} = \text{Tr}[\rho (\sigma_i \otimes \sigma_j)] \quad \Rightarrow \quad \text{spin-correlation of two-qubit}$$

Concurrence for Two-QUBITS

C.H. Bennet, D.P. DiVincenzo, J.A. Smolin, W.K. Wootters,
Phys. Rev. A 54 (1996), arXiv-quant-ph/9604024 [quant-ph]

● For **bi-partite systems A,B of qubit** the **concurrence** $\mathcal{C}[\rho]$ can be analytically computed

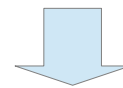
● built from the matrix $\rightarrow$

$$R = \rho (\sigma_2 \otimes \sigma_2) \rho^* (\sigma_2 \otimes \sigma_2)$$

(not hermitean)

has non-negative eigenvalues

$$\mathcal{C}[\rho] = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$



vanishes for separable states, **max value = 1**

$\lambda_i \rightarrow$ square root of R eigenvalues, $i=1,2,3,4$ with λ_1 the largest one

Bell inequality

for Two-Qubits

correlation matrix 3x3

unit 3-dim vectors $\hat{n}_{1,2,3,4}$

which correspond to the choices that A and B systems do for the axis where to measure the spin

$$\left| \hat{n}_1 \cdot C \cdot (\hat{n}_2 - \hat{n}_4) + \hat{n}_3 \cdot C \cdot (\hat{n}_2 + \hat{n}_4) \right| \leq 2$$

No need of suitable choice of $\hat{n}_{1,2,3,4}$ to maximize the Bell violation
maximization process optimized for a generic correlation matrix C

R. Horodecki *et al* , Phys. Lett. A200 (1995)

observable

$$M = C^T C$$

$$\mathfrak{m}_{12}[C] \equiv m_1 + m_2$$

the largest two eigenvalues of M

$$m_1 \geq m_2 \geq m_3$$

two-qubit state violates Bell inequality

IF AND ONLY IF

$$\mathfrak{m}_{12}[C] > 1$$

independently of $\hat{n}_{1,2,3,4}$

$$C = \begin{pmatrix} C_{nn} & C_{nr} & C_{nk} \\ C_{rn} & C_{rr} & C_{rk} \\ C_{kn} & C_{kr} & C_{kk} \end{pmatrix}$$

$$\text{C-eigenvalues} = \left\{ C_{nn}^2, \frac{1}{4} \left[C_{kk} + C_{rr} + \sqrt{(C_{kk}^2 - C_{rr})^2 + 4C_{kr}^2} \right]^2, \frac{1}{4} \left[C_{kk} + C_{rr} - \sqrt{(C_{kk}^2 - C_{rr})^2 + 4C_{kr}^2} \right]^2 \right\}$$

Two massive spin-1 particles

- A spin-1 particle has 3 polarizations states, corresponding to $S_z = +1, 0, -1$
- We talk of **qutrits** in this case
- At high energy (LHC) candidates for qutrits are the **weak gauge bosons** $W^{+/-}, Z^0$
- Entanglement and Bell inequality violation have been recently analyzed for the on-shell WW, ZZ production at the LHC
- also via Higgs boson decay $H \rightarrow W W^*$ or $H \rightarrow Z Z^*$ (where W^*, Z^* are off-shell)
- Polarizations of W, Z are reconstructed from angular dependence of their decay products
- From the polarization states of W, Z one can reconstruct the polarization density matrix

Two-QUTRITS

Example $V_1 V_2 = (WW, ZZ, WZ)$

System of two massive spin-1 particles

useful to decompose the **density matrix** on the basis of tensor products of **Gell-Mann matrices**

$$\{\mathbb{1} \otimes \mathbb{1}, \mathbb{1} \otimes T^a, T^a \otimes \mathbb{1}, T^a \otimes T^b\}$$

$T^a \rightarrow 3 \times 3$ Gell-Mann matrices

given by the Kronecker product of the matrix representations $[A \otimes B]_{ii'jj'} = A_{ii'} B_{jj'}$

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \left(\frac{1}{9} [\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [\mathbb{1} \otimes T^a] + \sum_a g_a [T^a \otimes \mathbb{1}] + \sum_{ab} h_{ab} [T^a \otimes T^b] \right)_{\lambda_1 \lambda'_1, \lambda_2 \lambda'_2}$$

9x9 matrix

$$f_a = \frac{1}{6} \text{Tr} [\rho (\mathbb{1} \otimes T^a)] , \quad g_a = \frac{1}{6} \text{Tr} [\rho (T^a \otimes \mathbb{1})] , \quad h_{ab} = \frac{1}{4} \text{Tr} [\rho (T^a \otimes T^b)]$$

● As for qubits, these coefficients are scalar quantities that depend on

$V_1 V_2$ invariant mass and scattering angle in c.m. frame Θ

● we can also extract them from data, using the angular distributions of their decay products

Quantifying entanglement for Two-Qutrits

- Difficult to compute the concurrence $\mathcal{C}[\rho]$

- analytical solution exists for the lower bound

$$(\mathcal{C}[\rho])^2 \geq \mathcal{C}_2[\rho]$$

Witness of Entanglement

Mintert, Buchleitner,
PRL 98 (2007)

$$\mathcal{C}_2[\rho] = 2 \max \left(0, \text{Tr} [\rho^2] - \text{Tr} [(\rho_A)^2], \text{Tr} [\rho^2] - \text{Tr} [(\rho_B)^2] \right)$$

- If non-vanishing unequivocally signal the presence of entanglement
- used in our analysis of entanglement for WW, ZZ and WZ productions
- Solution for upper bound

$$(\mathcal{C}[\rho])^2 \leq 2 \min \left(1 - \text{Tr} [(\rho_A)^2], 1 - \text{Tr} [(\rho_B)^2] \right)$$

- maximum value of Concurrence can be obtained for the maximum symmetric state

for two-qutrits $\longrightarrow \mathcal{C}[|\Psi_+\rangle] = 2/\sqrt{3}$ corresponding to the state $|\Psi_+\rangle = \frac{1}{\sqrt{3}} \sum_{i=1}^3 |i\rangle \otimes |i\rangle$

Lower bound on Concurrence (witness of entanglement)

$$\mathcal{C}_2 = 2 \max \left[-\frac{2}{9} - 12 \sum_a f_a^2 + 6 \sum_a g_a^2 + 4 \sum_{ab} h_{ab}^2, \right. \\ \left. -\frac{2}{9} - 12 \sum_a g_a^2 + 6 \sum_a f_a^2 + 4 \sum_{ab} h_{ab}^2 \right],$$

Quantify entanglement in pure states

- If the bipartite (A,B) system is a pure state it is possible to quantify its entanglement via

Entropy of entanglement

$$\mathcal{E}[\rho] = -\text{Tr} [\rho_A \log \rho_A] = -\text{Tr} [\rho_B \log \rho_B]$$

equality holds if and only if the bipartite is separable

$$0 \leq \mathcal{E}[\rho] \leq \ln 3$$

(Von Neumann Entropy)

- Very useful in the case of bipartite (A,B) system of qutrit for which there is not a solution for the general case

Bell inequalities for Two-QUTRITS

- take a bi-partite system with components A, B
- perform two measurements $\longrightarrow (A_1, A_2) \quad (B_1, B_2)$ each can take values $\rightarrow \{0, 1, 2\}$ on A, B
- consider the following correlator $\mathcal{I}_3$ for probability measurements

$$\begin{aligned} \mathcal{I}_3 = & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ & - P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1) \end{aligned}$$

$\mathcal{I}_3$ can be expressed as

$$\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}]$$

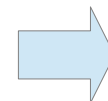
with $\mathcal{B}$ a suitable Bell operator

Collins, Gisin, Linden,
Massar, Popescu, PRL 88 (2002)

Bell inequalities for two-qutrits

For deterministic local models

$$\mathcal{I}_3 \leq 2$$



QM for qutrits can violate this inequality with upper bound = 4

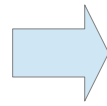
for the maximal entangled state

$$|\Psi_+\rangle = \frac{1}{\sqrt{3}} \sum_{i=1}^3 |i\rangle \otimes |i\rangle$$

$$\rho = |\Psi_+\rangle \langle \Psi_+|$$

● problem of finding optimal choice has been found

given by the Bell operator



$\mathcal{B}$

written on the basis of spin-operators, where

S_3 spin operator $\longrightarrow \text{diag}\{1, 0, -1\}$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

● still freedom to modify measured observables through unitary transformations on the Bell operator

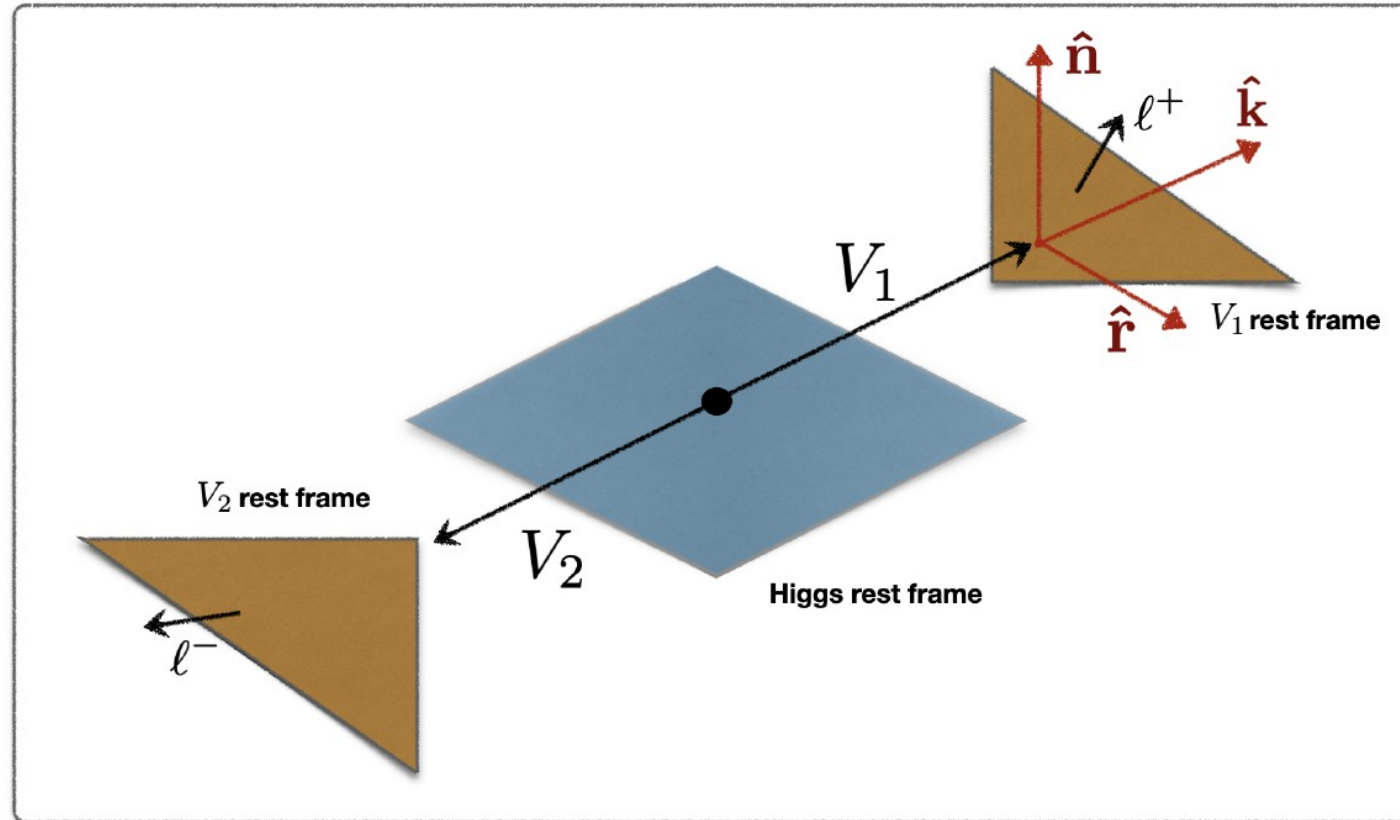
$$\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

$\longrightarrow$ U,V are unitary 3x3 matrices
(depend on the kinematic of the process)

in order to maximize the violation of Bell inequality

Di-boson production in Higgs boson decays

$$H \rightarrow V(k_1, \lambda_1) V^*(k_2, \lambda_2)$$



In the **Higgs boson rest frame** the density matrix of the bipartite **VV^*** system does not depend on the scattering angle, but only by the **Higgs mass**, the **V mass** and the off-shell **V^* mass**

Di-boson production in Higgs boson decays

A. Barr, PLB 825 (2022) ,
2106.01377 [hep-ph]

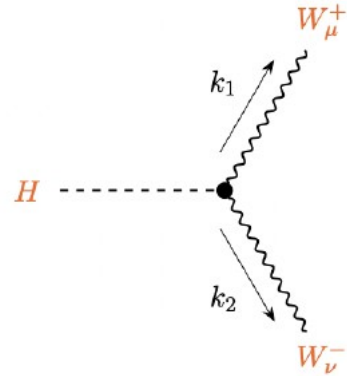
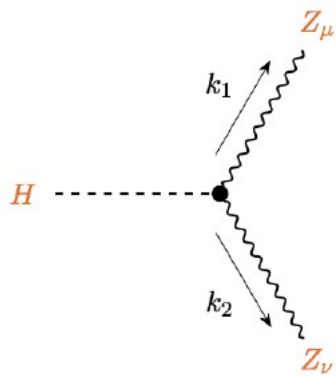
Aguilar-Saavedra, Bernal,
Casas, Moreno
2209.13441 [hep-ph]

$$H \rightarrow V(k_1, \lambda_1) V^*(k_2, \lambda_2)$$

Results based on
2302.00683 [hep-ph]

extracting density matrix from simulation at parton level

Analytic computation



$$\left\{ \begin{array}{l} \xi_W = 1, \text{ and } \xi_Z = 1/(2c_W), \text{ with } c_W = \cos \theta_W \\ g \text{ is the weak coupling} \end{array} \right.$$

Weinberg angle

V^* regarded as an off-shell vector boson with mass $M_V^* = f M_V$ $0 < f < 1$
mass of V boson M_V

Quantum Amplitude

$$\mathcal{M}_H(\lambda_1, \lambda_2) = g M_V \xi_V g_{\mu\nu} \varepsilon^{\mu*}(k_1, \lambda_1) \varepsilon^{\nu*}(k_2, \lambda_2)$$

$$\mathcal{M}_H(\lambda_1, \lambda_2) \mathcal{M}_H(\lambda'_1, \lambda'_2)^\dagger = g^2 M_V^2 \xi_V^2 g_{\mu\nu} g_{\mu'\nu'} \mathcal{P}_{\lambda_1 \lambda'_1}^{\mu\mu'}(k_1) \mathcal{P}_{\lambda_2 \lambda'_2}^{\nu\nu'}(k_2)$$

By projecting into the Gell-Mann matrix basis for 2-qutrits one can extract the **f,g** and **h** coefficients

covariant polarization vector of spin-1

rest frame limit

$$\varepsilon^\mu(p, \lambda) \xrightarrow{(\beta \rightarrow 0)} \psi_\pm, \psi_0$$

β is the velocity

helicity $\lambda = \pm 1, 0$

boosted base

$$n_1^\mu = (0, \hat{\mathbf{n}}), \quad n_2^\mu = (0, \hat{\mathbf{r}}), \quad n_3^\mu = \frac{E}{M}(\beta, \hat{\mathbf{k}})$$

Covariant Projector

$$\mathcal{P}_{\lambda\lambda'}^{\mu\nu}(p) = \varepsilon^\mu(p, \lambda)^* \varepsilon^\nu(p, \lambda')$$

master formula

$$\varepsilon^{0123} = 1$$

$$= \frac{1}{3} \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2} \right) \delta_{\lambda\lambda'} - \frac{i}{2M} \epsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta^i (S_i)_{\lambda\lambda'} - \frac{1}{2} n_i^\mu n_j^\nu (S_{ij})_{\lambda\lambda'}$$

$S_i, i \in \{1, 2, 3\} \rightarrow$ rotation matrices for spin-1 particle

H.S. Song, *Lett. Nuovo Cim.* 25 (1979)
S.Y. Choi, T. Lee, H.S. Song, *PRD* 40 (1989)
Fabbrichesi, Floreanini, EG, Marzola,
2302.00683 [hep-ph]

$$S_{ij} = S_i S_j + S_j S_i - \frac{4}{3} \mathbb{1} \delta_{ij} \quad (\text{see backup slides})$$

basis correspondence for $(S_i)_{\lambda\lambda'}$ $|+\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad |-\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ corresponding to eigenevalues $\lambda = \pm 1, 0$

Inserting the **f,g** and **h** coefficients into the Gell-Mann basis for 2-qutrits

$$\rho_H = 2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{16} & 0 & 2h_{33} & 0 & h_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\text{Tr} [\rho_H] = 1$$

density matrix is idempotent

$$\rho_H^2 = \rho_H$$

Signaling that $H \rightarrow VV^*$ is a **pure state**

$$\rho_H = |\Psi_H\rangle\langle\Psi_H|$$

using the basis

$$|\lambda \lambda'\rangle = |\lambda\rangle \otimes |\lambda'\rangle \text{ with } \lambda, \lambda' \in \{+, 0, -\}$$

where the pure state is

2302.00683 [hep-ph]

$$|\Psi_H\rangle = \frac{1}{\sqrt{2 + \varkappa^2}} [|+-\rangle - \varkappa |00\rangle + |-+\rangle]$$

$$\varkappa = 1 + \frac{m_H^2 - (1 + f)^2 M_V^2}{2f M_V^2}$$

**Aguilar-Saavedra, Bernal,
Casas, Moreno**
2209.13441 [hep-ph]

scalar/pseudoscalar decay in two vectors $S \rightarrow V_1 V_2$

- what have we learned from di-boson Higgs decays $H \rightarrow WW$ or ZZ ?
- $V_1 V_2$ comes out as a pure state
- this property applies to any scalar or pseudoscalar particle decaying into two spin-1 $V_1 V_2$ particles
→ same structure for the density matrix as for the Higgs boson (but different entries)

Fabbrichesi, Floreanini, EG, Marzola
2304.02403 [hep-ph]
2305.04982 [hep-ph]

Approach based on helicity amplitudes

generic scalar meson (for instance B meson)

helicity amplitude →

$$h_\lambda = \langle V_1(\lambda) V_2(-\lambda) | \mathcal{H} | B \rangle \quad \text{with} \quad \lambda = (+, 0, -)$$

There are 3 helicity amplitude combination

helicities defined with respect to
z-direction (spin quantization axis)
in rest frame of one of the two Vectors

$$|\Psi\rangle = \frac{1}{\sqrt{|H|^2}} \left[h_+ |\mathbf{V}_1(+) \mathbf{V}_2(-)\rangle + h_0 |\mathbf{V}_1(0) \mathbf{V}_2(0)\rangle + h_- |\mathbf{V}_1(-) \mathbf{V}_2(+)\rangle \right]$$

$$|H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2$$

Relative weight of $|0\ 0\rangle$ with respect to $|+\ -\rangle$ or $|-\ +\rangle$ controlled by angular momentum conservation

Density matrix for a pure $V_1 V_2$ state can be written as

$$\rho = |\Psi\rangle\langle\Psi|$$

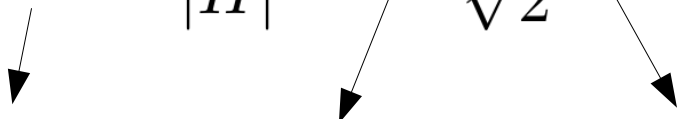
Fabbrichesi, Floreanini, EG, Marzola
2304.02403 [hep-ph]
2305.04982 [hep-ph]

Master formula

in helicity base

$$\rho = \frac{1}{|H|^2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+ h_+^* & 0 & h_+ h_0^* & 0 & h_+ h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0 h_+^* & 0 & h_0 h_0^* & 0 & h_0 h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_- h_+^* & 0 & h_- h_0^* & 0 & h_- h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

proved also using most general Lorentz invariant structures (see next slide)

$$\frac{h_0}{|H|} = A_0, \quad \frac{h_+}{|H|} = \frac{A_{\parallel} + A_{\perp}}{\sqrt{2}} \quad \text{and} \quad \frac{h_-}{|H|} = \frac{A_{\parallel} - A_{\perp}}{\sqrt{2}}$$


Polarization amplitudes directly measured in experimental analysis of B meson decays !

Example: use the most general Lagrangian of a Higgs H decaying in WW^* and ZZ^*

$$\mathcal{L}_{HVV} = g M_W W_\mu^+ W^{-\mu} H + \frac{g}{2 \cos \theta_W} M_Z Z_\mu Z^\mu H \\ - \frac{g}{M_W} \left[\frac{a_W}{2} W_{\mu\nu}^+ W^{-\mu\nu} + \frac{\tilde{a}_W}{2} W_{\mu\nu}^+ \tilde{W}^{-\mu\nu} + \frac{a_Z}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{\tilde{a}_Z}{4} Z_{\mu\nu} \tilde{Z}^{\mu\nu} \right] H$$

$$\text{Dual field} \rightarrow \tilde{V}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} V_{\rho\sigma}$$

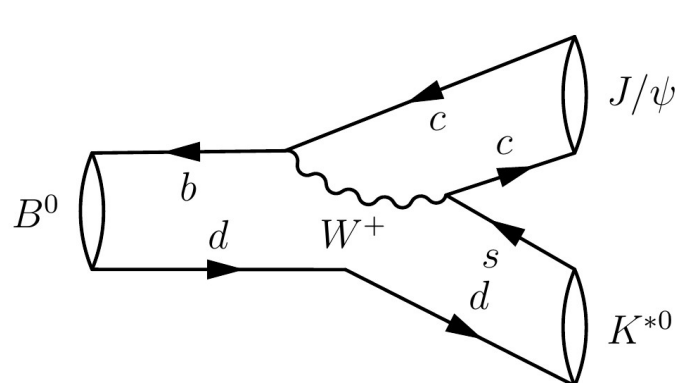
Helicity amplitudes are given by

$$\begin{aligned} h_0 &= -M_V x - a_V M_V (x^2 - 1), \\ h_+ &= -M_V - \tilde{a}_V M_V \sqrt{x^2 - 1}, \\ h_- &= -M_V + \tilde{a}_V M_V \sqrt{x^2 - 1}. \end{aligned} \quad \begin{aligned} x &= m_H^2 / (2f M_V^2) - 1 \\ M_V^* &= f M_V \end{aligned}$$

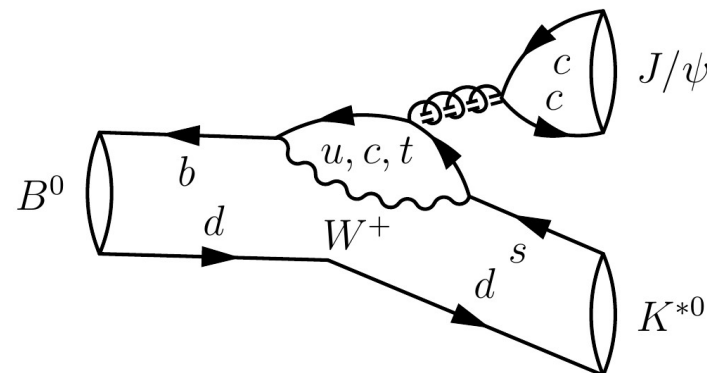
Relations between the two basis Gell-Mann and helicity amplitudes

$$\begin{aligned} h_+ h_+^* &= h_- h_+^* = h_- h_-^* &= \tilde{h}_{44} \\ h_- h_0^* &= h_+ h_0^* = h_- h_0^* &= \tilde{h}_{17} \\ h_0 h_0^* &= \tilde{h}_{33} \end{aligned}$$

Bell inequality violation in B meson decay $B \rightarrow J/\psi K^*$



(a) Tree



(b) Penguin

- from measurement of polarization amplitudes we can compute the I_3 observable for qutrits
- there are 4 independent parameters: 3 amplitudes modulus and 2 relative phases as input

$$A_{\parallel} = |A_{\parallel}| e^{i\delta_{\parallel}}$$

$$A_{\perp} = |A_{\perp}| e^{i\delta_{\perp}}$$

$$A_0 = |A_0| e^{i\delta_0}$$

$$J/\psi \rightarrow \mu^+ \mu^-$$

$$K^* \rightarrow K^+ \pi^-$$

We choose $\delta_0 = 0$

Complex because of final state interactions

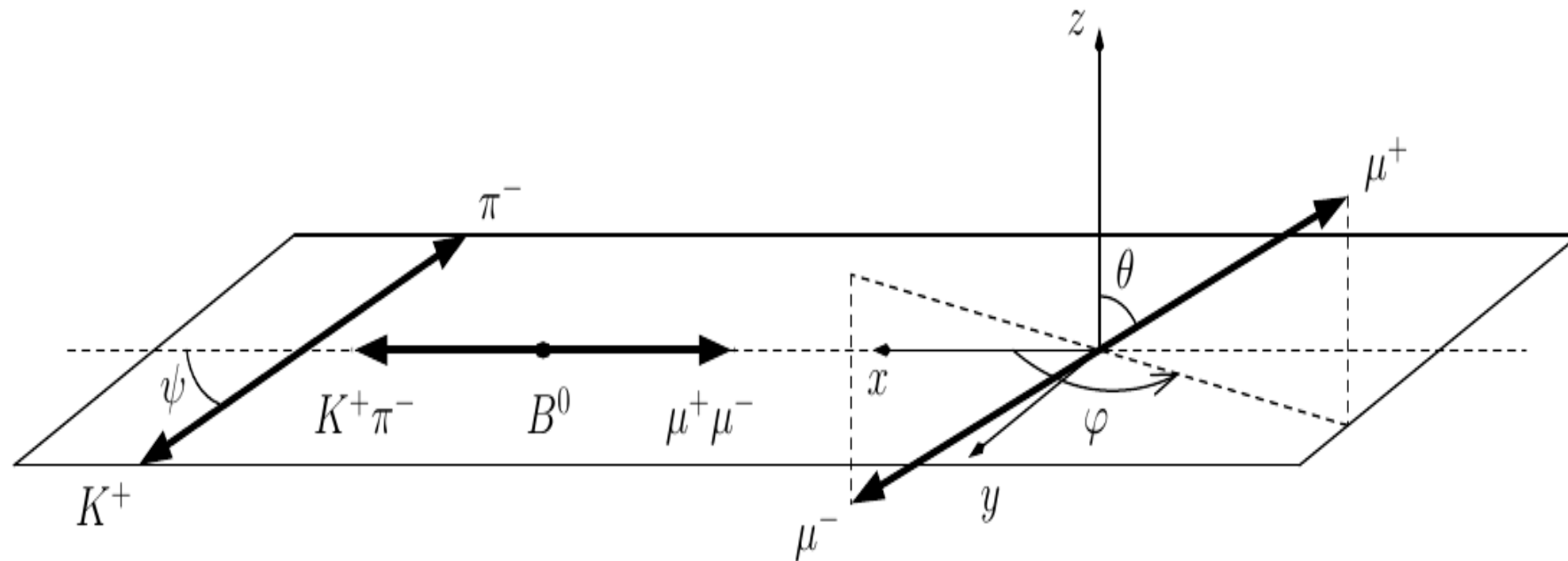
Phases induced by strong interactions due to rescattering

Distance when both have decayed can be estimated to be ~ 1100 fm

Decays take place outside the range of strong interaction at the time of their production (3×10^{-5} fm gluon exchange) as well as of final state interactions (1.5 fm exchange of pions)

From angular distribution one can extract the polarization amplitudes and phases by a fit of coefficients h_k (see next slide)

$$\frac{d^4\Gamma(B^0 \rightarrow J/\psi K^{*0})}{dt d\Omega} \propto e^{-\Gamma_d t} \sum_{k=1}^{10} h_k f_k(\Omega) \quad \Omega = \{\cos \theta, \cos \psi, \varphi\}$$



FIT of coefficients h_k

$$\Omega = \{\cos \theta, \cos \psi, \varphi\}$$

$$\frac{d^4\Gamma(B^0 \rightarrow J/\psi K^{*0})}{dt d\Omega} \propto e^{-\Gamma_d t} \sum_{k=1}^{10} h_k f_k(\Omega)$$

k	h_k	$f_k(\Omega)$
1	$ A_0 ^2$	$\frac{9}{32\pi} 2 \cos^2 \psi (1 - \sin^2 \theta \cos^2 \varphi)$
2	$ A_{\parallel} ^2$	$\frac{9}{32\pi} \sin^2 \psi (1 - \sin^2 \theta \sin^2 \varphi)$
3	$ A_{\perp} ^2$	$\frac{9}{32\pi} \sin^2 \psi \sin^2 \theta$
4	$ A_{\parallel} A_{\perp} \sin(\delta_{\perp} - \delta_{\parallel})$	$-\frac{9}{32\pi} \sin^2 \psi \sin 2\theta \sin \varphi$
5	$ A_0 A_{\parallel} \cos(\delta_{\parallel})$	$\frac{9}{32\pi\sqrt{2}} \sin 2\psi \sin^2 \theta \sin 2\varphi$
6	$ A_0 A_{\perp} \sin(\delta_{\perp})$	$\frac{9}{32\pi\sqrt{2}} \sin 2\psi \sin 2\theta \cos \varphi$
7	$ A_S ^2$	$\frac{3}{32\pi} 2(1 - \sin^2 \theta \cos^2 \varphi)$
8	$ A_{\parallel} A_S \cos(\delta_{\parallel} - \delta_S)$	$\frac{3}{32\pi} \sqrt{6} \sin \psi \sin^2 \theta \sin 2\varphi$
9	$ A_{\perp} A_S \sin(\delta_{\perp} - \delta_S)$	$\frac{3}{32\pi} \sqrt{6} \sin \psi \sin 2\theta \cos \varphi$
10	$ A_0 A_S \cos(\delta_S)$	$\frac{3}{32\pi} 4\sqrt{3} \cos \psi (1 - \sin^2 \theta \cos^2 \varphi)$

(A_S contribution from
non-resonant $J/\psi K^*$ amplitude)

$$|A_{\parallel}|^2 = 0.227 \pm 0.004 \text{ (stat)} \pm 0.011 \text{ (sys)}$$

$$|A_{\perp}|^2 = 0.201 \pm 0.004 \text{ (stat)} \pm 0.008 \text{ (sys)} \quad + \text{ correlations (taken also into account)}$$

$$\delta_{\parallel} [\text{rad}] = -2.94 \pm 0.02 \text{ (stat)} \pm 0.03 \text{ (sys)} \quad |A_0|^2 + |A_{\perp}|^2 + |A_{\parallel}|^2 = 1$$

$$\delta_{\perp} [\text{rad}] = 2.94 \pm 0.02 \text{ (stat)} \pm 0.02 \text{ (sys)},$$

WE GET **Fabbrichesi, Floreanini, EG, Marzola, 2305.04982 [hep-ph]**

$$\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}] \quad \text{we maximize the effect via rotations} \quad \mathcal{B} \rightarrow (U \otimes V)^{\dagger} \cdot \mathcal{B} \cdot (U \otimes V)$$

$$\mathcal{I}_3 = 2.548 \pm 0.015 \quad \Rightarrow B^0 \rightarrow J/\psi K^*(892)^0$$

Bell inequality $\mathcal{I}_3 < 2$ is violated with a significance of **36 σ**

$0 < \text{von Neumann entropy} < \ln[3]$	$\mathcal{E}$	$\mathcal{I}_3$	
• $B^0 \rightarrow J/\psi K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015	
• $B^0 \rightarrow \phi K^*(892)^0$ [20]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$	$\rightarrow 1.1\sigma$
• $B^0 \rightarrow \rho K^*(892)^0$ [21]	$0.450 \pm 0.067^*$	$2.208 \pm 0.129^*$	$\rightarrow 1.6\sigma$
• $B_s \rightarrow \phi \phi$ [22]	$0.734 \pm 0.050^*$	$2.525 \pm 0.084^*$	$\rightarrow 6.2\sigma$
• $B_s \rightarrow J/\psi \phi$ [23]	0.731 ± 0.032	2.462 ± 0.080	$\rightarrow 5.8\sigma$

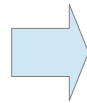
Summary

- This result shows for the first time the Bell inequality violation in **Weak** and **Strong** interactions and at **high energies** ($\sim$ **GeV**) and in **qutrit** systems
- LHCb experiment **was not designed** to test the Bell inequalities as for instance the experiments with entangled photons

Closing the locality loophole (LL)

- LL exploits events not separated by a space-like interval as in the case of $B \rightarrow J/\Psi \ K^*$
- One must consider decays in which the produced particles are identical as in the $B \rightarrow \Phi \ \Phi$
(so their life time is also the same)

we need to check how many events satisfies the space-like condition



$$\frac{|t_1 - t_2| c}{(t_1 + t_2) v} < 1$$

$t_{1,2}$ → time of decays

decay times follow the PDF distribution $P(t) \sim \text{Exp}[-\gamma \beta t]$

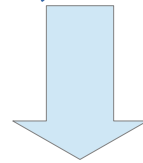
β → the velocity in unit of c
 γ → the Lorentz factor

- we find that almost 90% of events satisfies this condition
- the two bases used in measuring the polarization are arbitrarily chosen (U V diagonalization)
- → provides a set-up where orientations of polarimeters can be freely and arbitrarily chosen

● So locality loophole can be closed!

Closing the detection loophole (DL)

- DL exploits the fact that detectors are not 100% efficient
- Already for qubit the DL is closed if efficiency is more than 80%
- This requirement above is even lower for states belonging to larger Hilbert space as qutrits
- The efficiency of LHCb detector for pion, Kaons, and muons is more than 90%



- So also detection loophole is closed for LHCb !

Tests of Bell inequality violation in other relevant processes at the LHC

$pp \rightarrow \text{Higgs} \rightarrow WW \text{ and } ZZ$

$pp \rightarrow \text{top antitop}$

$pp \rightarrow \text{tau anti-tau}$

Summary of results for $H \rightarrow WW^*$ and $H \rightarrow ZZ^*$

significance S for rejecting the null hypothesis $\mathcal{I}_3 \leq 2$

1000 pseudo experiments

$S(WW^*) \rightarrow$ uncertainty dominated by the **systematic** error

$S(ZZ^*) \rightarrow$ uncertainty dominated by the **statistical** error

LHC Run 2

1

1.3

LHC Hi-Lumi

1

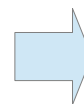
5.6

$\rightarrow$ larger due to smaller uncertainty and higher mean value (2.9)

ZZ^* promising channel at LHC Hi-Lumi

In real simulation a **reduced significance** is expected (systematic uncertainties due to unfolding, background, and detector were neglected or only partially modeled here)

Results are also consistent with analysis of



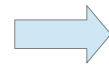
A. Barr, PLB 825 (2022) ,
2106.01377 [hep-ph]

Aguilar-Saavedra, Bernal, Casas,
Moreno 2209.13441 [hep-ph]

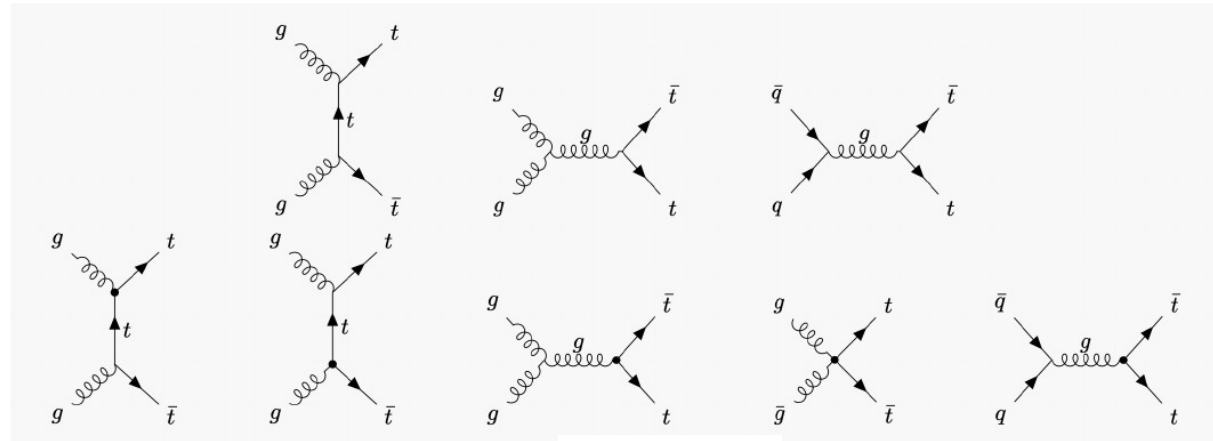
where simulations of events have been adopted for the reconstruction of the density matrix

top- anti-top production

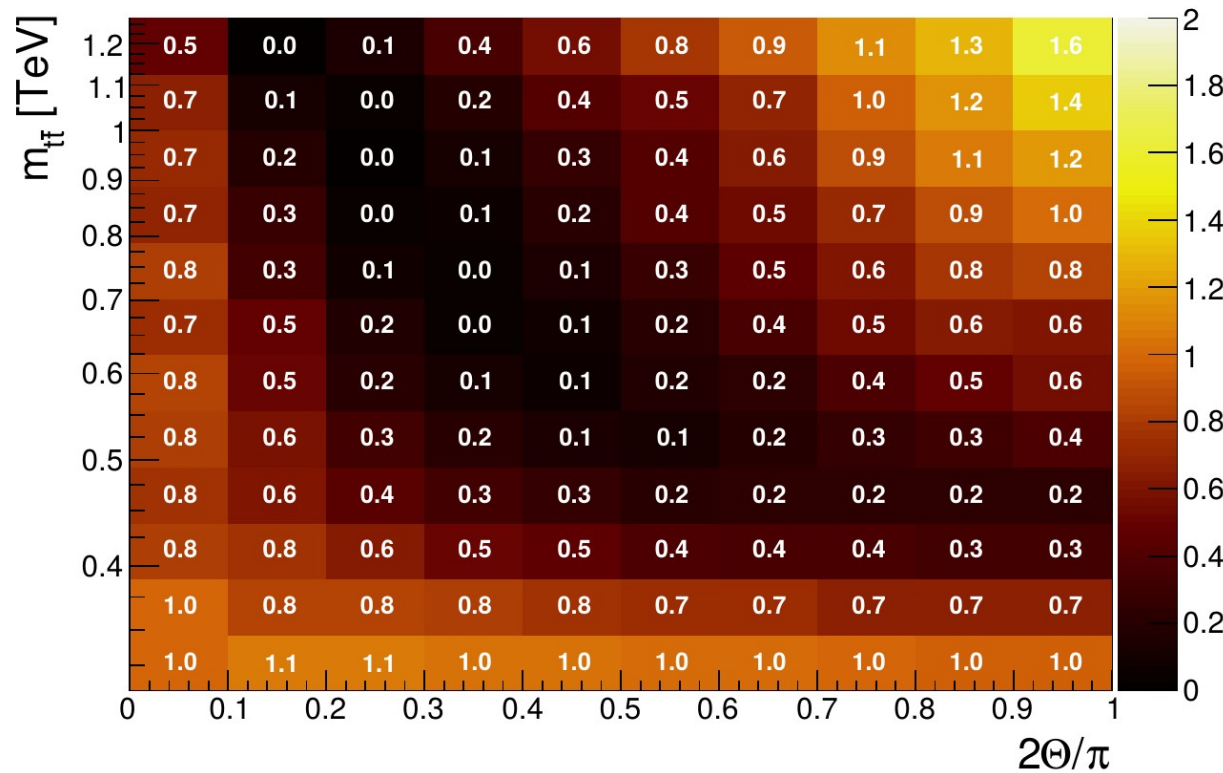
Fabbrichesi, Floreanini, Panizzo, PRL 127 (2021), 2102.11883 [hep-ph]



First analysis of Bell inequalities where correlation matrix C_{ij} is extracted from event simulation, including ATLAS detector resolution (DELEPHES), acceptance, migration and efficiency effects.



$m_{12}[C]$



Null hypothesis of Bell inequalities

$$\left| \hat{n}_1 \cdot C \cdot (\hat{n}_2 - \hat{n}_4) + \hat{n}_3 \cdot C \cdot (\hat{n}_2 + \hat{n}_4) \right| \leq 2$$

Violation of Bell inequalities requires

$$m_{12}[C] > 1$$

Violation of null hypothesis can be assessed:

- at 2 σ level with present Run 2 Luminosity
- at 4 σ with projected full Run 3 Luminosity

Sensitivity to NP (EFT) studied in

Aoude, Madge, Maltoni, Mantani, PRD 106 (2022), 2203.05619 [hep-ph]

Severi, Vryonidou, JHEP 01 (2023), 2210.09339 [hep-ph]

Fabbrichesi, EG, Floreanini, EPJC 83 (2023), 2302.00683 [hep-ph]

$$p p \rightarrow \tau^+ \tau^- + jets$$

(run 2) $\mathcal{L} = 140 \text{ fb}^{-1}$

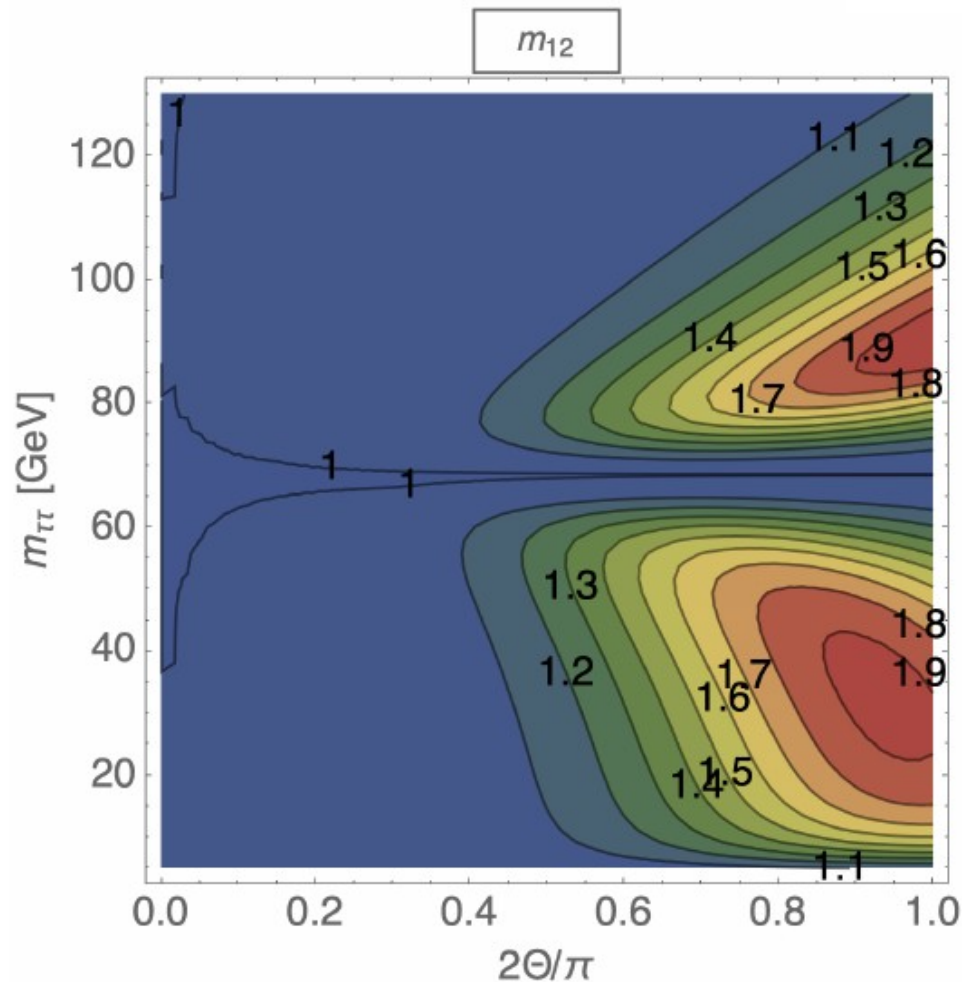
(Hi-Lumi) $\mathcal{L} = 3 \text{ ab}^{-1}$

events

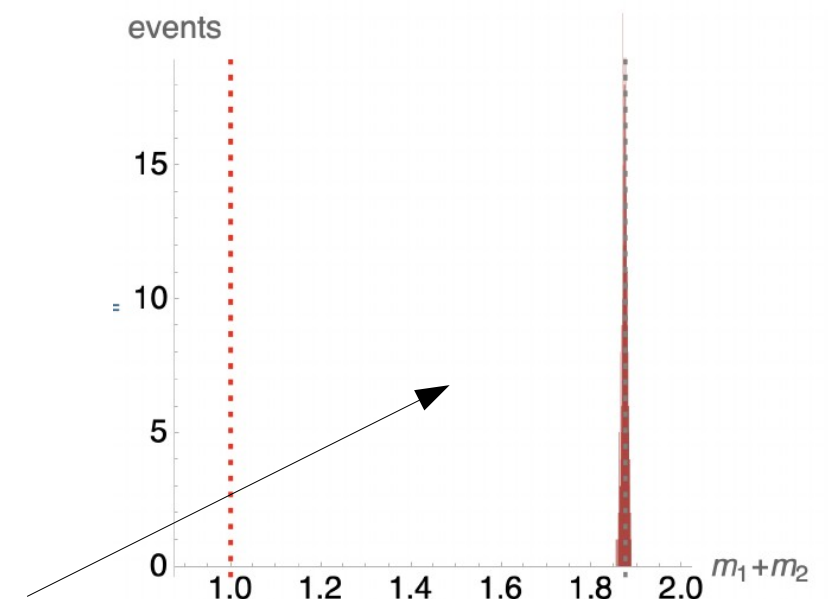
1.1×10^6

2.2×10^7

Fabbrichesi, EG, Floreanini,
EPJC 83 (2023), 2302.00683 [hep-ph]



$$20 < m_{\tau\bar{\tau}} < 45 \text{ GeV and } 2\Theta/\pi > 0.80$$



using toy Montecarlo → only statistical error assumed

Statistical distribution for 100 events around mean value of **1.88** with dispersion given by $\sigma = 0.006$

- Statistical significance to reject Bell's violation null hypothesis is huge → more than 100σ due to the large number of events (opposite than in the top-pair case due to scarcity of events)
- only statistical error assumed: including systematic errors could significantly reduce these expectations

Conclusions

Bell inequality is violated in $B \rightarrow J/\Psi K^*$ at 36σ level !

M. Fabbrichesi, R. Floreanini, EG,
L. Marzola [2305.04982 \[hep-ph\]](#)

- Shown to take place for the first time in **qutrits systems** and involving also **strong and EW interactions** and at high energies (**~ 5 GeV**)
- Locality loophole is closed in $B \rightarrow \Phi \Phi$ (Bell inequality violated at about **6σ level**)
- Detection loophole is also closed for LHCb

The race is on for higher energies !

● **Qubit** $\rightarrow$ tau production at the Z peak at the run2 LHC

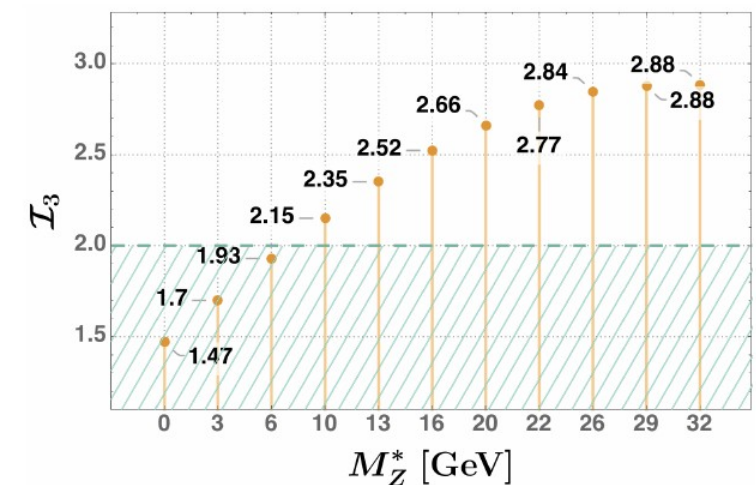
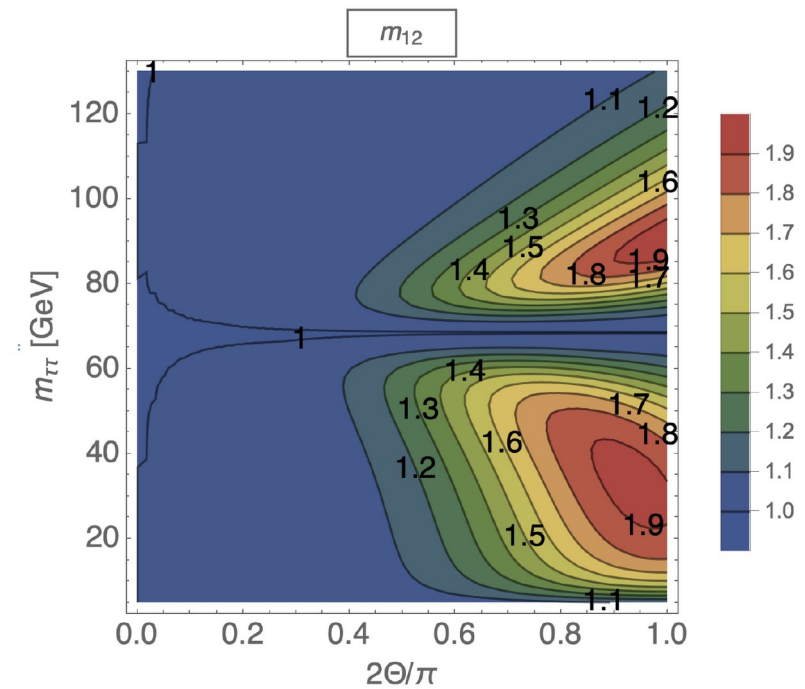
Fabbrichesi, EG, Floreanini, [EPJC 83 \(2023\), 2302.00683 \[hep-ph\]](#)

● **Qutrit** $\rightarrow$ Higgs decay $H \rightarrow ZZ^*$ at Hi-Lumi LHC

Aguilar-Saavedra, Bernal, Casas, Moreno, [2209.13441 \[hep-ph\]](#)

Ashby-Pickering, Barr, Wierzchucka, [2209.13990 \[quant-ph\]](#)

Fabbrichesi, Floreanini, EG, Marzola, [2302.00683 \[hep-ph\]](#)



Backup slides

Extract the density matrix for **Two-Qutrits** from data

Ashby-Pickering, Barr, Wierzhucka, 2209.13990 [quant-ph]

W W

$$p p \rightarrow V_1 + V_2 + X \rightarrow \ell^+ \ell^- + \text{jets} + E_T^{\text{miss}}$$

Differential cross section

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega^+ d\Omega^-} = \left(\frac{3}{4\pi} \right)^2 \text{Tr} \left[\rho_{V_1 V_2} (\Gamma_+ \otimes \Gamma_-) \right]$$

depend on the invariant mass m_{VV} (or velocity β) and **scattering angle** Θ in the $V_1 V_2$ cm frame

Rahaman, Singh, NPB 984 (2022), 2109.09345 [hep-ph]

$$d\Omega^\pm = \sin \theta^\pm d\theta^\pm d\phi^\pm \longrightarrow$$

$\downarrow$ polar angle $\ell^\pm$ $\downarrow$ azimuthal angle $\ell^\pm$

phase space written in terms of the spherical coordinates (with independent polar axis) for the momenta of the final charged leptons in the respective rest frames of the decaying spin-1 particles

$$\rho_{V_1 V_2} = \text{density matrix of } V_1 V_2$$

$$\Gamma_\pm \longrightarrow \text{Density matrices that describe the polarization of the two decaying } W \text{ into final leptons (the charged ones assumed to be massless)}$$

these are projectors in the case of the W-bosons because of their chiral couplings to leptons

can be computed by **rotating** to an arbitrary polar axis the spin states of gauge bosons from the ones given in the **k-direction** quantization axis

$$\Gamma_{\pm} = \frac{1}{3} \mathbb{1} + \sum_{i=1}^8 q_{\pm}^a T^a$$

→ Density matrices for W-bosons

$q_{\pm}^a$
 ↓
 set of polynomials
 of spherical
 coordinates
 (see backup slide)

(Winer q-symbols)

are functions of the corresponding spherical coordinates

$$h_{ab} = \frac{1}{\sigma} \int \int \frac{d\sigma}{d\Omega^+ d\Omega^-} p_+^a p_-^b d\Omega^+ d\Omega^-$$

$$f_a = \frac{1}{\sigma} \int \frac{d\sigma}{d\Omega^+} p_+^a d\Omega^+$$

$$g_a = \frac{1}{\sigma} \int \frac{d\sigma}{d\Omega^-} p_-^a d\Omega^-$$

$p_{\pm}^n$ a particular set of orthogonal functions
 (see next slide)

→

$\left(\frac{3}{4\pi}\right) \int p_{\pm}^n q_{\pm}^m d\Omega^{\pm} = \delta^{nm}$

For **ZZ** case, the set of functions are linear combinations of $q_{\pm}^a$ → see backup slides

Wigner's *Q* symbols

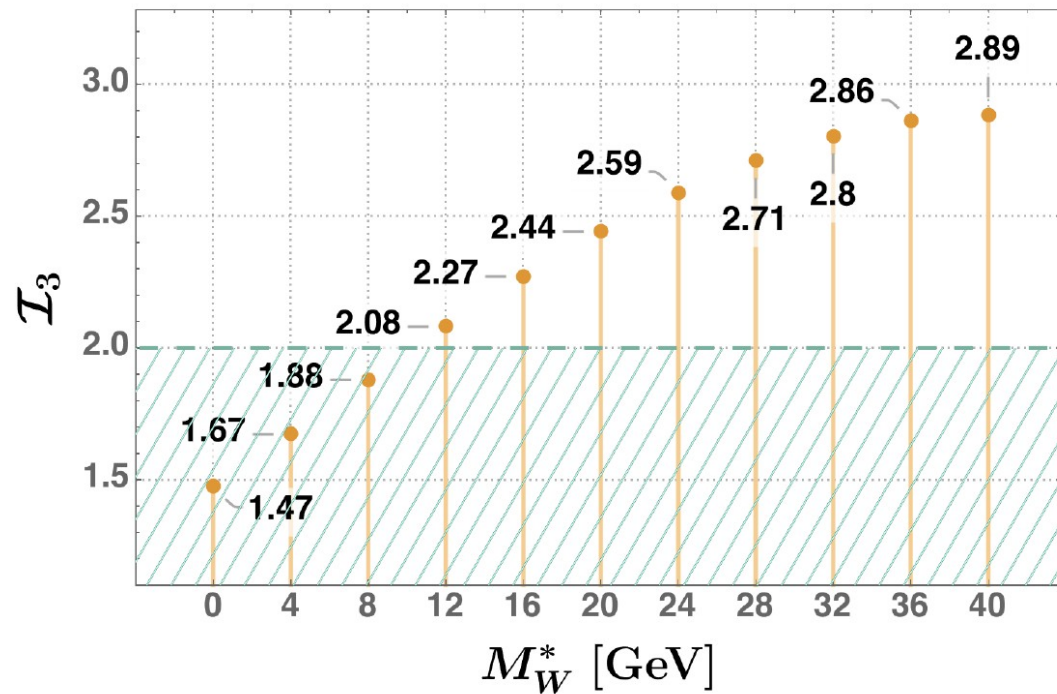
$$\begin{aligned}q_{\pm}^1 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(\cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\q_{\pm}^2 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(\cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\q_{\pm}^3 &= \frac{1}{8} \left(1 \pm 4 \cos \theta^{\pm} + 3 \cos 2\theta^{\pm} \right), \\q_{\pm}^4 &= \frac{1}{2} \sin^2 \theta^{\pm} \cos 2\phi^{\pm}, \\q_{\pm}^5 &= \frac{1}{2} \sin^2 \theta^{\pm} \sin 2\phi^{\pm}, \\q_{\pm}^6 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(-\cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\q_{\pm}^7 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(-\cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\q_{\pm}^8 &= \frac{1}{8\sqrt{3}} \left(-1 \pm 12 \cos \theta^{\pm} - 3 \cos 2\theta^{\pm} \right),\end{aligned}$$

$$\begin{aligned}p_{\pm}^1 &= \sqrt{2} \sin \theta^{\pm} \left(5 \cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\p_{\pm}^2 &= \sqrt{2} \sin \theta^{\pm} \left(5 \cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\p_{\pm}^3 &= \frac{1}{4} \left(5 \pm 4 \cos \theta^{\pm} + 15 \cos 2\theta^{\pm} \right), \\p_{\pm}^4 &= 5 \sin^2 \theta^{\pm} \cos 2\phi^{\pm}, \\p_{\pm}^5 &= 5 \sin^2 \theta^{\pm} \sin 2\phi^{\pm}, \\p_{\pm}^6 &= \sqrt{2} \sin \theta^{\pm} \left(-5 \cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\p_{\pm}^7 &= \sqrt{2} \sin \theta^{\pm} \left(-5 \cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\p_{\pm}^8 &= \frac{1}{4\sqrt{3}} \left(-5 \pm 12 \cos \theta^{\pm} - 15 \cos 2\theta^{\pm} \right).\end{aligned}$$

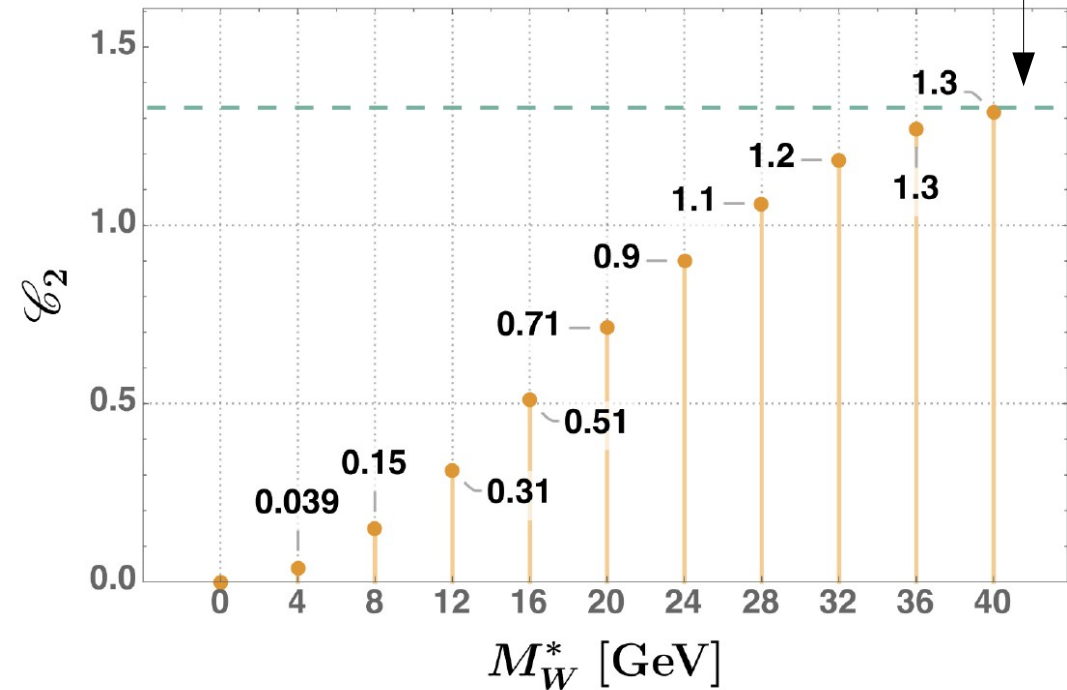
$$H \rightarrow W W^*$$

maximum value 4/3
for a pure state

Bell inequality test



Quantum entanglement (witness)



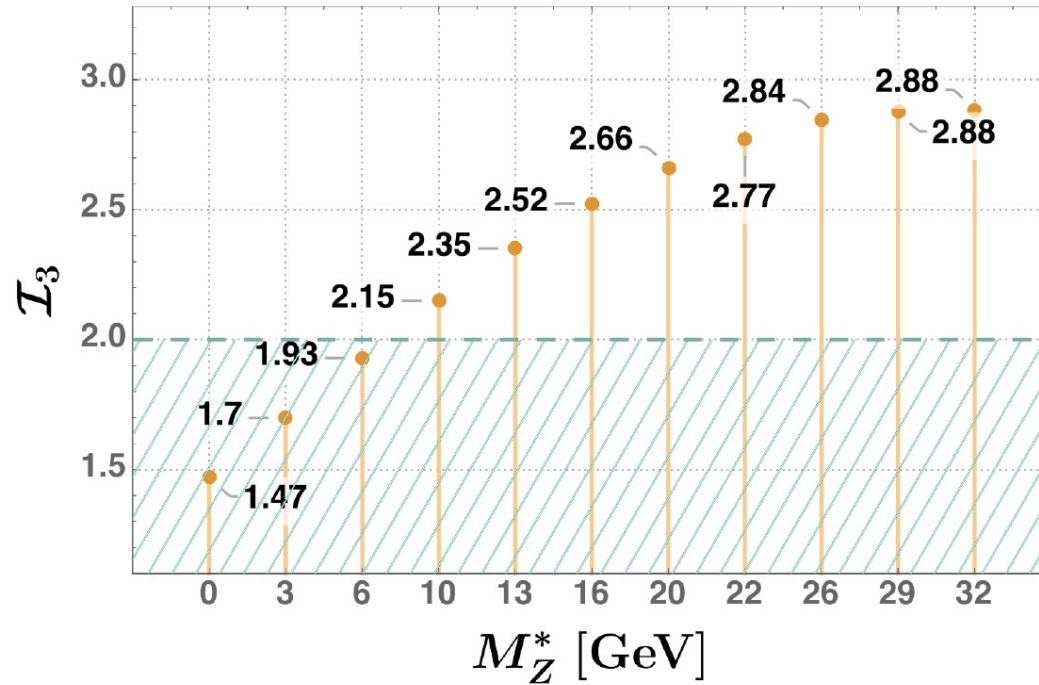
Bell inequality violation condition $\mathcal{I}_3 > 2$

Maximization of $\mathcal{I}_3$ performed point by point as function of M_{W^*}
(see backup slides for solution of “optimized” U,V matrices in the region of max entanglement)

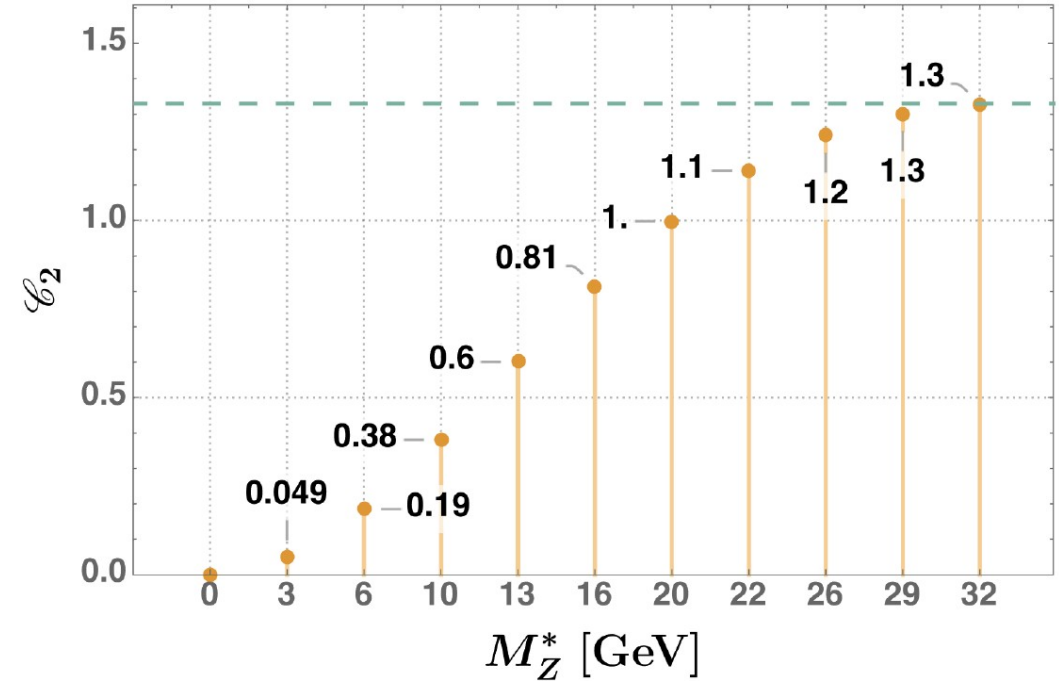
$$\mathcal{C}_2 = \frac{32f^2 M_V^4 \left[m_H^4 - 2(1 + f^2)m_H^2 M_V^2 + (1 + 4f^2 + f^4)M_V^4 \right]}{\left[m_H^4 - 2(1 + f^2)m_H^2 M_V^2 + (1 + 10f^2 + f^4)M_V^4 \right]^2}$$

$$H \rightarrow Z Z^*$$

Bell inequality test



Quantum entanglement (witness)



Maximization of $\mathcal{I}_3$ performed point by point as a function of M_{Z^*}
 (see backup slides for optimized U,V matrices in the region of max entanglement)

Include statistical and systematic errors (include lepton tagging efficiency). For WW systematic error dominates (due to undetected neutrino momentum → about 5 GeV uncertainty on momentum reconstruction)

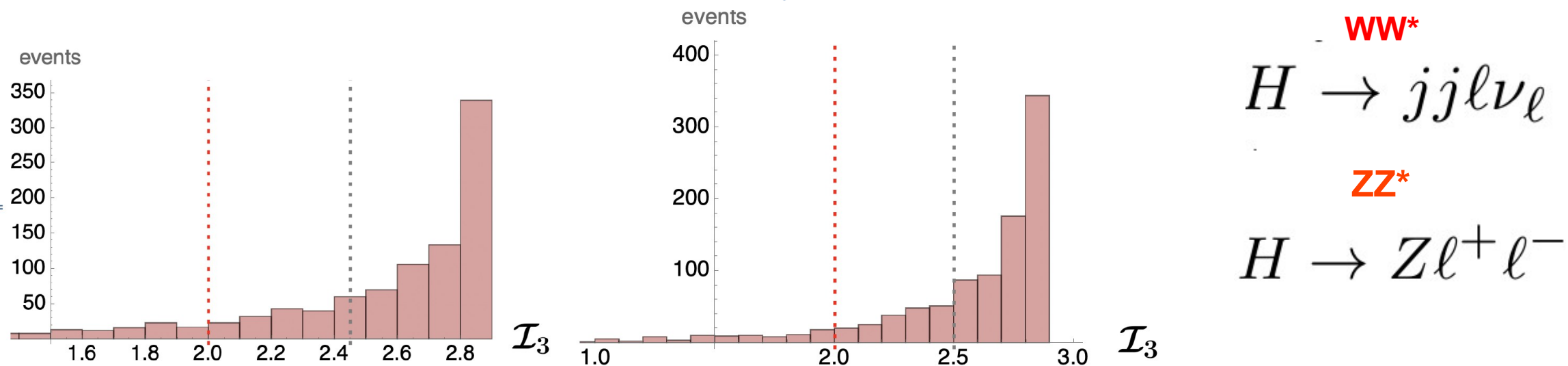


Fig. 6 Distribution of the events at the LHC run 2 for the $H \rightarrow W^+\ell^-\bar{\nu}_\ell$ and $H \rightarrow Z\ell^+\ell^-$ processes. The set of events for WW^* has mean value $\mathcal{I}_3 = 2.4$, that for ZZ^* has mean value $\mathcal{I}_3 = 2.5$. The threshold value of 2 for Bell inequality violation is shown as a dashed red line.

Running 1000 pseudo experiments

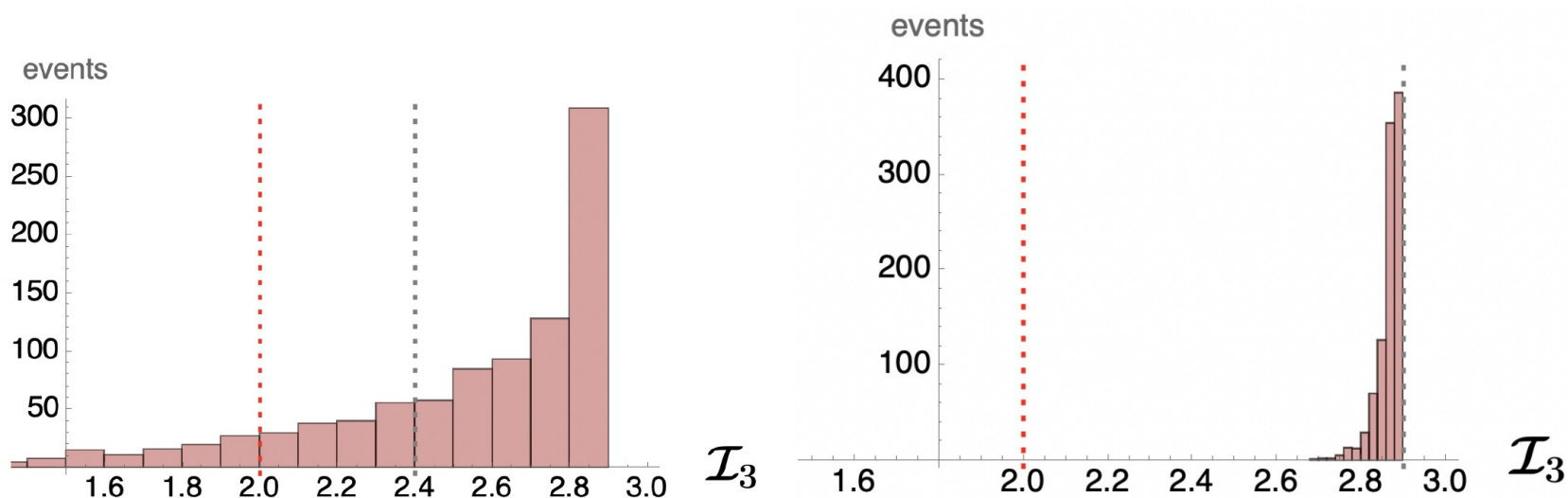


Fig. 7 Distribution of the events at the LHC Hi-Lumi for the $H \rightarrow W^+\ell^-\bar{\nu}_\ell$ and $H \rightarrow Z\ell^+\ell^-$ processes. The set of events for WW^* has mean value $\mathcal{I}_3 = 2.5$, that for ZZ^* has mean value $\mathcal{I}_3 = 2.9$. The threshold value of 2 for Bell inequality violation is shown as a dashed red line.

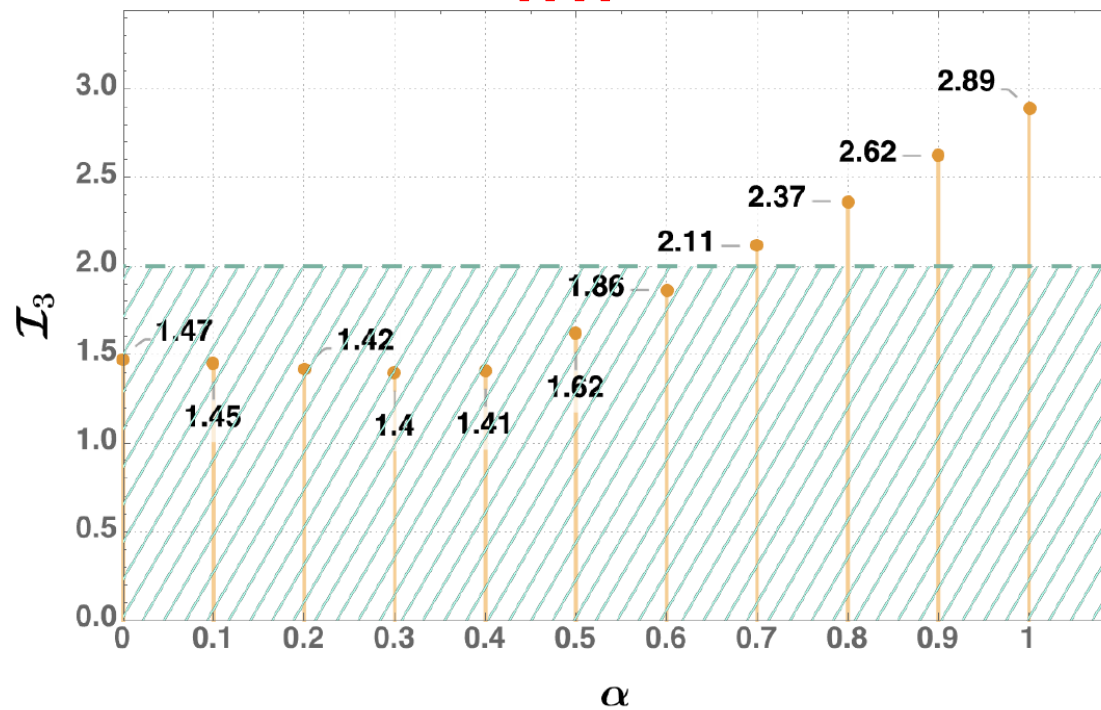
$$\rho_{WW,ZZ} = \alpha \rho_{H \rightarrow WW,ZZ} + (1 - \alpha) \rho_{BCKG}$$

$$H \rightarrow W^+ \ell^- \bar{\nu}_\ell$$

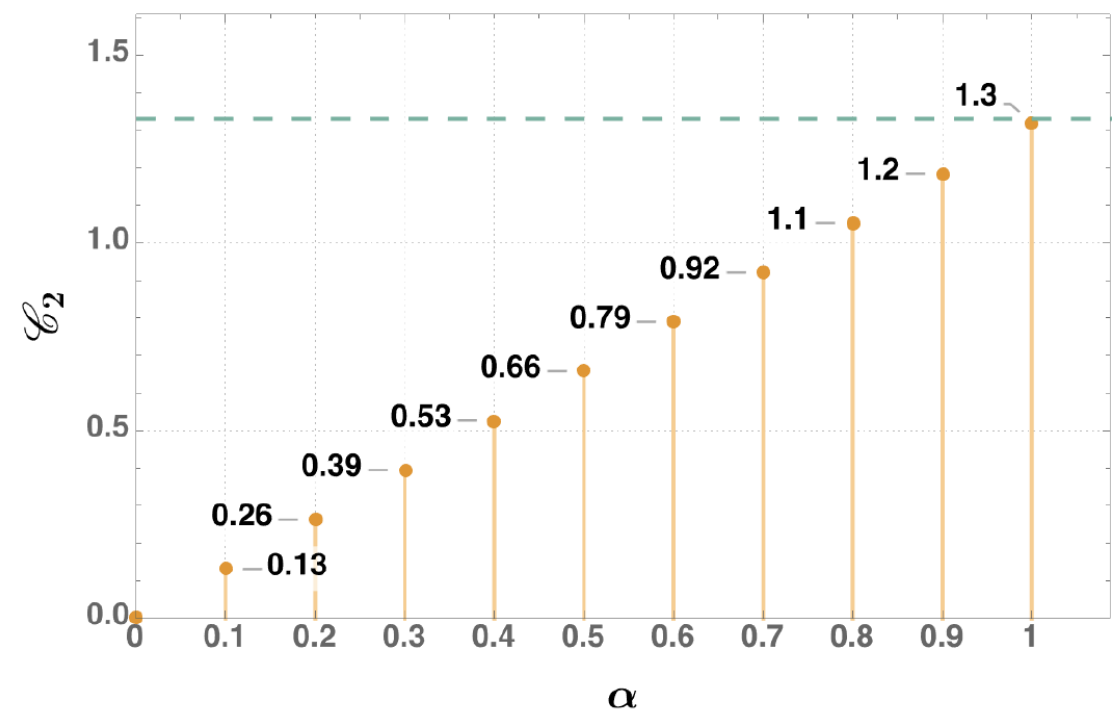
For WW^* dominant bckg is reducible (t tbar and Wt) ρ is factorizable

For ZZ^* irred. bckg dominates and comes from continuum, we included it exactly

WW*



ZZ*



Below $\alpha < 0.7$ the mixing of bckg decreases the entanglement beyond possibility of assessing Bell inequality violation