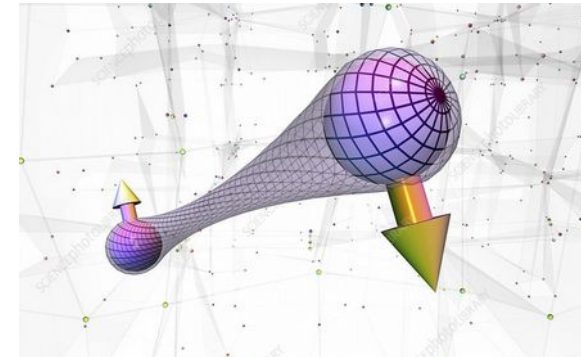


Quantum Entanglement and Bell Inequalities tests at LHC and future colliders

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based on: 2305.04982 [hep-ph]; 2304.02403 [hep-ph]; 2302.00683 [hep-ph];
EPJC 83 (2023) 2, 162, arXiv: 2208.11723 [hep-ph]

Outline

- Polarization **density** matrix for **Qubits** (fermions pair) and **Qutrits** (di-boson) spin systems
- How to extract density matrix from data (Qubits and Qutrits systems)
- Quantum tomography to test **entanglement** and **Bell violation observables** at the LHC:
 - ▶ $p p \rightarrow \text{top anti-top}$
 - ▶ $p p \rightarrow \text{tau}(+) \text{ tau}(-)$
 - ▶ Higgs decays: $H \rightarrow WW^*, ZZ^*$
 - ▶ Drell-Yan processes $q q' \rightarrow WW, ZZ, WZ$
 - ▶ $B \rightarrow J/\Psi K^*$ **NEW** → Bell's inequality is violated

● Conclusions

Quantum tomography of two particles of same spin ($\frac{1}{2}$, 1)

- Requires the knowledge of the **polarization density matrix** of two-particles **A,B** production
- it can be fully reconstructed from the angular distributions of the single **A,B decay products**
- **but it can also be computed analytically**
- so far experimental analyses have been focused on the spin correlations (top-quark pairs, di-boson)
- not the same thing than measuring the whole density matrix
- main targets → top-quark pair, tau-lepton pair and di-boson production
- knowledge of the full **polarization density matrix** allows to study many interesting phenomena

- ▶ Quantum Entanglement
- ▶ Violation of Bell's inequalities
- ▶ Sensitivity to New Physics

Polarization density matrix of **two-Qubit** systems

Consider production of **two spin-1/2** particles (top-quark pair)

$$\bar{q}(p_1) q(p_2) \rightarrow t(k_1, \lambda_1) \bar{t}(k_2, \lambda_2)$$

discrete degrees of freedom $\rightarrow$ **Qubit**

$\downarrow$ spin eigenvalues

$$\lambda_i = \pm \frac{1}{2}, \quad i = 1, 2$$

● right-handed basis

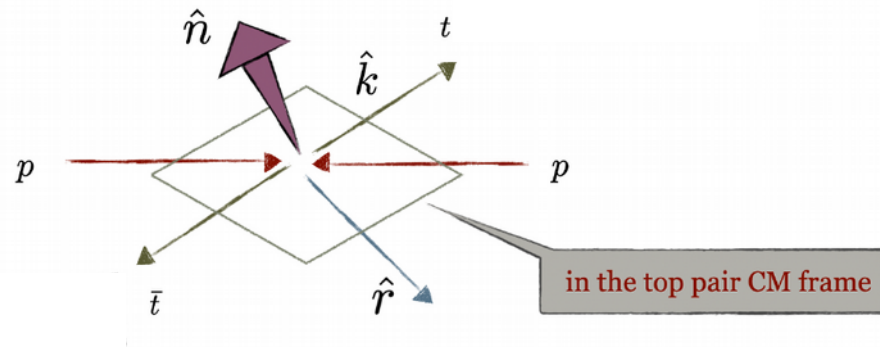
$$\{\hat{n}, \hat{r}, \hat{k}\} \quad \hat{n} = \hat{r} \times \hat{k}$$

$\hat{k}$ = axis of spin-quantization for particle at rest



● then **boost** to the c.m. frame
top anti-top system along $\hat{k}$
 $\rightarrow$ **helicity basis**

Spin defined on the corresponding top or anti-top rest frame



Quantum amplitude $\mathcal{M}(\lambda_1, \lambda_2)$

Polarization density matrix is defined as

Qubit
4x4 matrix

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \frac{\mathcal{M}(\lambda_1, \lambda_2) \mathcal{M}^\dagger(\lambda'_1, \lambda'_2)}{|\overline{\mathcal{M}}|^2}$$

$$\Rightarrow \text{Tr}[\rho] = 1$$

ρ depends only on scattering angle in c.m. frame and t-tbar invariant mass

Unpolarized square amplitude

Two-Qubit systems

Useful to decompose polarization density matrix along the Pauli-matrices σ_i basis

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \frac{1}{4} \left[\mathbb{1} \otimes \mathbb{1} + \sum_i B_i^+ (\sigma_i \otimes \mathbb{1}) + \sum_j B_j^- (\mathbb{1} \otimes \sigma_j) + \sum_{ij} C_{ij} (\sigma_i \otimes \sigma_j) \right]_{\lambda_1 \lambda'_1, \lambda_2 \lambda'_2}$$

Tensorial product $[A \otimes B]_{ii'jj'} = A_{ii'} B_{jj'}$

$$B_i^+ = \text{Tr}[\rho(\sigma_i \otimes \mathbb{1})] \quad B_j^- = \text{Tr}[\rho(\mathbb{1} \otimes \sigma_j)] \quad \Rightarrow \quad \text{polarization coefficients of the two-qubits}$$

$$C_{ij} = \text{Tr}[\rho(\sigma_i \otimes \sigma_j)] \quad \Rightarrow \quad \text{spin-correlation of two-qubit}$$

How to experimentally access the polarization density matrix for Qubits

Consider pp production of top anti-top system (helicity base)

right-handed basis

$$\{\hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}\} \quad \hat{\mathbf{n}} = \hat{\mathbf{r}} \times \hat{\mathbf{k}}$$

spin-quantization axis of top

lepton(+) momenta
direction in top rest frame

Look at

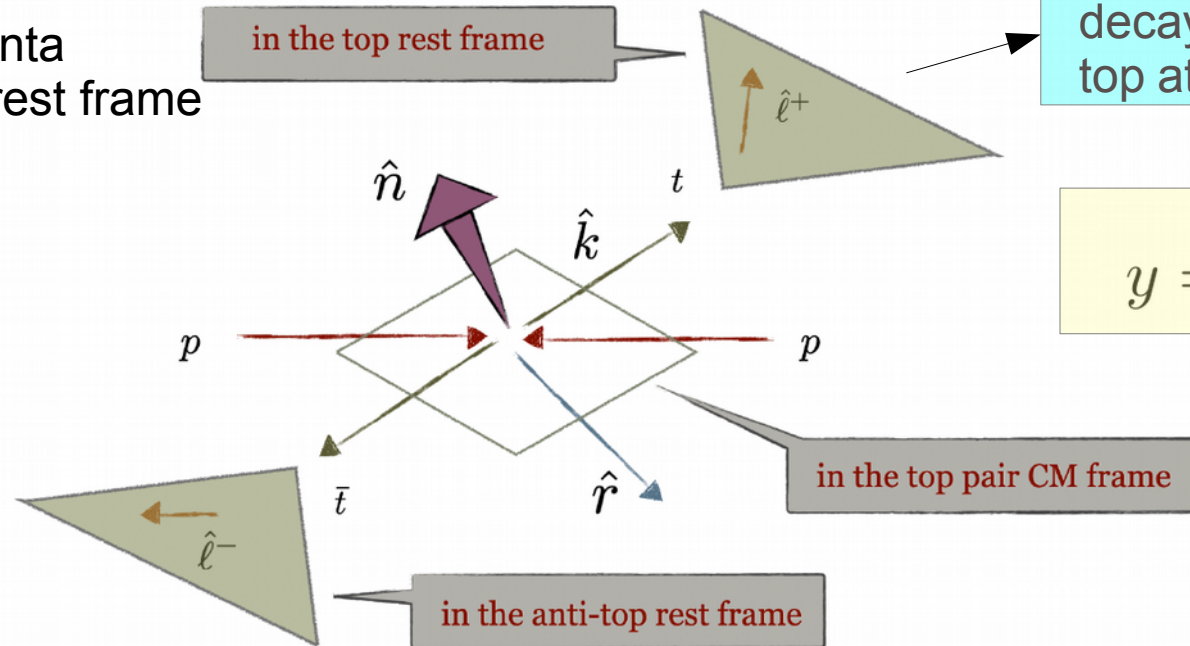
$$\cos \theta_+^a = \hat{\ell}_+ \cdot \hat{\mathbf{a}}$$

$$\cos \theta_-^b = \hat{\ell}_- \cdot \hat{\mathbf{b}}$$

a and $b \in \{k, n, r\}$

$$\hat{p} = (0, 0, 1), \quad \hat{r} = \frac{1}{r}(\hat{p} - y\hat{k}), \quad \hat{n} = \frac{1}{r}(\hat{p} \times \hat{k})$$

$$\xi_{ab} = \cos \theta_+^a \cos \theta_-^b$$



decay plane of
top at rest

$$y = \hat{p} \cdot \hat{k} = \cos \Theta$$

top scattering-angle
in c.m. top anti-top

For example for specific axes

$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_+ d \cos \theta_-} = \frac{1}{4} \left(1 + B_1 \cos \theta_+ + B_2 \cos \theta_- - C \cos \theta_+ \cos \theta_- \right)$$

$$C_{ab} [\sigma(m_{t\bar{t}}, \cos \Theta)] = -9 \frac{1}{\sigma} \int d\xi_{ab} \frac{d\sigma}{d\xi_{ab}} \xi_{ab}$$

average of ξ_{ab}

Polarization density matrix of two spin-1 $V_1 V_2$

$$\bar{q}(p_1) q(p_2) \rightarrow V_1(k_1, \lambda_1) V_2(k_2, \lambda_2)$$

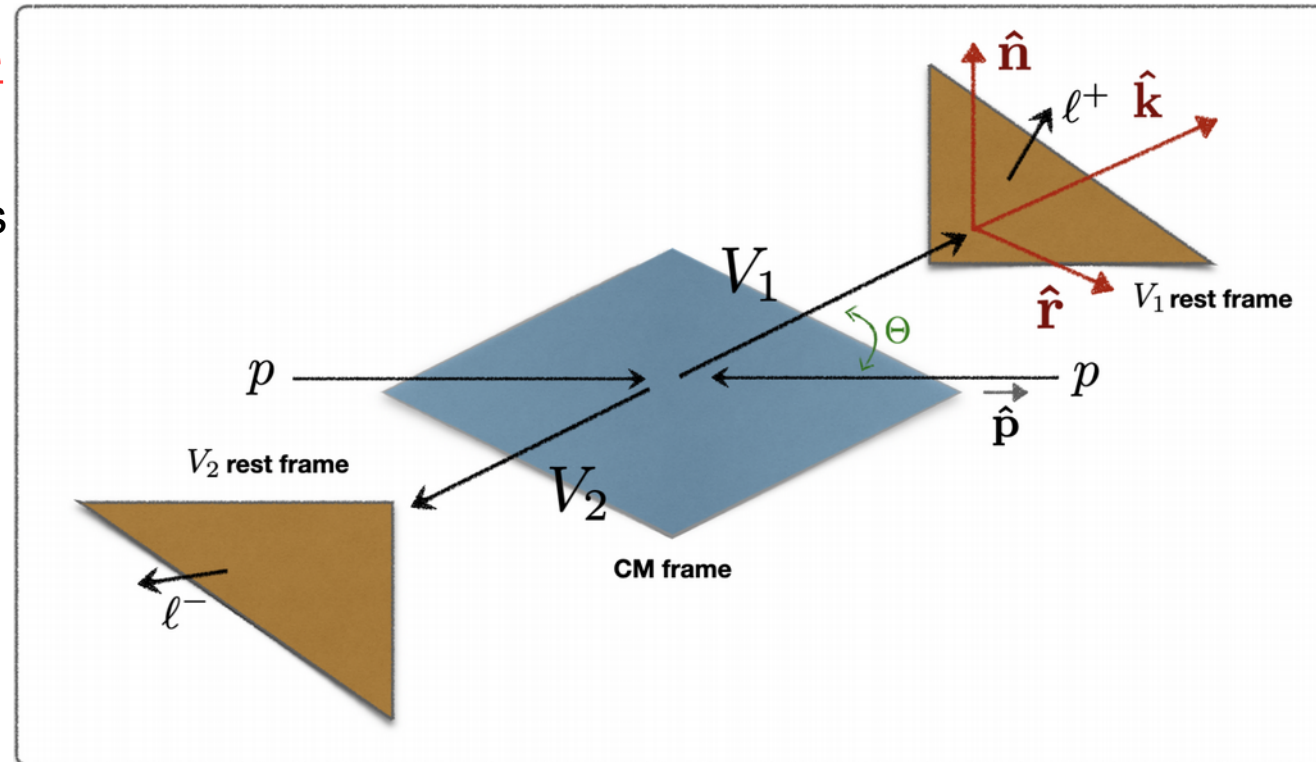
Spin eigenstates embedded in covariant polarization vectors

in the center of mass frame

$\{\hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}\}$ right-handed basis

same procedure as for Qubits
but for spin-1 $\rightarrow$ **3 polarizations**
 $\rightarrow$ **Qutrits**

more complicated !



$$n_m^\mu(1)$$

$$n_m^\mu(2)$$

$m = 1, 2, 3$

$$\hat{\mathbf{r}} = \frac{1}{\sin \Theta} (\hat{\mathbf{p}} - \cos \Theta \hat{\mathbf{k}}), \quad \hat{\mathbf{n}} = \frac{1}{\sin \Theta} (\hat{\mathbf{p}} \times \hat{\mathbf{k}})$$

$\nwarrow$ **scattering angle**

quarks assumed massless

Boosted (n,r,k) basis in c.m. frame $\rightarrow$ **n(1)** and **n(2)** stands for particle V_1 and V_2

$$\begin{aligned} n_1^\mu(1) &= n_1^\mu(2) = (0, \hat{\mathbf{n}}), & n_2^\mu(1) &= n_2^\mu(2) = (0, \hat{\mathbf{r}}) \\ n_3^\mu(1) &= \gamma(\beta, \hat{\mathbf{k}}), & n_3^\mu(2) &= \gamma(-\beta, \hat{\mathbf{k}}), \end{aligned}$$

β = velocity in c.m. frame

- **Matrix element of two-Vector Boson production** $\bar{q}(p_1) q(p_2) \rightarrow V_1(k_1, \lambda_1) V_2(k_2, \lambda_2)$

$$\mathcal{M}(\lambda_1, \lambda_2) = \mathcal{M}_{\mu\nu} \varepsilon^{\mu*}(k_1, \lambda_1) \varepsilon^{\nu*}(k_2, \lambda_2)$$

Density matrix

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) =$$

9 x 9 matrix

covariant expression

$$\frac{\mathcal{M}_{\mu\nu} \mathcal{M}_{\mu'\nu'}^\dagger \mathcal{P}_{\lambda_1\lambda'_1}^{\mu\mu'}(k_1) \mathcal{P}_{\lambda_2\lambda'_2}^{\nu\nu'}(k_2)}{|\overline{\mathcal{M}}|^2}$$

↓ unpolarized matrix element square

$$\mathcal{P}_{\lambda\lambda'}^{\mu\nu}(p) = \varepsilon^\mu(p, \lambda)^* \varepsilon^\nu(p, \lambda')$$

(spin-1 eigenstates **at rest**)

k=axis of spin-quantization $\longrightarrow \psi_{\pm} = -\frac{1}{\sqrt{2}} (\pm \hat{\mathbf{n}} + i \hat{\mathbf{r}}) \quad \text{and} \quad \psi_0 = \hat{\mathbf{k}}$

- How to related spin-1 eigenstates at rest to the boosted frame ? (see next slide)

covariant polarization vector of spin-1

rest frame limit

$$\varepsilon^\mu(p, \lambda) \xrightarrow{(\beta \rightarrow 0)} \psi_\pm, \psi_0$$

helicity $\lambda = \pm 1, 0$

β is the velocity

boosted base

$$n_1^\mu = (0, \hat{\mathbf{n}}), \quad n_2^\mu = (0, \hat{\mathbf{r}}), \quad n_3^\mu = \frac{E}{M}(\beta, \hat{\mathbf{k}})$$

Covariant Projector

$$\mathcal{P}_{\lambda\lambda'}^{\mu\nu}(p) = \varepsilon^\mu(p, \lambda)^* \varepsilon^\nu(p, \lambda')$$

master formula

$$\varepsilon^{0123} = 1$$

$$= \frac{1}{3} \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2} \right) \delta_{\lambda\lambda'} - \frac{i}{2M} \epsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta^i (S_i)_{\lambda\lambda'} - \frac{1}{2} n_i^\mu n_j^\nu (S_{ij})_{\lambda\lambda'}$$

$S_i, i \in \{1, 2, 3\} \rightarrow$ rotation matrices for spin-1 particle

H.S. Song, *Lett. Nuovo Cim.* 25 (1979)
S.Y. Choi, T. Lee, H.S. Song, *PRD* 40 (1989)
Fabbrichesi, Floreanini, EG, Marzola,
2302.00683 [hep-ph]

$$S_{ij} = S_i S_j + S_j S_i - \frac{4}{3} \mathbb{1} \delta_{ij} \quad (\text{see backup slides})$$

basis correspondence for $(S_i)_{\lambda\lambda'}$ $|+\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad |-\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ corresponding to eigenevalues $\lambda = \pm 1, 0$

Two-QUTRITS

(WW, ZZ, WZ)

useful to decompose the **density matrix** on the basis of tensor products of **Gell-Mann matrices**

$$\{\mathbb{1} \otimes \mathbb{1}, \mathbb{1} \otimes T^a, T^a \otimes \mathbb{1}, T^a \otimes T^b\} \quad T^a \rightarrow 3 \times 3 \text{ Gell-Mann matrices}$$

given by the Kronecker product of the matrix representations $[A \otimes B]_{ii'jj'} = A_{ii'} B_{jj'}$

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \left(\frac{1}{9} [\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [\mathbb{1} \otimes T^a] + \sum_a g_a [T^a \otimes \mathbb{1}] + \sum_{ab} h_{ab} [T^a \otimes T^b] \right)_{\lambda_1 \lambda'_1, \lambda_2 \lambda'_2}$$

9x9 matrix

$$f_a = \frac{1}{6} \text{Tr} [\rho (\mathbb{1} \otimes T^a)] , \quad g_a = \frac{1}{6} \text{Tr} [\rho (T^a \otimes \mathbb{1})] , \quad h_{ab} = \frac{1}{4} \text{Tr} [\rho (T^a \otimes T^b)]$$

● As for qubits, these coefficients are scalar quantities that depend on

VV' invariant mass and **scattering angle** in c.m. frame Θ

● we can also **extract them from data**, using the decay products of final VB (see next slides)

Extract the density matrix for **Two-Qutrits** from data

Ashby-Pickering, Barr, Wierchucka, 2209.13990 [quant-ph]

W W

$$p p \rightarrow V_1 + V_2 + X \rightarrow \ell^+ \ell^- + \text{jets} + E_T^{\text{miss}}$$

Differential cross section

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega^+ d\Omega^-} = \left(\frac{3}{4\pi} \right)^2 \text{Tr} \left[\rho_{V_1 V_2} (\Gamma_+ \otimes \Gamma_-) \right]$$

depend on the invariant mass m_{VV} (or velocity β) and **scattering angle** Θ in the $V_1 V_2$ cm frame

Rahaman, Singh, NPB 984 (2022), 2109.09345 [hep-ph]

$$d\Omega^\pm = \sin \theta^\pm d\theta^\pm d\phi^\pm \longrightarrow$$

$\downarrow$ polar angle $\ell^\pm$ $\downarrow$ azimuthal angle $\ell^\pm$

phase space written in terms of the spherical coordinates (with independent polar axis) for the momenta of the final charged leptons in the respective rest frames of the decaying spin-1 particles

$$\rho_{V_1 V_2} = \text{density matrix of } V_1 V_2$$

$$\Gamma_\pm \longrightarrow \text{Density matrices that describe the polarization of the two decaying } W \text{ into final leptons (the charged ones assumed to be massless)}$$

these are projectors in the case of the W-bosons because of their chiral couplings to leptons

can be computed by **rotating** to an arbitrary polar axis the spin states of gauge bosons from the ones given in the **k-direction** quantization axis

$$\Gamma_{\pm} = \frac{1}{3} \mathbb{1} + \sum_{i=1}^8 q_{\pm}^a T^a$$

→ Density matrices for W-bosons

$q_{\pm}^a$
 ↓
 set of polynomials
 of spherical
 coordinates
 (see backup slide)

(Winer q-symbols)

are functions of the corresponding spherical coordinates

$$h_{ab} = \frac{1}{\sigma} \int \int \frac{d\sigma}{d\Omega^+ d\Omega^-} p_+^a p_-^b d\Omega^+ d\Omega^-$$

$$f_a = \frac{1}{\sigma} \int \frac{d\sigma}{d\Omega^+} p_+^a d\Omega^+$$

$$g_a = \frac{1}{\sigma} \int \frac{d\sigma}{d\Omega^-} p_-^a d\Omega^-$$

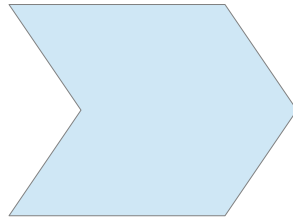
$p_{\pm}^n$ a particular set of orthogonal functions
 (see next slide)

→

$\left(\frac{3}{4\pi}\right) \int p_{\pm}^n q_{\pm}^m d\Omega^{\pm} = \delta^{nm}$

For **ZZ** case, the set of functions are linear combinations of $q_{\pm}^a$ → see backup slides

Quantum Entanglement and Bell's inequality violations in Qubits and Qutrits systems



Quantifying entanglement

difficult task → simplifies in the case of pure states

✓ Pure states (or pure ensables): the system is composed only by **one** quantum state $|\psi\rangle$

Density matrix $\rho = |\psi\rangle\langle\psi|$ for pure states

✓ in real life we do not have pure states → but **mixture of states** $|\psi_n\rangle_{n=1,2,\dots}$

$$\rho = \sum_n p_n |\psi_n\rangle\langle\psi_n|, \quad \sum_n p_n = 1 \quad \Rightarrow \quad \text{Tr}[\rho] = 1$$

$\langle A \rangle = \text{Tr}[\rho A]$ → expectation value of operators (A) → can be computed in any base since Trace is base independent

✓ For pure states

$$\rho \doteq \begin{pmatrix} 0 & & & & & & 0 \\ & 0 & & & & & \\ & & \ddots & & & & \\ & & & 0 & & & \\ & & & & 1 & & \\ & & & & & 0 & \\ & & & & & & 0 \\ & & & & & & & 0 \\ & & & & & & & \ddots & \\ & & & & & & & & 0 \end{pmatrix}$$

(diagonal form)

$$\rho^2 = \rho$$

$$\text{Tr}[\rho^2] = 1$$

- A suitable observable (**concurrence**) to quantify entanglement in a bi-partite (A,B) system for a pure state $|\Psi\rangle$ (with matrix density $\rho = |\Psi\rangle\langle\Psi|$) is defined as

concurrence

$$\mathcal{C}[|\Psi\rangle] = \sqrt{1 - \text{Tr}[(\rho_r)^2]} \quad r = A \text{ or } B$$

Rungta, Buzek, Caves, Hillery, Milburn, PRA 64 (2001)

$$\rho_A = \text{Tr}_B[|\Psi\rangle\langle\Psi|] \text{ and similar for } \rho_B$$

Trace performed in the subsystem B

vanishes for separable states $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$

pure states

- For **mixed states** with matrix density $\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$, $p_i \geq 0$, $\sum_i p_i = 1$

$$\mathcal{C}[\rho] = \inf_{\{|\Psi_i\rangle\}} \sum_i p_i \mathcal{C}[|\Psi_i\rangle]$$

infimum taken over all possible decompositions in pure states

$\mathcal{C}[\rho]$ vanishes for separable states

Concurrence for Two-QUBITS

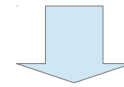
● For **bi-partite systems A,B of qubit** the concurrence $\mathcal{C}[\rho]$ can be analytically computed

● built from the matrix $\rightarrow$

$$R = \rho (\sigma_2 \otimes \sigma_2) \rho^* (\sigma_2 \otimes \sigma_2) \quad (\text{not hermitean})$$

has non-negative eigenvalues

$$\mathcal{C}[\rho] = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$



vanishes for separable states, **max value = 1**

$\lambda_i \rightarrow$ square root of R eigenvalues, $i=1,2,3,4$ with λ_1 the largest one

Quantifying entanglement for Two-Qutrits

- Difficult to compute the concurrence $\mathcal{C}[\rho]$

- analytical solution exists for the lower bound

$$(\mathcal{C}[\rho])^2 \geq \mathcal{C}_2[\rho]$$

Witness of Entanglement

Mintert, Buchleitner,
PRL 98 (2007)

$$\mathcal{C}_2[\rho] = 2 \max \left(0, \text{Tr} [\rho^2] - \text{Tr} [(\rho_A)^2], \text{Tr} [\rho^2] - \text{Tr} [(\rho_B)^2] \right)$$

- If non-vanishing unequivocally signal the presence of entanglement
- used in our analysis of entanglement for WW, ZZ and WZ productions
- Solution for upper bound

$$(\mathcal{C}[\rho])^2 \leq 2 \min \left(1 - \text{Tr} [(\rho_A)^2], 1 - \text{Tr} [(\rho_B)^2] \right)$$

- maximum value of Concurrence can be obtained for the maximum symmetric state

for two-qutrits $\longrightarrow \mathcal{C}[|\Psi_+\rangle] = 2/\sqrt{3}$ corresponding to the state $|\Psi_+\rangle = \frac{1}{\sqrt{3}} \sum_{i=1}^3 |i\rangle \otimes |i\rangle$

Bell's inequalities

spin - $\frac{1}{2}$ systems

(qubit)

$$H = H_1 \otimes H_2$$

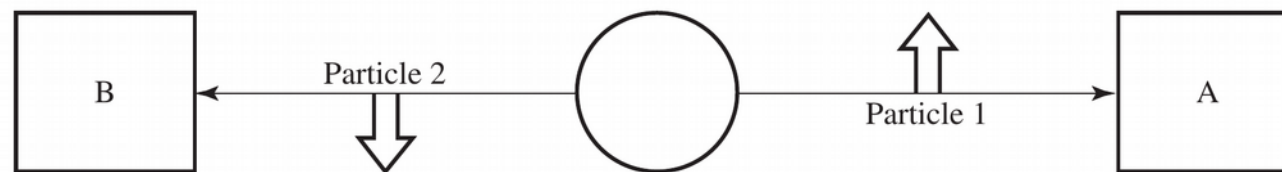
$$\dim H_1 = \dim H_2 = 2$$

example

$$H \rightarrow \mu^+ + \mu^-$$

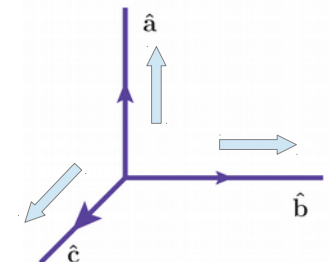
total spin of $\mu^+ \mu^-$ is zero

- measurement of spin in particle 1 induces correlation on spin measurement of particle 2
- measuring spin correlations of particles 1 and 2 along same directions just test property of angular momentum conservation → QM gives same expectations as in classical theory



- to check departure from deterministic nature of reality → require to perform correlated measurements of spin-projection in different directions (where spin operators $\mathbf{S}$ do not commute)

$$\{\hat{a}, \hat{b}, \hat{c}\} \Rightarrow [\mathbf{S}_{\hat{a}}, \mathbf{S}_{\hat{b}}] \neq 0 \quad [\mathbf{S}_{\hat{a}}, \mathbf{S}_{\hat{c}}] \neq 0 \quad [\mathbf{S}_{\hat{b}}, \mathbf{S}_{\hat{c}}] \neq 0$$



Classical theory predictions (**Bell's inequality**)

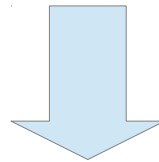
J. Bell, Physics 1, 195-200 (1964)

for correlated probabilities P

$$P(\hat{\mathbf{a}}+; \hat{\mathbf{b}}+) \leq P(\hat{\mathbf{a}}+; \hat{\mathbf{c}}+) + P(\hat{\mathbf{c}}+; \hat{\mathbf{b}}+)$$

spin+ along **a** for particle 1

spin+ along **b** for particle 1



Entangled states can violates Bell's inequality !

- maximization of the Bell's inequality violation is an optimization problem
 - ▶ require a suitable choice of the axes where measure the spins of the bipartite system
 - ▶ **optimization problem** solved **for Two-Qubits** systems (see next slides)

Bell's inequality

for Two-Qubits

correlation matrix 3x3

unit 3-dim vectors $\hat{n}_{1,2,3,4}$

which correspond to the choices that Bob and Alice do for the axis where to measure the spin

$$\left| \hat{n}_1 \cdot C \cdot (\hat{n}_2 - \hat{n}_4) + \hat{n}_3 \cdot C \cdot (\hat{n}_2 + \hat{n}_4) \right| \leq 2$$

No need of suitable choice of $\hat{n}_{1,2,3,4}$ to maximize the Bell violation

maximization process optimized for a generic correlation matrix C R. Horodecki *et al*, Phys. Lett. A200 (1995)

observable

$$M = C^T C$$

$$\mathfrak{m}_{12}[C] \equiv m_1 + m_2$$

the largest two eigenvalues of M

$$m_1 \geq m_2 \geq m_3$$

two-qubit state violates Bell's inequality

IF AND ONLY IF

$$\mathfrak{m}_{12}[C] > 1$$

independently of $\hat{n}_{1,2,3,4}$

$$C = \begin{pmatrix} C_{nn} & C_{nr} & C_{nk} \\ C_{rn} & C_{rr} & C_{rk} \\ C_{kn} & C_{kr} & C_{kk} \end{pmatrix}$$

$$\text{C-eigenvalues} = \left\{ C_{nn}^2, \frac{1}{4} \left[C_{kk} + C_{rr} + \sqrt{(C_{kk}^2 - C_{rr})^2 + 4C_{kr}^2} \right]^2, \frac{1}{4} \left[C_{kk} + C_{rr} - \sqrt{(C_{kk}^2 - C_{rr})^2 + 4C_{kr}^2} \right]^2 \right\}$$

Bell's inequality

for Two-Qutrits

- consider the following correlator $\mathcal{I}_3$ for probability measurements

Collins, Gisin, Linden,
Massar, Popescu, PRL 88 (2002)

$$\mathcal{I}_3 = \text{Tr} [\rho \mathcal{B}]$$

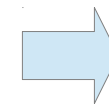
with $\mathcal{B}$ a suitable Bell operator

Optimized Bell's inequality for qutrits (CGLMP)

For deterministic local models

CGLMP

$$\mathcal{I}_3 \leq 2$$



QM qutrits can violate this inequality

with upper bound $(\mathcal{I}_3)^{\max} = 4$

- For the case of **maximally entangled** state $\rho = |\Psi_+\rangle\langle\Psi_+|$ optimal choice of measurements has been found $\rightarrow$ giving a specific form of $\mathcal{B}$

Acin, Durt, Gisin, Latorre,
PRA 65 (2002), quant-ph/0111143

- optimization of the Bell violation not found in general case (contrary to the case of two-qubit)
- still freedom to modify measured observables through **unitary transformations** U, V on $\mathcal{B}$

$$\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$



U, V unitary 3x3 matrices
depending on the kinematic
of the process

in order to **maximize** the **violation** of CGLMP **Bell's inequality** for two-qutrits

Testing entanglement and Bell's violations at colliders

(recent activity → starting from 2021)

+ top-quark pair production

SM →

- Afik, de Nova, Euro Phys. J Plus 136 (2021) 2003.02280 [quant-ph]
Fabbrichesi, Floreanini, Panizzo, PRL 127 (2021), 2102.11883 [hep-ph]
Severi, Boschi, Maltoni, Sioli, EPJC 82 (2022), 2110.10112 [hep-ph]
Afik, de Nova, Quantum 6 (2022), 2203.05582 [quant-ph]
Aguilar-Saavedra, Casas, EPJC 82 (2022), 2205.00542 [hep-ph]

New Physics →

- Fabbrichesi, EG, Floreanini, EPJC 83 (2023), 2302.00683 [hep-ph]
Aoude, Madge, Maltoni, Mantani, PRD 106 (2022), 2203.05619 [hep-ph]
Severi, Vryonidou, JHEP 01 (2023), 2210.09339 [hep-ph]

+ tau-pair production (Drell-Yan)

- Fabbrichesi, EG, Floreanini, EPJC 83 (2023), 2302.00683 [hep-ph]

+ Higgs boson → weak-boson pairs

- Alan Barr, PLB 285 (2022), 2106.01377 [hep-ph]
Barr, Caban, Rembielinski, 2204.11063 [hep-ph]
Aguilar-Saavedra, Bernal, Casas, Moreno, 2209.13441 [hep-ph]
Aguilar-Saavedra, 2209.14033 [hep-ph]
Fabbrichesi, Floreanini, EG, Marzola, 2302.00683 [hep-ph]

H → tau pair, two photons

- Fabbrichesi, Gabrielli, Floreanini, EPJC 83 (2023), 2208.11723 [hep-ph]
Altakach, Lambda, Maltoni, Mawatari, Sakurai, 2211.10513 [hep-ph]

+ WW, ZZ, WZ (Drell-Yan)

- Ashby-Pickering, Barr, Wierchucka, 2209.13990 [quant-ph]
Fabbrichesi, Floreanini, EG, Marzola, 2302.00683 [hep-ph]

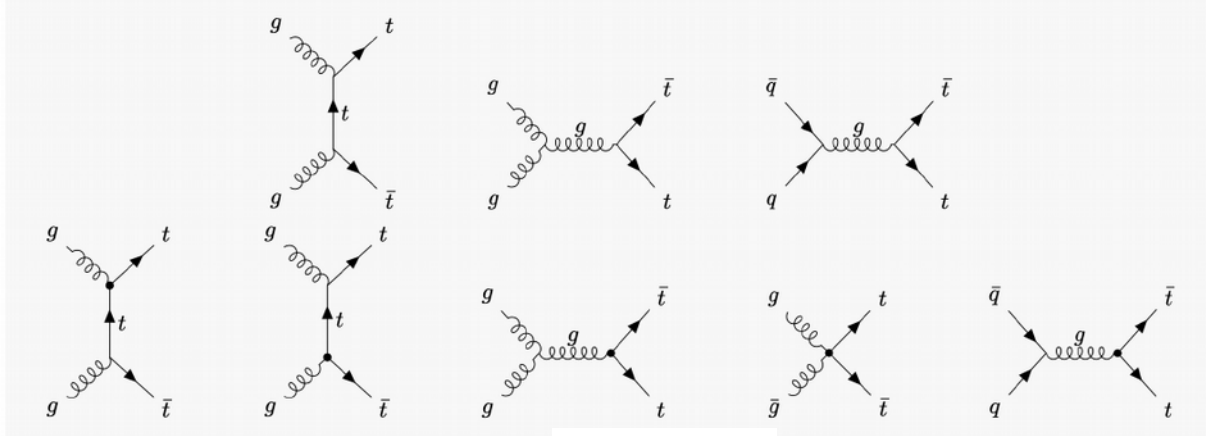
+ Λ -hyperons

- Gong, Parida, Tu, Venugopalan, 2107.13007 [hep-ph]

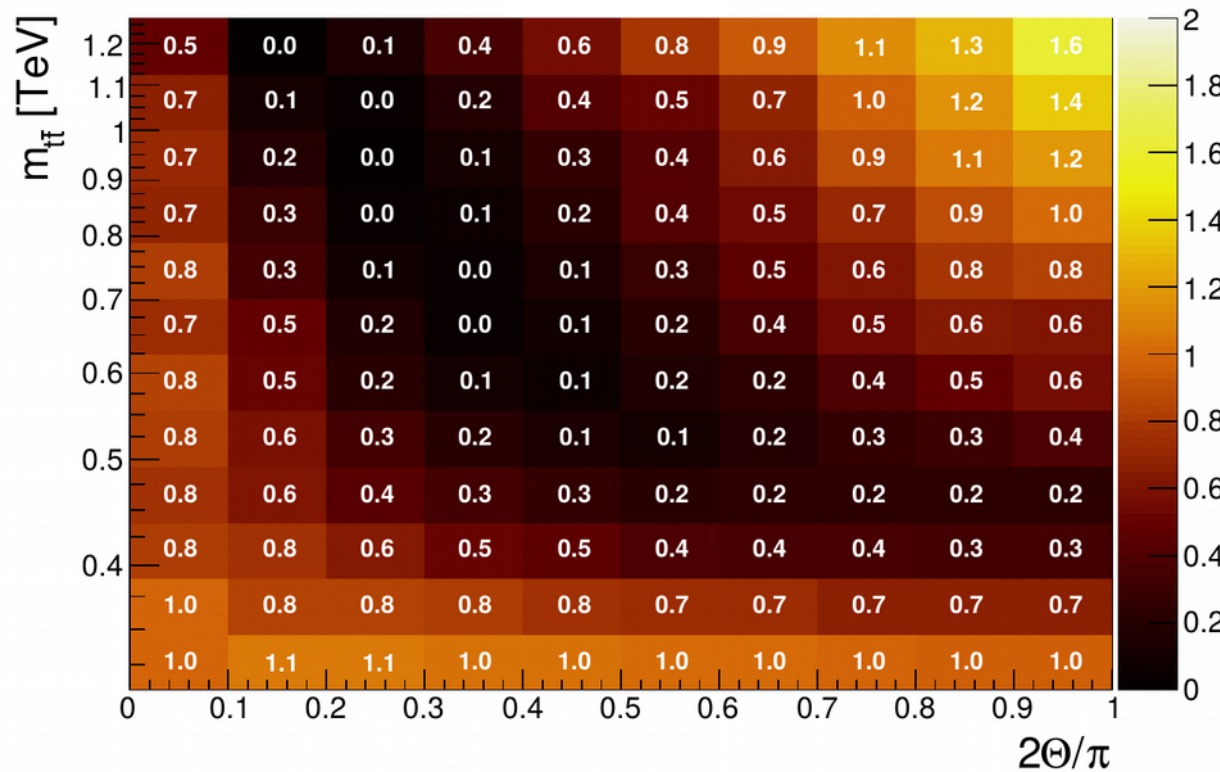
top- anti-top production

Fabbrichesi, Floreanini, Panizzo, PRL 127 (2021), 2102.11883 [hep-ph]

First analysis of Bell's inequalities where correlation matrix C_{ij} is extracted from event simulation, including ATLAS detector resolution (DELEPHES), acceptance, migration and efficiency effects.



$m_{12}[C]$



Null hypothesis of Bell's inequalities

$$\left| \hat{n}_1 \cdot C \cdot (\hat{n}_2 - \hat{n}_4) + \hat{n}_3 \cdot C \cdot (\hat{n}_2 + \hat{n}_4) \right| \leq 2$$

Violation of Bell's inequalities requires

$$m_{12}[C] > 1$$

Violation of null hypothesis can be assessed:

- at 2σ level with present Run 2 Luminosity
- at 4σ with projected full Run 3 Luminosity

Sensitivity to NP (EFT) studied in

Aoude, Madge, Maltoni, Mantani, PRD 106 (2022), 2203.05619 [hep-ph]
Severi, Vryonidou, JHEP 01 (2023), 2210.09339 [hep-ph]

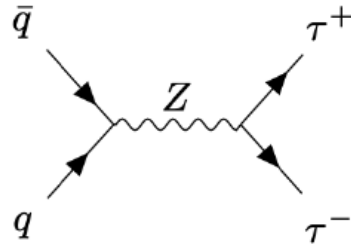
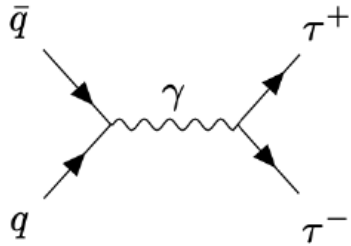
Fabbrichesi, EG, Floreanini, EPJC 83 (2023), 2302.00683 [hep-ph]

Drell-Yan tau-lepton pair production

Fabbrichesi, EG, Floreanini,
2302.00683 [hep-ph]

SM

$$p p \rightarrow \tau^+ \tau^- + jets$$



$m_{\tau\bar{\tau}} \rightarrow$ invariant mass
of tau-pairs

partonic contribution to Correlation coefficients

$$C_{ij}[m_{\tau\bar{\tau}}, \Theta] = \frac{L^{uu}(\tau) \tilde{C}_{ij}^{uu}[m_{\tau\bar{\tau}}, \Theta] + [L^{dd}(\tau) + L^{ss}(\tau)] \tilde{C}_{ij}^{dd}[m_{\tau\bar{\tau}}, \Theta]}{L^{uu}(\tau) \tilde{A}^{uu}[m_{\tau\bar{\tau}}, \Theta] + [L^{dd}(\tau) + L^{ss}(\tau)] \tilde{A}^{dd}[m_{\tau\bar{\tau}}, \Theta]}$$

unpolarized partonic contribution $\sim$ (square-amplitude)

Parton density

PDF4LHC21

$$\tilde{C}_{ij}^{qq} = C_{ij}^{qq} \tilde{A}_{ij}^{qq}$$

$$L^{qq}(\tau) = \frac{4\tau}{\sqrt{s}} \int_{\tau}^{1/\tau} \frac{dz}{z} q_q(\tau z) q_{\bar{q}}\left(\frac{\tau}{z}\right)$$

Unpolarized differential cross-section

$$\frac{d\sigma}{d\Omega dm_{\tau\bar{\tau}}} = \frac{\alpha^2 \beta_{\tau}}{64\pi^2 m_{\tau\bar{\tau}}^2} \left\{ L^{uu}(\tau) \tilde{A}^{uu}[m_{\tau\bar{\tau}}, \Theta] + [L^{dd}(\tau) + L^{ss}(\tau)] \tilde{A}^{dd}[m_{\tau\bar{\tau}}, \Theta] \right\}$$

$$p p \rightarrow \tau^+ \tau^- + jets$$

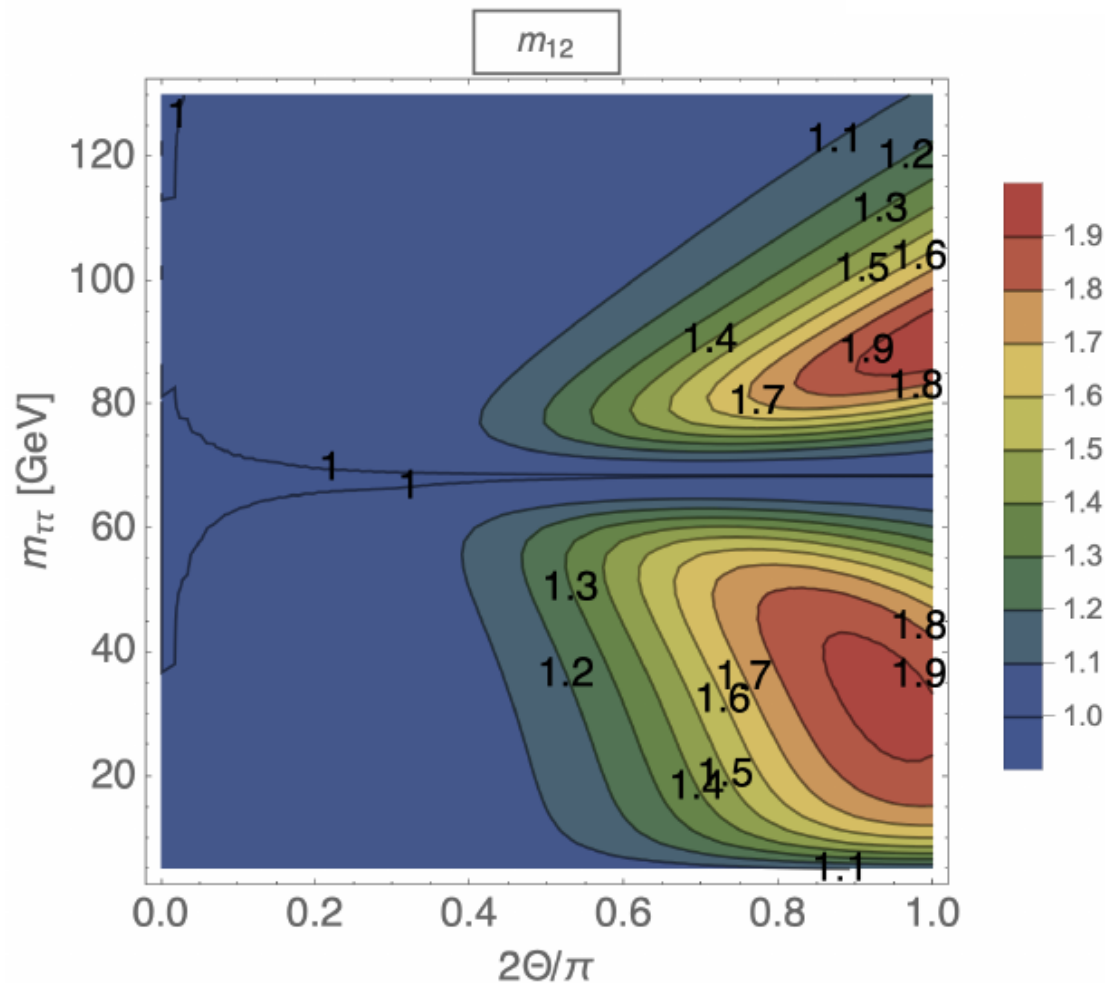
$$(\text{run 2}) \mathcal{L} = 140 \text{ fb}^{-1}$$

$$(\text{Hi-Lumi}) \mathcal{L} = 3 \text{ ab}^{-1}$$

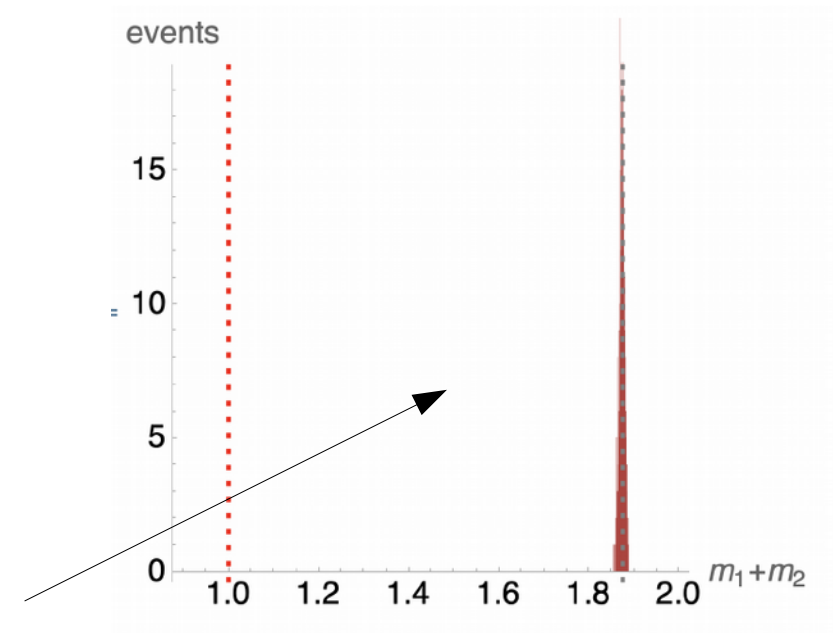
events

$$1.1 \times 10^6$$

$$2.2 \times 10^7$$



$$20 < m_{\tau\bar{\tau}} < 45 \text{ GeV and } 2\Theta/\pi > 0.80$$



using toy Montecarlo → only statistical error assumed

Statistical distribution for 100 events around mean value of **1.88** with dispersion given by **$\sigma = 0.006$**

- Statistical significance to reject Bell's violation null hypothesis is huge → more than **100 σ** due to the large number of events (opposite than in the top-pair case due to scarcity of events)
- only statistical error assumed: including systematic errors could significantly reduce these expectations

The τ -lepton decays

$$\tau^- \rightarrow A + \nu_\tau$$

$$\hookrightarrow A = e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \pi^-, \rho^-, a_1 \dots$$

$$\text{BR}(\ell^- \bar{\nu}_\ell \nu_\tau) \simeq 35\%,$$

$$\text{BR}(\pi^- \nu_\tau) \simeq 11\%,$$

$$\text{BR}(\rho^- / a_1 \nu_\tau) \simeq 26\%$$

Angular distribution of decay products **can be used** to determine the **polarization** of the **τ leptons**



$$\tau^- \rightarrow \pi^- \nu_\tau$$

$$\frac{1}{\Gamma} \frac{d\Gamma}{dz} = \frac{1}{2} (1 + P_\tau z) \quad z = \cos \theta \longrightarrow$$

defined with respect to the charged final states

$\swarrow$ same distribution as top-quark pair

$$\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$$

$$\frac{1}{\Gamma} \frac{d\Gamma}{dz} = \frac{1}{2} (1 - z) \left[(5 + 5z - 4z^2) + P_\tau (1 + z - 8z^2) \right]$$

weak dependence on the polarization

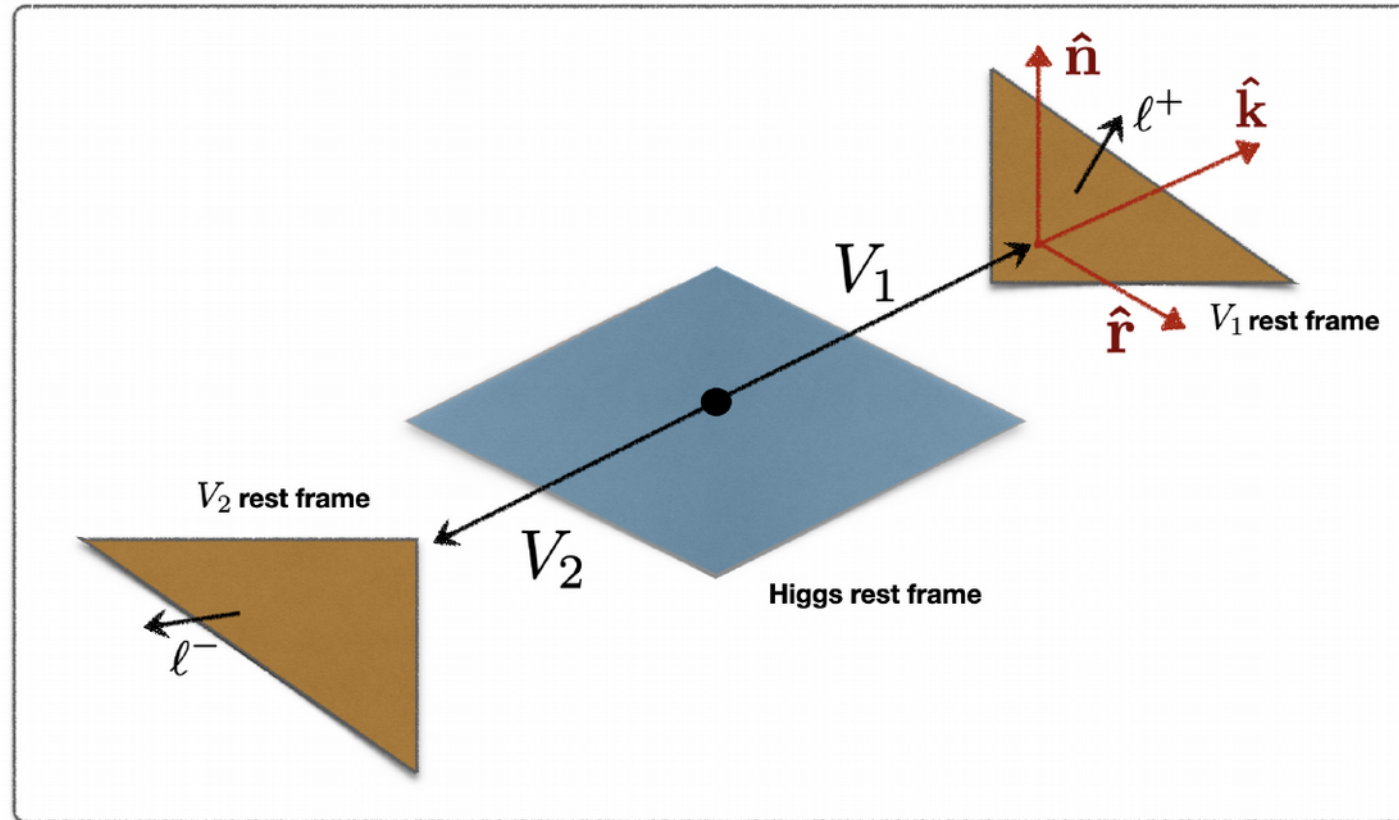


$$\tau \rightarrow \rho \nu_\tau \quad \tau \rightarrow a_1 \nu_\tau$$

Waiting for the full simulation

Di-boson production in Higgs boson decays

$$H \rightarrow V(k_1, \lambda_1) V^*(k_2, \lambda_2)$$



In the **Higgs boson rest frame** the density matrix of the bipartite **VV^*** system does not depend on the scattering angle, but only by the **Higgs mass**, the **V mass** and the off-shell **V^* mass**

Di-boson production in Higgs boson decays

A. Barr, PLB 825 (2022) ,
2106.01377 [hep-ph]

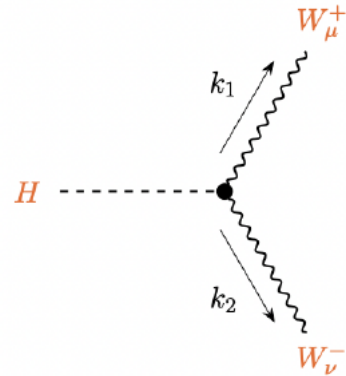
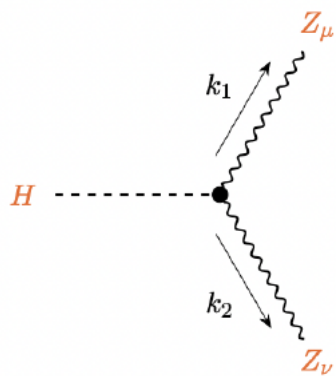
Aguilar-Saavedra, Bernal,
Casas, Moreno
2209.13441 [hep-ph]

$$H \rightarrow V(k_1, \lambda_1) V^*(k_2, \lambda_2)$$

Results based on
2302.00683 [hep-ph]

extracting density matrix from simulation at parton level

Analytic computation



$$\left\{ \begin{array}{l} \xi_W = 1, \text{ and } \xi_Z = 1/(2c_W), \text{ with } c_W = \cos \theta_W \\ g \text{ is the weak coupling} \end{array} \right.$$

Weinberg angle

V^* regarded as an off-shell vector boson with mass $M_V^* = f M_V$ $0 < f < 1$
mass of V boson M_V

Quantum Amplitude

$$\mathcal{M}_H(\lambda_1, \lambda_2) = g M_V \xi_V g_{\mu\nu} \varepsilon^{\mu*}(k_1, \lambda_1) \varepsilon^{\nu*}(k_2, \lambda_2)$$

$$\mathcal{M}_H(\lambda_1, \lambda_2) \mathcal{M}_H(\lambda'_1, \lambda'_2)^\dagger = g^2 M_V^2 \xi_V^2 g_{\mu\nu} g_{\mu'\nu'} \mathcal{P}_{\lambda_1 \lambda'_1}^{\mu\mu'}(k_1) \mathcal{P}_{\lambda_2 \lambda'_2}^{\nu\nu'}(k_2)$$

By projecting into the Gell-Mann matrix basis for 2-qutrits one can extract the **f**, **g** and **h** coefficients

Inserting the **f,g** and **h** coefficients into the Gell-Mann basis for 2-utrits

$$\rho_H = 2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{16} & 0 & 2h_{33} & 0 & h_{16} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\text{Tr} [\rho_H] = 1$$

density matrix is idempotent

$$\rho_H^2 = \rho_H$$

Signaling that $H \rightarrow VV^*$ is a **pure state**

$$\rho_H = |\Psi_H\rangle\langle\Psi_H|$$

using the basis

$$|\lambda \lambda'\rangle = |\lambda\rangle \otimes |\lambda'\rangle \text{ with } \lambda, \lambda' \in \{+, 0, -\}$$

where the pure state is

2302.00683 [hep-ph]

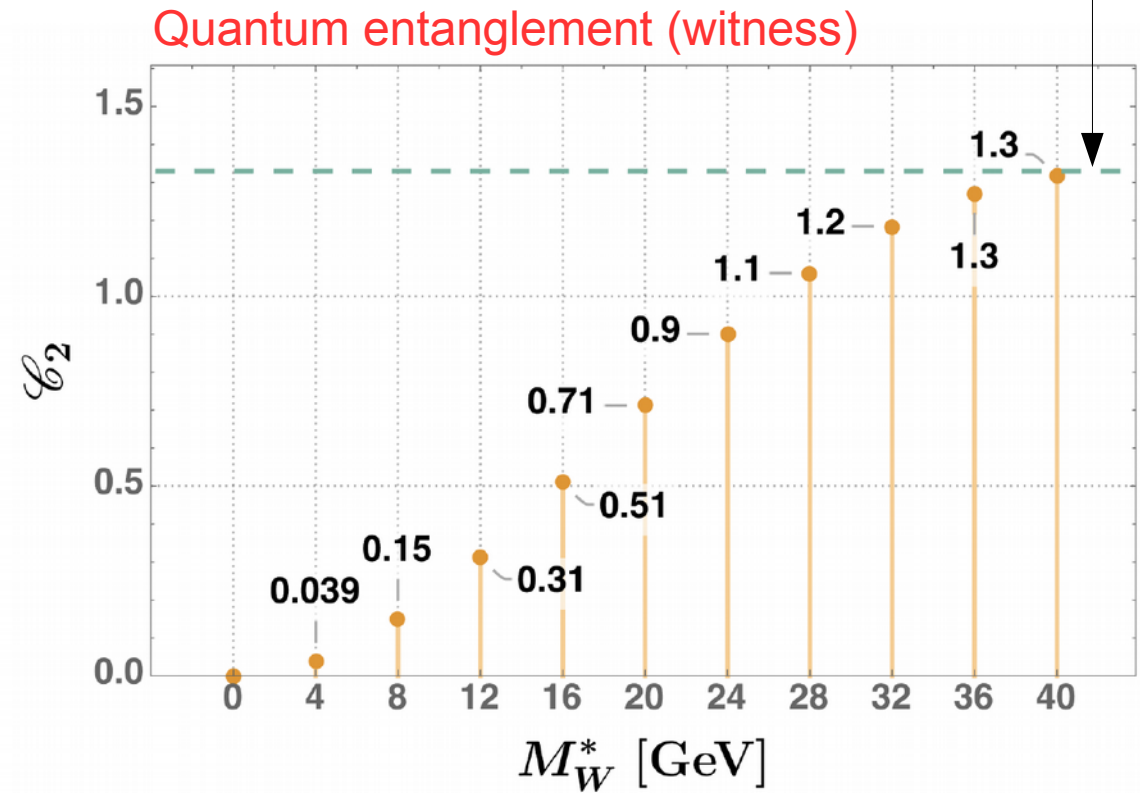
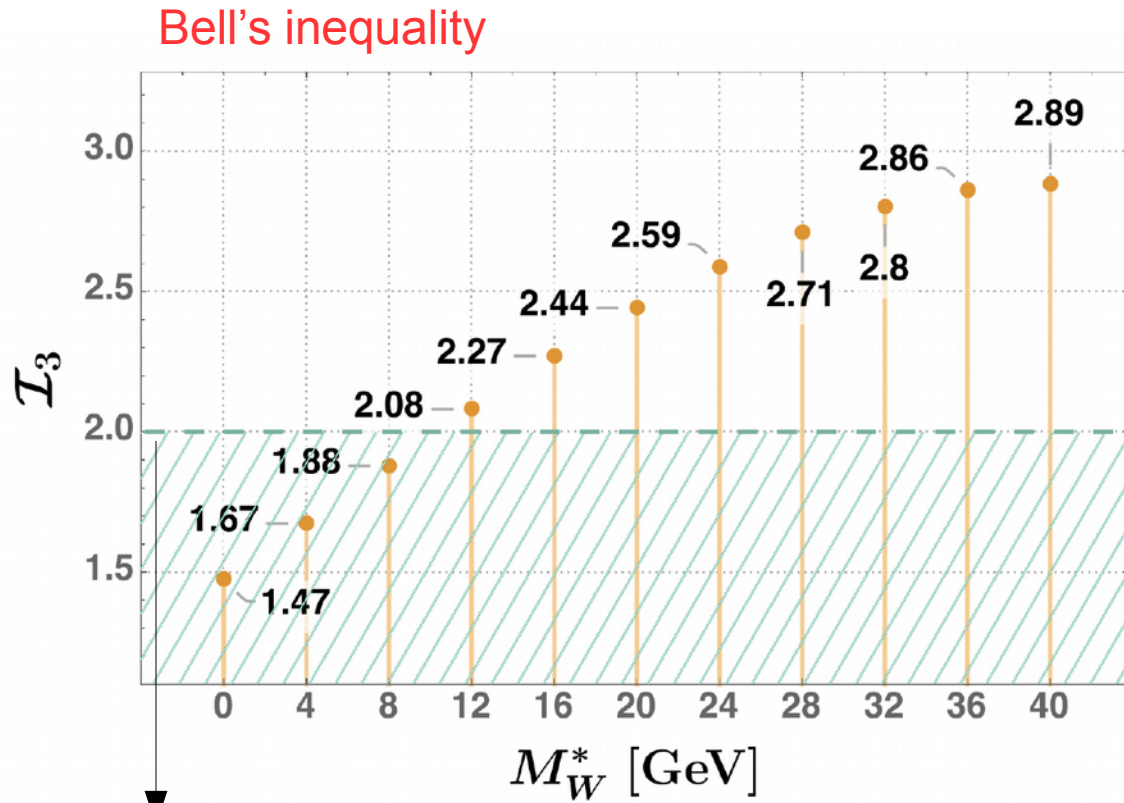
$$|\Psi_H\rangle = \frac{1}{\sqrt{2 + \varkappa^2}} [|+-\rangle - \varkappa |00\rangle + |-+\rangle]$$

$$\varkappa = 1 + \frac{m_H^2 - (1 + f)^2 M_V^2}{2f M_V^2}$$

**Aguilar-Saavedra, Bernal,
Casas, Moreno**
2209.13441 [hep-ph]

$$H \rightarrow W W^*$$

maximum value 4/3
for a pure state



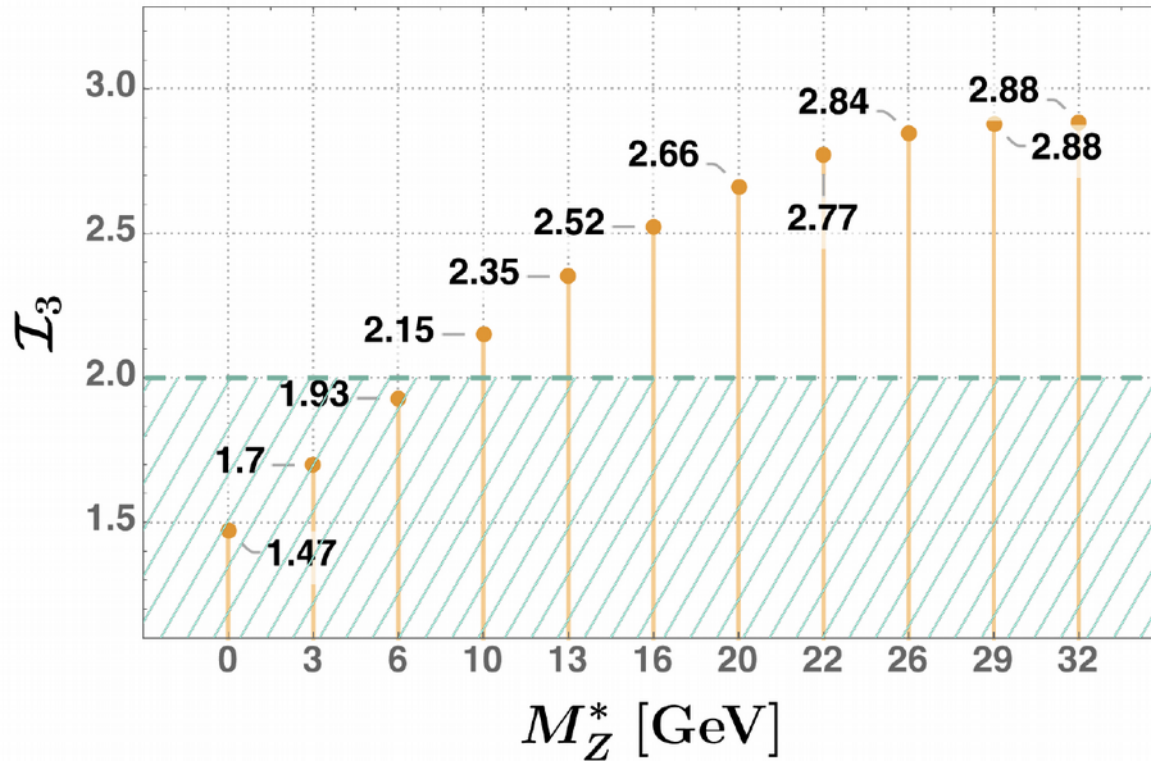
Bell's inequality violation condition $\mathcal{I}_3 > 2$

Maximization of $\mathcal{I}_3$ performed point by point as function of M_{W^*}
(see backup slides for solution of “optimized” U,V matrices in the region of max entanglement)

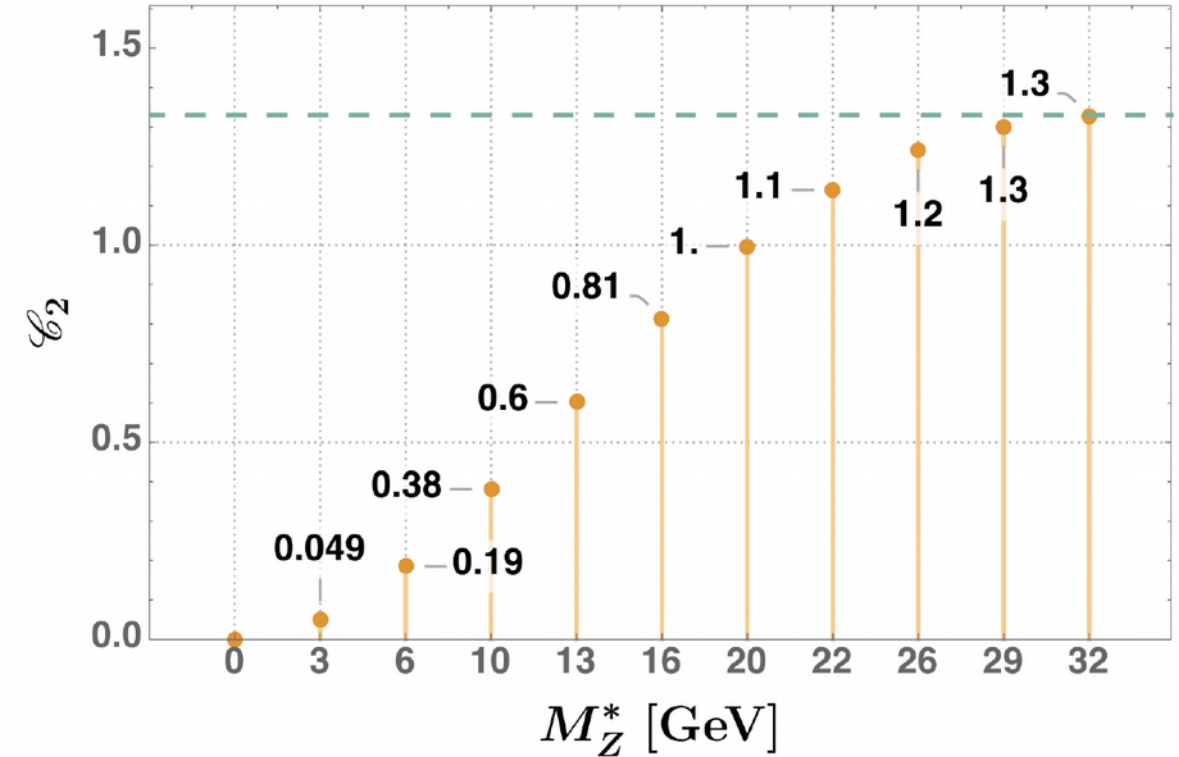
$$\mathcal{C}_2 = \frac{32f^2 M_V^4 \left[m_H^4 - 2(1 + f^2)m_H^2 M_V^2 + (1 + 4f^2 + f^4)M_V^4 \right]}{\left[m_H^4 - 2(1 + f^2)m_H^2 M_V^2 + (1 + 10f^2 + f^4)M_V^4 \right]^2}$$

$$H \rightarrow Z Z^*$$

Bell's inequality



Quantum entanglement (witness)



Maximization of $\mathcal{I}_3$ performed point by point as a function of M_{Z^*}
 (see backup slides for optimized U,V matrices in the region of max entanglement)

Entropy of entanglement for $H \rightarrow VV^*$

● If the bipartite (A,B) system is a **pure state** (as in the $H \rightarrow VV$ case)

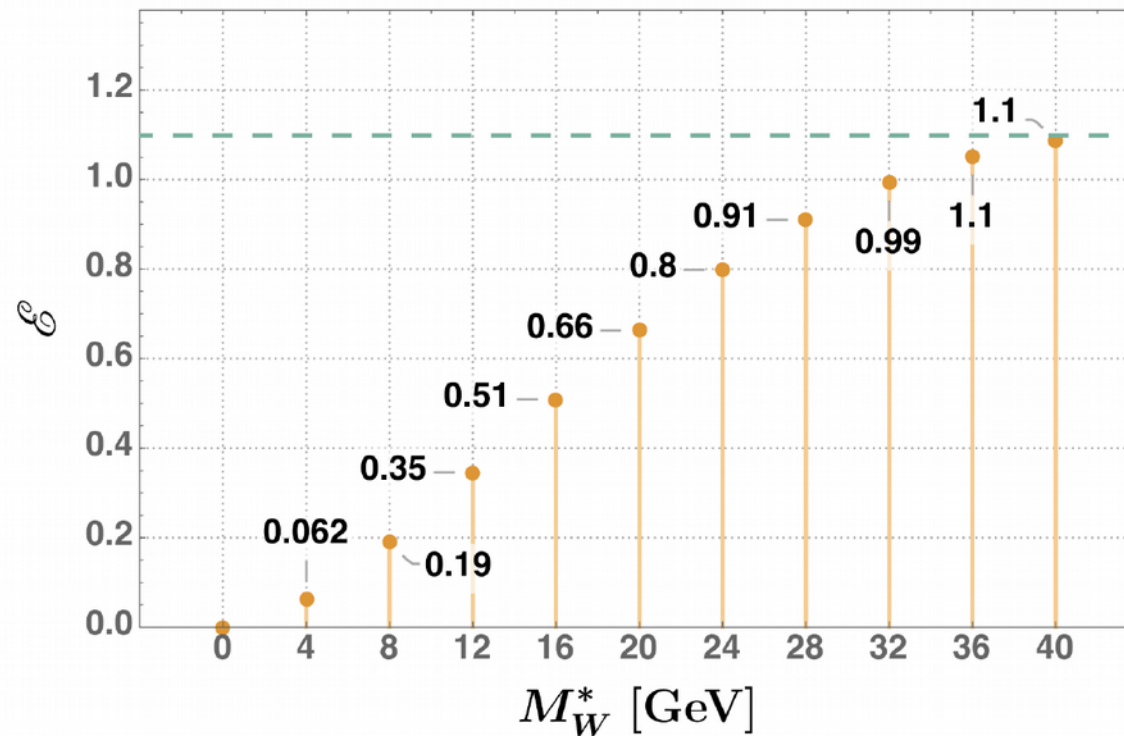
it is possible to **quantify** its entanglement via **Entropy of entanglement** (Von Neumann Entropy)

$$\mathcal{E}[\rho] = -\text{Tr} [\rho_A \log \rho_A] = -\text{Tr} [\rho_B \log \rho_B]$$

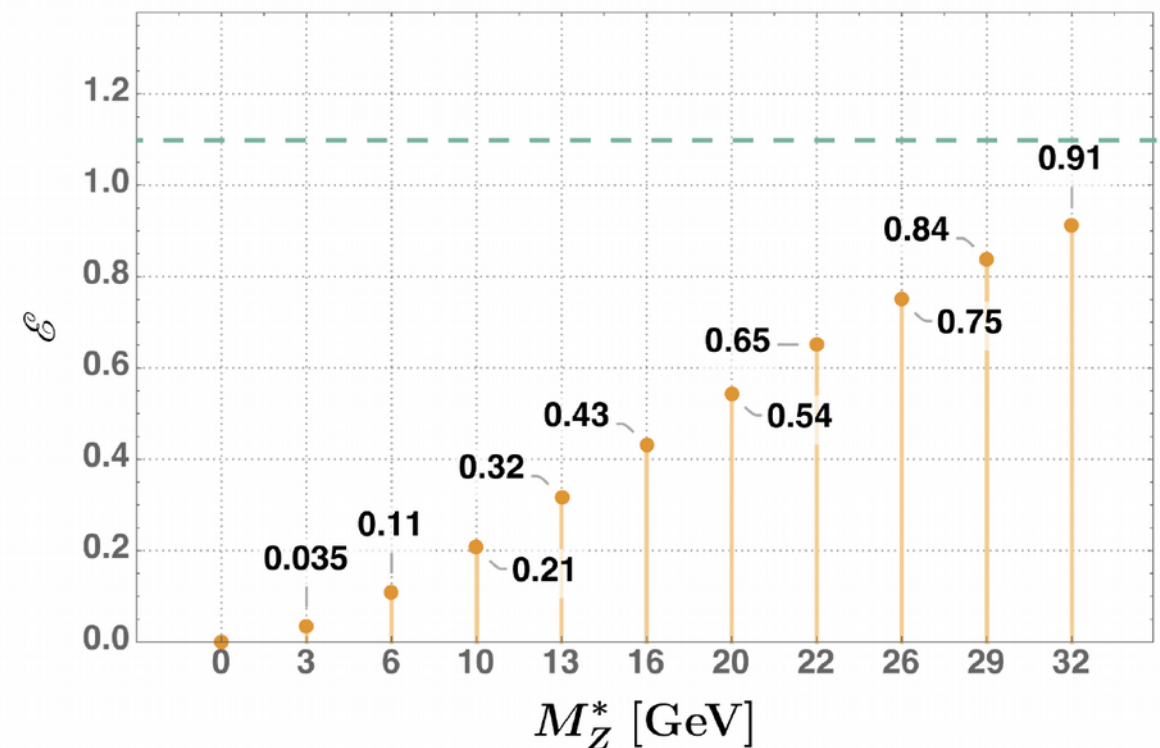
$$0 \leq \mathcal{E}[\rho] \leq \ln 3$$

equality holds if and only if the bipartite is separable

$H \rightarrow WW^*$



$H \rightarrow ZZ^*$



Sensitivity to the Bell violation

$$H \rightarrow WW^* \quad ZZ^*$$

		$\ell^+ \nu_\ell \ell^- \bar{\nu}_\ell$	$\ell^- \ell^+ \ell^- \ell^+$
<u>LHC run2</u>	$(\mathcal{L} = 140 \text{ fb}^{-1})$	4571	28
<u>Hi-Lumi</u>	$(\mathcal{L} = 3 \text{ ab}^{-1})$	9.8×10^3	589

cuts in V^* invariant mass: 40 GeV for WW^* and 30 GeV for ZZ^*

- we used **MADGRAPH5 @ LO** corrected by k factors at **N3LO + N3LL**
Bonvini, Marzani, Muselli, Rottoli, JHEP 08, 105 (2016) 1603.08000 [hep-ph]
- take into account **efficiency** (70%) in the identification of final leptons
- events reduced of **25% for ZZ^*** and **50% for WW^***

- backgrounds can be efficiently reduced after considering topology of final states and use of suitable cuts
- we neglect all background effects in our assessment of significance
- We model uncertainty as a Gaussian dispersion in the determination of the **mass of the off-shell weak boson** controlled by the number of events
- For the WW^* we add a **systematic error** taking into account uncertainty in the reconstruction of **W^* mass**, taking as benchmark value = **5 GeV** for the **systematic error** in reconstruction of neutrino momenta

Summary of results for $H \rightarrow WW^*$ and $H \rightarrow ZZ^*$

significance S for rejecting the null hypothesis $\mathcal{I}_3 \leq 2$

1000 pseudo experiments

$S(WW^*) \rightarrow$ uncertainty dominated by the systematic error

$S(ZZ^*) \rightarrow$ uncertainty dominated by the statistical error

LHC Run 2

1

1.3

LHC Hi-Lumi

1

5.6

$\rightarrow$ larger due to smaller uncertainty and higher mean value (2.9)

ZZ^* promising channel at LHC Hi-Lumi

In real simulation a reduced significance is expected (systematic uncertainties due to unfolding, background, and detector were neglected or only partially modeled here)

Results are also consistent with analysis of $\Rightarrow$ A. Barr, PLB 825 (2022), 2106.01377 [hep-ph] Aguilar-Saavedra, Bernal, Casas, Moreno 2209.13441 [hep-ph] where simulations of events have been adopted for the reconstruction of the density matrix

Testing Bell's inequalities in Di-boson production via Drell-Yan processes

2302.00683 [hep-ph]

$$pp \rightarrow W^+W^- \quad pp \rightarrow ZZ \quad pp \rightarrow WZ$$

(backup slides for details)

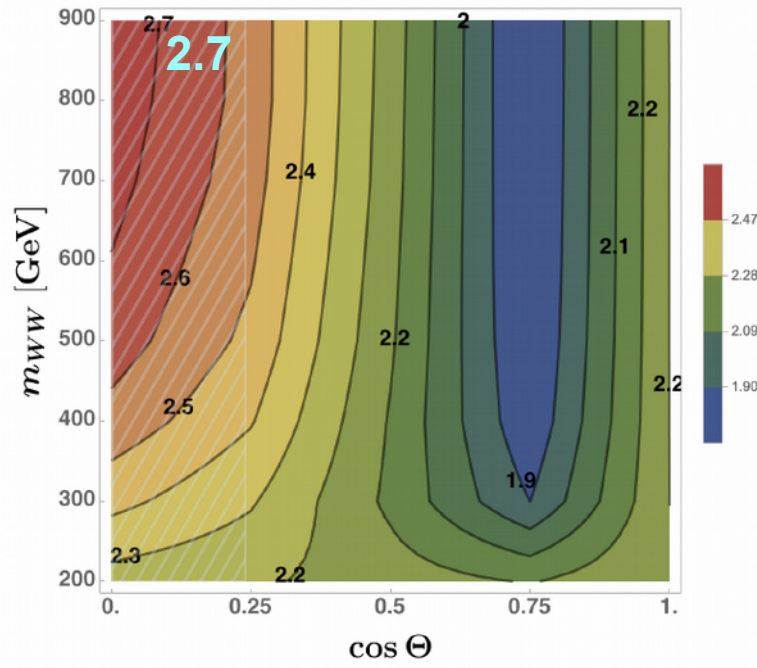
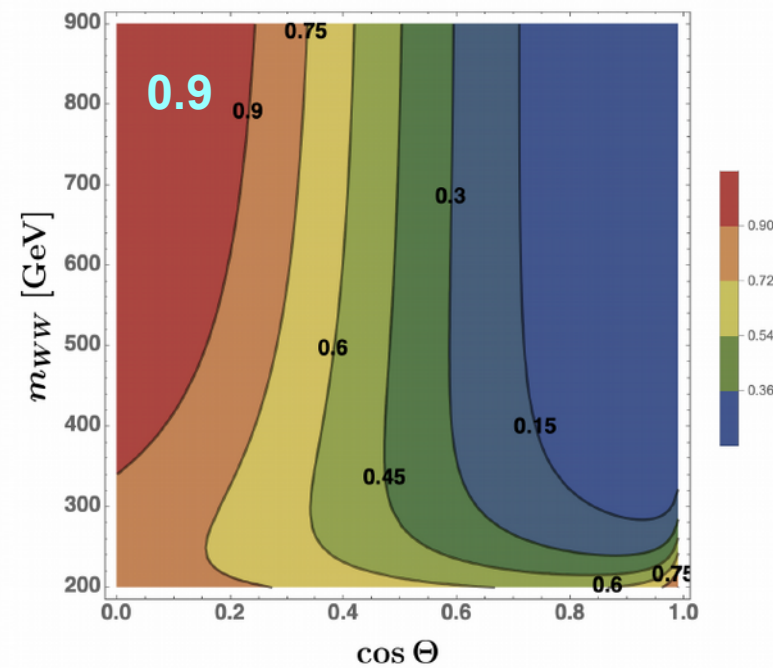
- not sufficient N. of events to test Bell's inequalities with large significance in the relevant kinematic region
- $pp \rightarrow WW$ largely affect by systematic error induced by the missing neutrino momentum reconstruction
- $pp \rightarrow ZZ$ is the most promising channel

At Hi-lumi **significance more than 2** to reject the null hypothesis

- estimates of significance expected to be reduced by a full simulation analysis

Lepton colliders

$$e^+ e^- \rightarrow W^+ W^-$$

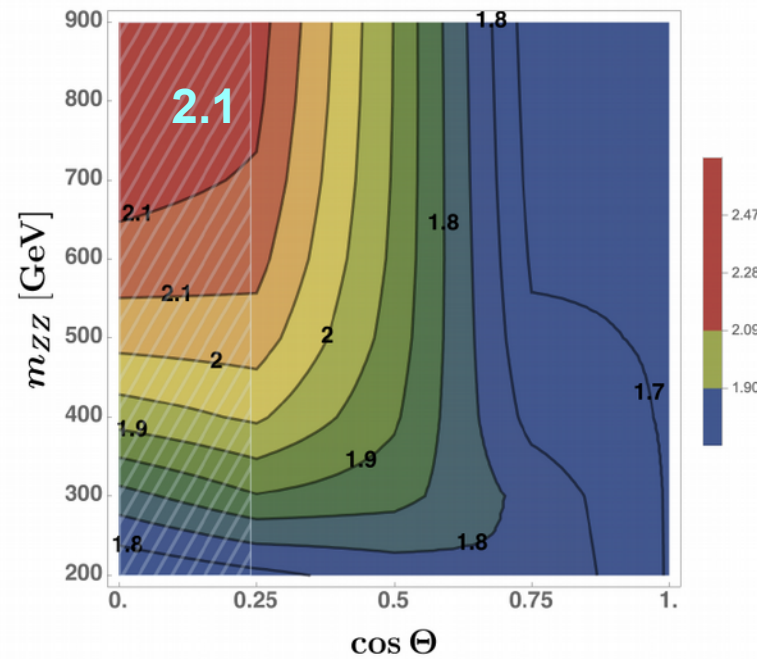
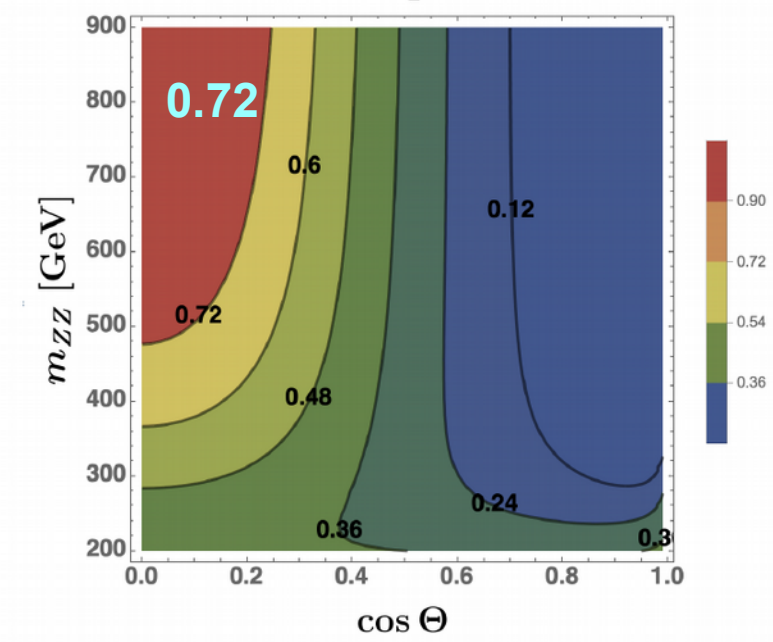
 $\mathcal{I}_3$

 $\mathcal{C}_2$


$$h_{ab}[m_{WW}, \Theta] = \frac{\tilde{h}_{ab}^{\ell\bar{\ell}}[m_{WW}, \Theta]}{A^{\ell\bar{\ell}}[m_{WW}, \Theta]},$$

$$f_a[m_{WW}, \Theta] = \frac{\tilde{f}_a^{\ell\bar{\ell}}[m_{WW}, \Theta]}{A^{\ell\bar{\ell}}[m_{WW}, \Theta]},$$

$$g_a[m_{WW}, \Theta] = \frac{\tilde{g}_a^{\ell\bar{\ell}}[m_{WW}, \Theta]}{A^{\ell\bar{\ell}}[m_{WW}, \Theta]},$$

absence of PDF increases entanglement and Bell violations in WW

 $\mathcal{I}_3$

 $\mathcal{C}_2$


$$e^+ e^- \rightarrow Z Z$$

ZZ channel not much different than LHC → due to the fact that PDF almost factorize (t,u channels)

Lepton colliders : events and sensitivity

$$e^+ e^- \rightarrow W^+ W^-$$

$$\sqrt{s} = 1 \text{ TeV}$$

$$\sqrt{s} = 368 \text{ GeV}$$

$$(\text{muon}) \mathcal{L} = 1 \text{ ab}^{-1}$$

$$(\text{FCC}) \mathcal{L} = 1.5 \text{ ab}^{-1}$$

events

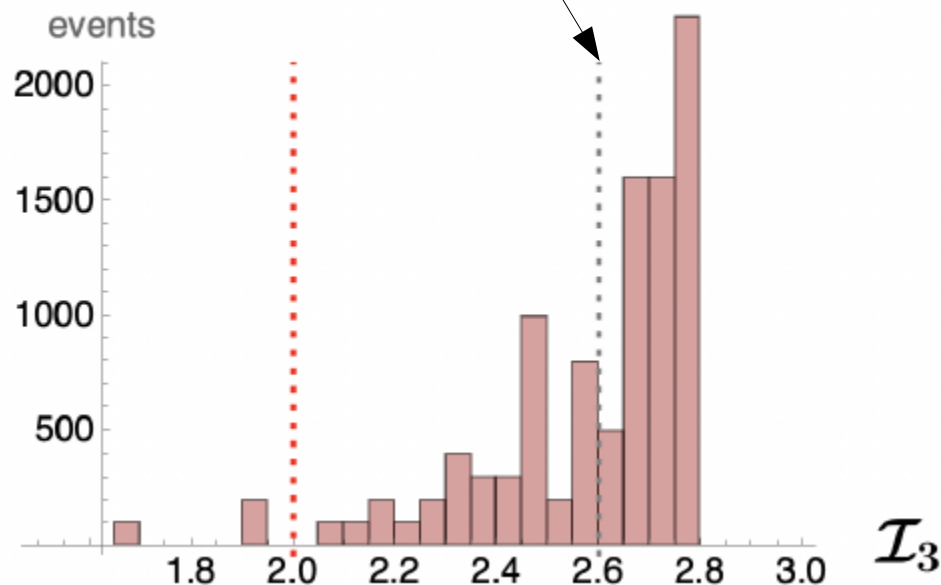
$$3.6 \times 10^3$$

$$5.8 \times 10^4$$

$$m_{WW} > 200 \text{ GeV and } \cos \Theta < 0.25$$

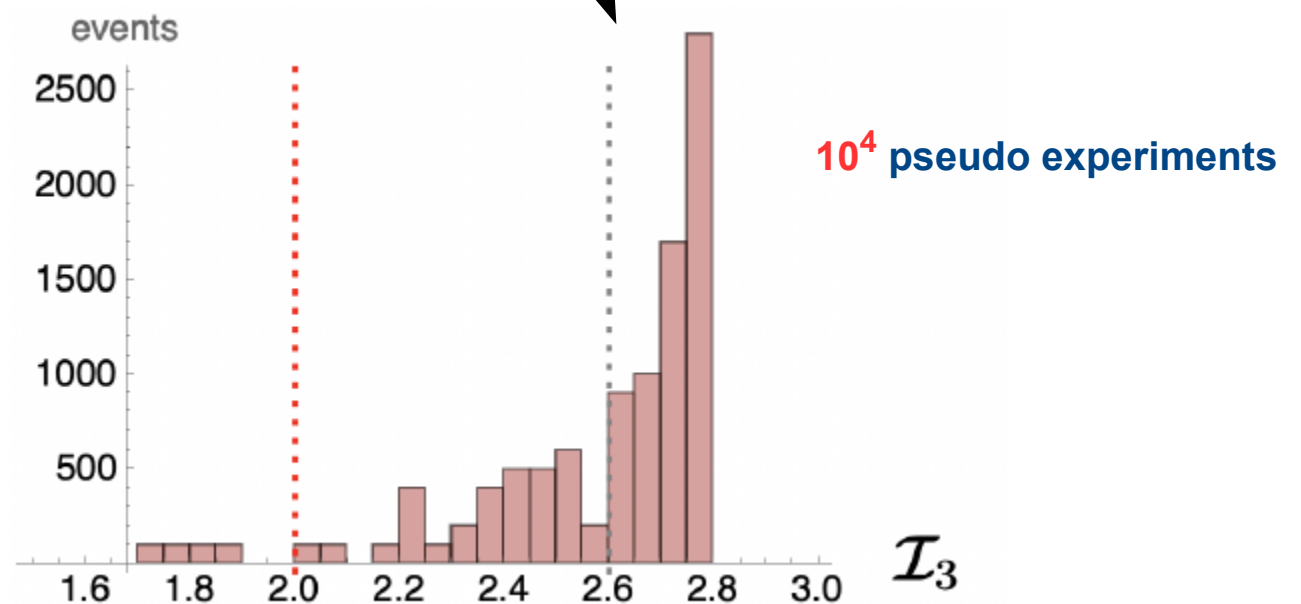
Significance = 2 to reject the null hypothesis
both at muon collider and FCC

mean value = 2.6



- Events computed with MADGRAPH@LO
- Efficiency 70% in each lepton identification
→ n. of events reduced accordingly
- systematic error due to missing neutrino
in WW mass reconstruction
largely dominates against statistical one
- no background considered

mean value = 2.6



Lepton colliders : events and sensitivity

$$e^+ e^- \rightarrow Z Z$$

promising channel

$$\sqrt{s} = 1 \text{ TeV}$$

$$\sqrt{s} = 368 \text{ GeV}$$

$$(\text{muon}) \mathcal{L} = 1 \text{ ab}^{-1}$$

$$(\text{FCC}) \mathcal{L} = 1.5 \text{ ab}^{-1}$$

events

44

748

$$m_{WW} > 200 \text{ GeV and } \cos \Theta < 0.25$$

Significance to reject null hypothesis

2 @ muon collider and > 4 @ FCC

Events computed with MADGRAPH@LO

Efficiency 70% in each lepton identification
→ n. of events reduced accordingly

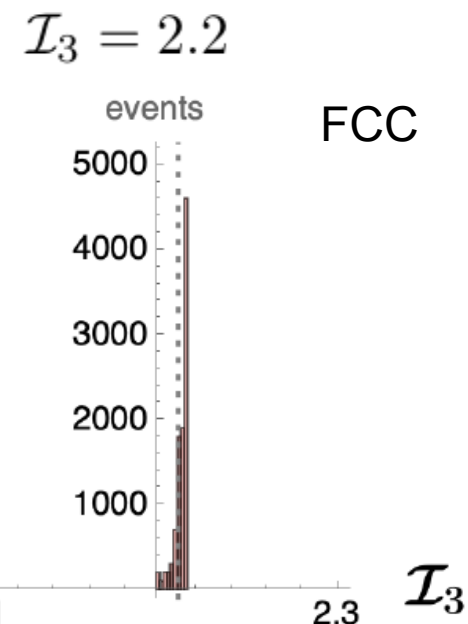
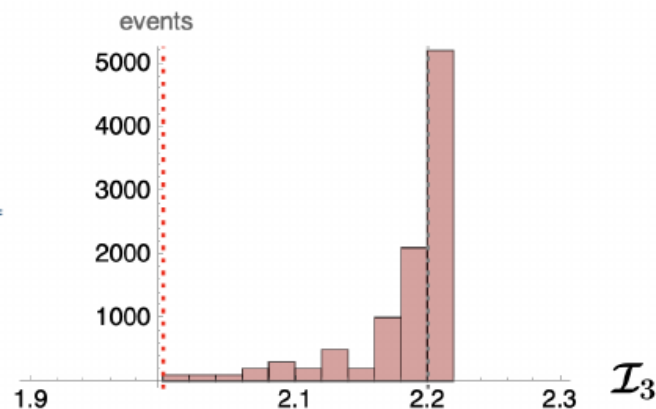
only statistic error assumed

no background considered

FCC significance is larger than muon collider due to larger n. of events at FCC (statistical error reduced)

10^4 pseudo experiments

$\mathcal{I}_3 = 2.2$
Muon collider



scalar/pseudoscalar decay in two vectors $S \rightarrow V_1 V_2$

- what have we learned from di-boson Higgs decays $H \rightarrow WW$ or ZZ ?
- VV comes out as a pure state
- this property applies to any scalar or pseudoscalar particle decaying into two spin-1 $V_1 V_2$ particles

Fabbrichesi, Floreanini, EG, Marzola
2304.02403 [hep-ph]
2305.04982 [hep-ph]

generic scalar meson

Define helicity amplitude

$$h_\lambda = \langle V_1(\lambda) V_2(-\lambda) | \mathcal{H} | B \rangle \quad \text{with} \quad \lambda = (+, 0, -)$$

helicities defined with respect to
z-direction (spin quantization axis)
in rest frame of one of the two Vectors

There are 3 helicity amplitude combination

$$|\Psi\rangle = \frac{1}{\sqrt{|H|^2}} \left[h_+ |V_1(+) V_2(-)\rangle + h_0 |V_1(0) V_2(0)\rangle + h_- |V_1(-) V_2(+)\rangle \right]$$

$$|H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2$$

Relative weight of $|0\ 0\rangle$ with respect to $|+\ -\rangle$ or $|-\ +\rangle$ controlled by angular momentum conservation

Density matrix for a pure $V_1 V_2$ state can be written as

$$\rho = |\Psi\rangle\langle\Psi|$$

Fabbrichesi, Floreanini, EG, Marzola
2304.02403 [hep-ph]
2305.04982 [hep-ph]

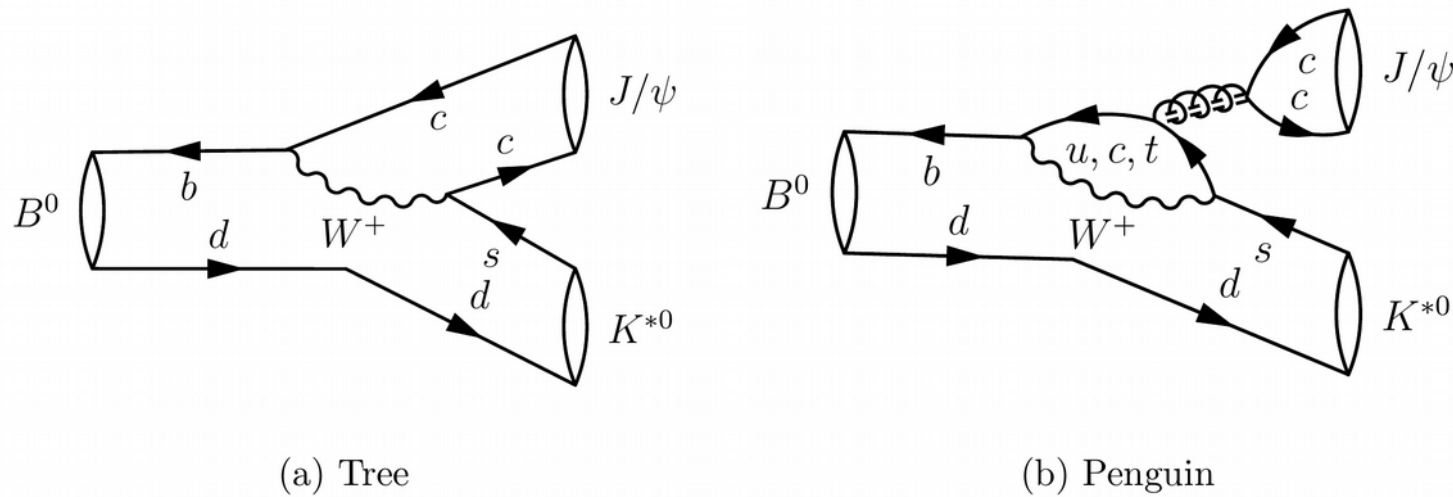
in helicity base

$$\rho = \frac{1}{|H|^2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+ h_+^* & 0 & h_+ h_0^* & 0 & h_+ h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0 h_+^* & 0 & h_0 h_0^* & 0 & h_0 h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_- h_+^* & 0 & h_- h_0^* & 0 & h_- h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\frac{h_0}{|H|} = A_0, \quad \frac{h_+}{|H|} = \frac{A_{\parallel} + A_{\perp}}{\sqrt{2}} \quad \text{and} \quad \frac{h_-}{|H|} = \frac{A_{\parallel} - A_{\perp}}{\sqrt{2}}$$

quantities measured in experimental analysis of B meson decays !

Bell's inequality violation in B meson decay $B \rightarrow J/\psi K^*$



- from measurement of helicity amplitudes we can compute the I_3 observable for qutrits
- there are 4 independent parameters: 3 amplitudes modulus and 2 relative phases as input

$$A_{\parallel} = |A_{\parallel}| e^{i\delta_{\parallel}}$$

$$A_{\perp} = |A_{\perp}| e^{i\delta_{\perp}}$$

$$A_0 = |A_0| e^{i\delta_0}$$

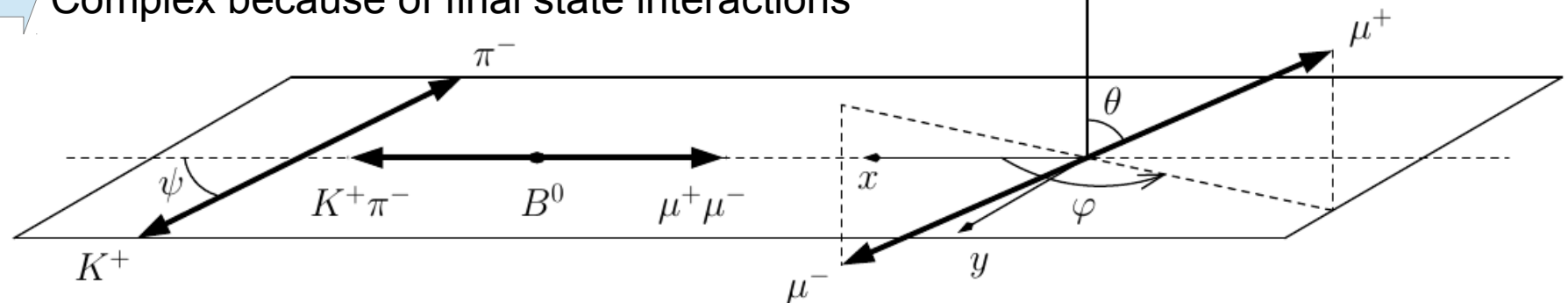
$$J/\psi \rightarrow \mu^+ \mu^-$$

$$K^* \rightarrow K^+ \pi^-$$

$$\Omega = \{\cos \theta, \cos \psi, \varphi\}$$

We choose $\delta_0 = 0$

Complex because of final state interactions



Measured by several collaborations, most recent one
LHCb Collaboration, PRD, 88 (2013) 052002, 1307.2782 [hep-ex]

From angular distribution one can extract the helicity amplitudes and phases by a fit of coefficients h_k as follows

$$\Omega = \{\cos \theta, \cos \psi, \varphi\}$$

$$\frac{d^4\Gamma(B^0 \rightarrow J/\psi K^{*0})}{dt d\Omega} \propto e^{-\Gamma_{dt}} \sum_{k=1}^{10} h_k f_k(\Omega)$$

k	h_k	$f_k(\Omega)$
1	$ A_0 ^2$	$\frac{9}{32\pi} 2 \cos^2 \psi (1 - \sin^2 \theta \cos^2 \varphi)$
2	$ A_{ } ^2$	$\frac{9}{32\pi} \sin^2 \psi (1 - \sin^2 \theta \sin^2 \varphi)$
3	$ A_{\perp} ^2$	$\frac{9}{32\pi} \sin^2 \psi \sin^2 \theta$
4	$ A_{ } A_{\perp} \sin(\delta_{\perp} - \delta_{ })$	$-\frac{9}{32\pi} \sin^2 \psi \sin 2\theta \sin \varphi$
5	$ A_0 A_{ } \cos(\delta_{ })$	$\frac{9}{32\pi\sqrt{2}} \sin 2\psi \sin^2 \theta \sin 2\varphi$
6	$ A_0 A_{\perp} \sin(\delta_{\perp})$	$\frac{9}{32\pi\sqrt{2}} \sin 2\psi \sin 2\theta \cos \varphi$
7	$ A_S ^2$	$\frac{3}{32\pi} 2(1 - \sin^2 \theta \cos^2 \varphi)$
8	$ A_{ } A_S \cos(\delta_{ } - \delta_S)$	$\frac{3}{32\pi} \sqrt{6} \sin \psi \sin^2 \theta \sin 2\varphi$
9	$ A_{\perp} A_S \sin(\delta_{\perp} - \delta_S)$	$\frac{3}{32\pi} \sqrt{6} \sin \psi \sin 2\theta \cos \varphi$
10	$ A_0 A_S \cos(\delta_S)$	$\frac{3}{32\pi} 4\sqrt{3} \cos \psi (1 - \sin^2 \theta \cos^2 \varphi)$

(A_S contribution from
non-resonant J/Y K^* amplitude)

$$|A_{\parallel}|^2 = 0.227 \pm 0.004 \text{ (stat)} \pm 0.011 \text{ (sys)}$$

$$|A_{\perp}|^2 = 0.201 \pm 0.004 \text{ (stat)} \pm 0.008 \text{ (sys)}$$

+ correlations (taken also into account)

$$\delta_{\parallel} [\text{rad}] = -2.94 \pm 0.02 \text{ (stat)} \pm 0.03 \text{ (sys)}$$

$$|A_0|^2 + |A_{\perp}|^2 + |A_{\parallel}|^2 = 1$$

$$\delta_{\perp} [\text{rad}] = 2.94 \pm 0.02 \text{ (stat)} \pm 0.02 \text{ (sys)},$$

WE GET

Fabbrichesi, Floreanini, EG, Marzola, 2305.04982 [hep-ph]

$$\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}]$$

After maximizing the effect via

$$\mathcal{B} \rightarrow (U \otimes V)^{\dagger} \cdot \mathcal{B} \cdot (U \otimes V)$$

(U,V given in backup slides)

$$\mathcal{I}_3 = 2.548 \pm 0.015$$

$$\Rightarrow B^0 \rightarrow J/\psi K^*(892)^0$$

Bell's inequality $\mathcal{I}_3 < 2$ is violated with a significance of **36σ**

0 < von Neumann entropy < ln[3]	$\mathcal{E}$	$\mathcal{I}_3$	
• $B^0 \rightarrow J/\psi K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015	
• $B^0 \rightarrow \phi K^*(892)^0$ [20]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$	→ 1.1σ
• $B^0 \rightarrow \rho K^*(892)^0$ [21]	$0.450 \pm 0.067^*$	$2.208 \pm 0.129^*$	→ 1.6σ
• $B_s \rightarrow \phi \phi$ [22]	$0.734 \pm 0.050^*$	$2.525 \pm 0.084^*$	→ 6.2σ
• $B_s \rightarrow J/\psi \phi$ [23]	0.731 ± 0.032	2.462 ± 0.080	→ 5.8σ

Conclusions

Bell's inequality is violated in $B \rightarrow J/\Psi K^*$ with 36σ !!!

2305.04982 [hep-ph]

- Shown to take place for the first time in **qutrits systems** and involving also **strong** and **EW interactions** and at higher energies ($\sim$ **5 GeV**) $\rightarrow$ well above eV scale (polarization of low-energy photons) !
- Updated analysis at LHCb could strongly improve the sensitivity, waiting for run2 data

The race is on for higher energies !

● **Qubit** $\rightarrow$ tau production at the Z peak at the run2 LHC

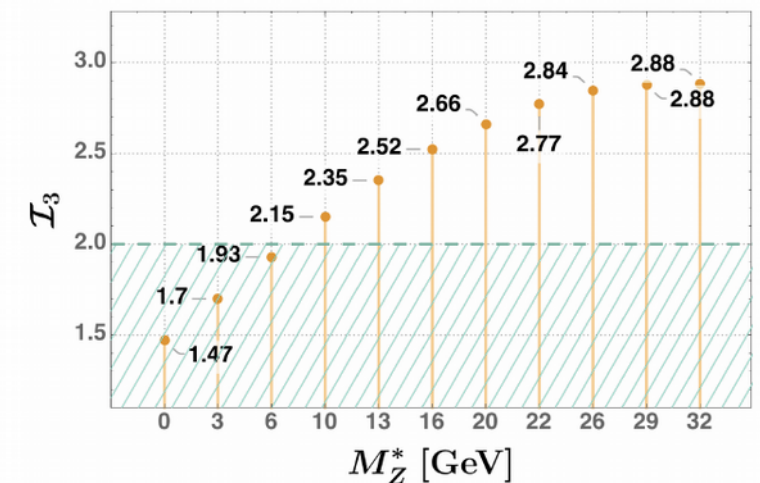
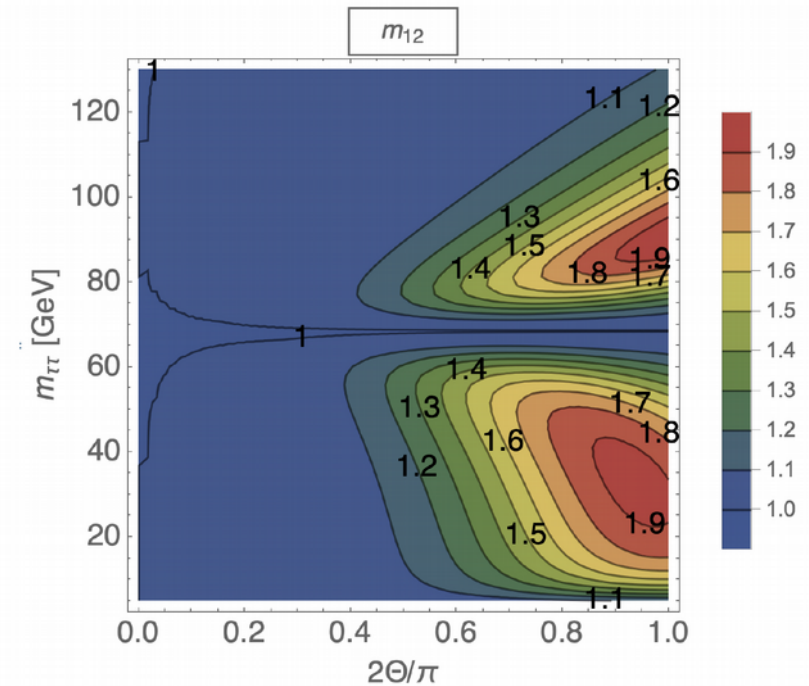
Fabbrichesi, EG, Floreanini, EPJC 83 (2023), 2302.00683 [hep-ph]

● **Qutrit** $\rightarrow$ Higgs decay $H \rightarrow ZZ^*$ at Hi-Lumi LHC

Aguilar-Saavedra, Bernal, Casas, Moreno, 2209.13441 [hep-ph]

Ashby-Pickering, Barr, Wierzchucka, 2209.13990 [quant-ph]

Fabbrichesi, Floreanini, EG, Marzola, 2302.00683 [hep-ph]



backup slides

Spin-1 matrices

$$S_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Expressed as a function of **Gell-Mann matrices**

$$S_1 = \frac{1}{\sqrt{2}} (T^1 + T^6), \quad S_2 = \frac{1}{\sqrt{2}} (T^2 + T^7), \quad S_3 = \frac{1}{2} T^3 + \frac{\sqrt{3}}{2} T^8$$

$$\begin{aligned} S_{31} = S_{13} &= \frac{1}{\sqrt{2}} (T^1 - T^6), \\ S_{12} = S_{21} &= T^5, \\ S_{23} = S_{32} &= \frac{1}{\sqrt{2}} (T^2 - T^7) \\ S_{11} &= \frac{1}{2\sqrt{3}} T^8 + T^4 - \frac{1}{2} T^3, \\ S_{22} &= \frac{1}{2\sqrt{3}} T^8 - T^4 - \frac{1}{2} T^3, \\ S_{33} &= T^3 - \frac{1}{\sqrt{3}} T^8, \end{aligned}$$

Gell-Mann basis

$$\begin{aligned} T^1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & T^2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & T^3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ T^4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & T^5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, & T^6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\ T^7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, & T^8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \end{aligned}$$

$\mathbb{1}$ being the 3×3 unit matrix

Wigner's *Q* symbols

$$\begin{aligned}
 q_{\pm}^1 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(\cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 q_{\pm}^2 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(\cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 q_{\pm}^3 &= \frac{1}{8} \left(1 \pm 4 \cos \theta^{\pm} + 3 \cos 2\theta^{\pm} \right), \\
 q_{\pm}^4 &= \frac{1}{2} \sin^2 \theta^{\pm} \cos 2\phi^{\pm}, \\
 q_{\pm}^5 &= \frac{1}{2} \sin^2 \theta^{\pm} \sin 2\phi^{\pm}, \\
 q_{\pm}^6 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(-\cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 q_{\pm}^7 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(-\cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 q_{\pm}^8 &= \frac{1}{8\sqrt{3}} \left(-1 \pm 12 \cos \theta^{\pm} - 3 \cos 2\theta^{\pm} \right),
 \end{aligned}$$

$$\begin{aligned}
 p_{\pm}^1 &= \sqrt{2} \sin \theta^{\pm} \left(5 \cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 p_{\pm}^2 &= \sqrt{2} \sin \theta^{\pm} \left(5 \cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 p_{\pm}^3 &= \frac{1}{4} \left(5 \pm 4 \cos \theta^{\pm} + 15 \cos 2\theta^{\pm} \right), \\
 p_{\pm}^4 &= 5 \sin^2 \theta^{\pm} \cos 2\phi^{\pm}, \\
 p_{\pm}^5 &= 5 \sin^2 \theta^{\pm} \sin 2\phi^{\pm}, \\
 p_{\pm}^6 &= \sqrt{2} \sin \theta^{\pm} \left(-5 \cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 p_{\pm}^7 &= \sqrt{2} \sin \theta^{\pm} \left(-5 \cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 p_{\pm}^8 &= \frac{1}{4\sqrt{3}} \left(-5 \pm 12 \cos \theta^{\pm} - 15 \cos 2\theta^{\pm} \right).
 \end{aligned}$$

For the **ZZ production** the density matrices $\Gamma_{\pm}$ are not projector due to the Z boson coupling

$$\mathcal{L} \supset -i \frac{g}{\cos \theta_W} \left[g_L (1 - \gamma^5) \gamma_\mu + g_R (1 + \gamma^5) \gamma_\mu \right] Z^\mu$$

$$\tilde{\mathbf{q}}^n = \frac{1}{g_R^2 + g_L^2} \left[g_R^2 \mathbf{q}_+^n + g_L^2 \mathbf{q}_-^n \right]$$

$$\tilde{\mathbf{p}}^n = \sum_m \mathbf{a}_m^n \mathbf{p}_+^m$$

$$\mathbf{a}_m^n = \frac{1}{g_L^2 - g_R^2} \begin{pmatrix} g_R^2 & 0 & 0 & 0 & 0 & g_L^2 & 0 & 0 \\ 0 & g_R^2 & 0 & 0 & 0 & 0 & g_L^2 & 0 \\ 0 & 0 & g_R^2 - \frac{1}{2} g_L^2 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} g_L^2 \\ 0 & 0 & 0 & g_R^2 - g_L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_R^2 - g_L^2 & 0 & 0 & 0 \\ g_L^2 & 0 & 0 & 0 & 0 & g_R^2 & 0 & 0 \\ 0 & g_L^2 & 0 & 0 & 0 & 0 & g_R^2 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{2} g_L^2 & 0 & 0 & 0 & 0 & \frac{1}{2} g_L^2 - g_R^2 \end{pmatrix}.$$

For the **ZZ production** the density matrices $\Gamma_{\pm}$ are not projector due to the Z boson coupling

$$\mathcal{L} \supset -i \frac{g}{\cos \theta_W} \left[g_L (1 - \gamma^5) \gamma_\mu + g_R (1 + \gamma^5) \gamma_\mu \right] Z^\mu$$

$$\tilde{\mathbf{q}}^n = \frac{1}{g_R^2 + g_L^2} \left[g_R^2 \mathbf{q}_+^n + g_L^2 \mathbf{q}_-^n \right]$$

$$\tilde{\mathbf{p}}^n = \sum_m \mathbf{a}_m^n \mathbf{p}_+^m$$

$$\mathbf{a}_m^n = \frac{1}{g_L^2 - g_R^2} \begin{pmatrix} g_R^2 & 0 & 0 & 0 & 0 & g_L^2 & 0 & 0 \\ 0 & g_R^2 & 0 & 0 & 0 & 0 & g_L^2 & 0 \\ 0 & 0 & g_R^2 - \frac{1}{2} g_L^2 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} g_L^2 \\ 0 & 0 & 0 & g_R^2 - g_L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_R^2 - g_L^2 & 0 & 0 & 0 \\ g_L^2 & 0 & 0 & 0 & 0 & g_R^2 & 0 & 0 \\ 0 & g_L^2 & 0 & 0 & 0 & 0 & g_R^2 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{2} g_L^2 & 0 & 0 & 0 & 0 & \frac{1}{2} g_L^2 - g_R^2 \end{pmatrix}.$$

U,V matrices maximizing the Bell observable $\mathcal{I}_3$ in $H \rightarrow WW^*, ZZ^*$

for region $M_W^* = 40$ GeV and $M_Z^* = 32$ GeV

$$U_W = \begin{pmatrix} \frac{4}{11} + \frac{i}{14} & \frac{1}{6} + \frac{9i}{13} & \frac{3}{5} + \frac{i}{14} \\ -\frac{1}{9} - \frac{6i}{7} & 0 & \frac{1}{10} + \frac{i}{2} \\ \frac{4}{11} + \frac{i}{12} & -\frac{1}{7} - \frac{7i}{10} & \frac{3}{5} + \frac{i}{10} \end{pmatrix}, \quad V_W = \begin{pmatrix} -\frac{1}{7} - \frac{7i}{12} & -\frac{7}{10} - \frac{i}{10} & -\frac{1}{9} - \frac{6i}{17} \\ \frac{11}{21} + \frac{i}{17} & 0 & -\frac{6}{7} - \frac{i}{26} \\ -\frac{1}{8} - \frac{3i}{5} & \frac{7}{10} + \frac{i}{8} & -\frac{1}{10} - \frac{5i}{14} \end{pmatrix}$$

$$U_Z = \begin{pmatrix} -\frac{1}{2} + \frac{3i}{11} & \frac{7}{13} + \frac{5i}{11} & \frac{4}{13} - \frac{3i}{10} \\ -\frac{1}{2} + \frac{3i}{8} & 0 & -\frac{15}{31} + \frac{5i}{8} \\ -\frac{1}{5} + \frac{10i}{19} & -\frac{5}{7} & +\frac{1}{22} - \frac{3i}{7} \end{pmatrix}, \quad V_Z = \begin{pmatrix} -\frac{1}{7} - \frac{5i}{12} & \frac{7}{11} + \frac{2i}{7} & \frac{1}{25} - \frac{5i}{9} \\ \frac{2}{11} + \frac{10i}{13} & 0 & \frac{2}{7} + \frac{6i}{11} \\ \frac{1}{6} + \frac{2i}{5} & -\frac{11}{16} + \frac{i}{5} & -\frac{1}{3} - \frac{4i}{9} \end{pmatrix}$$

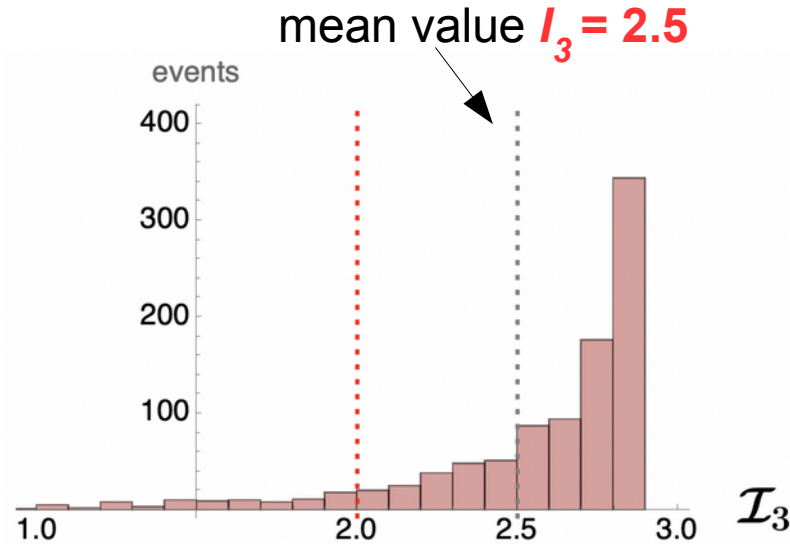
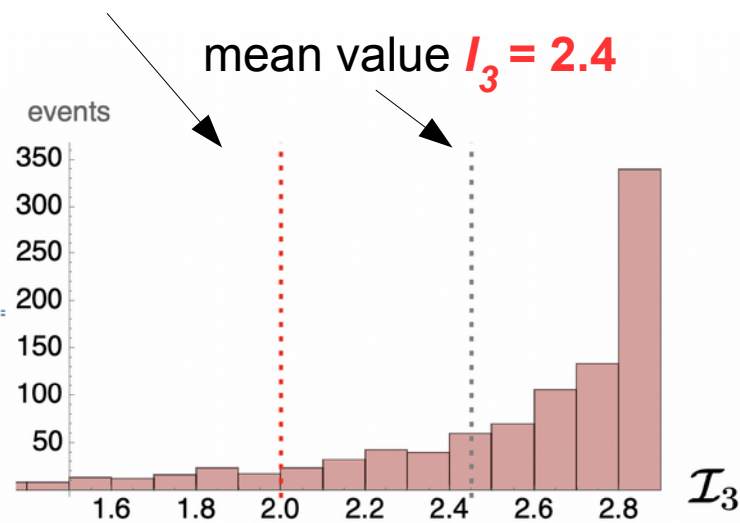
approximated matrices within 1% , unitary barring $O(10^{-2})$

$H \rightarrow WW^*$

$H \rightarrow ZZ^*$

WW* dominated by the large systematic error in reconstructing electron momenta due to neutrinos

Bell violation

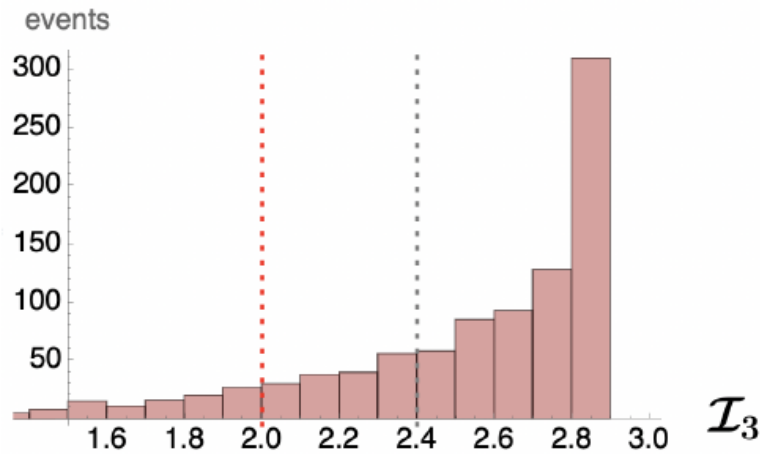


1000 pseudo experiments

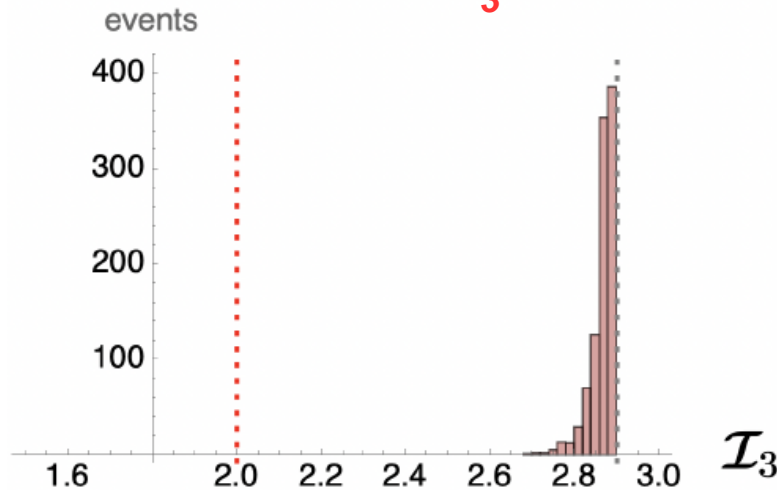
LHC (run2)

$$\mathcal{L} = 140 \text{ fb}^{-1}$$

mean value $I_3 = 2.4$



mean value $I_3 = 2.9$



LHC (Hi-lumi)

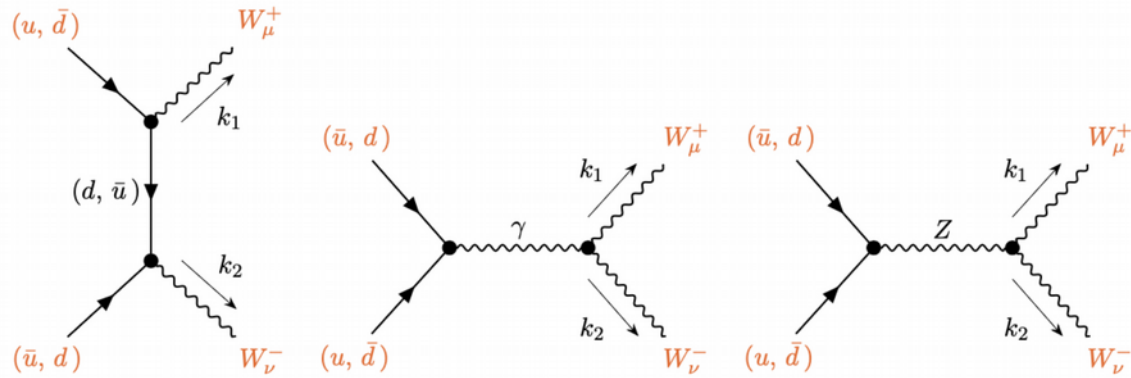
$$\mathcal{L} = 3 \text{ ab}^{-1}$$

$$\Phi(x) = \frac{1}{2} \left[1 + \text{erf} \left(\frac{x}{\sqrt{2}} \right) \right]$$

for significance $\rightarrow \mathcal{Z} = \Phi^{-1}(1 - p) \rightarrow$

N. of standard deviation in the approx. in which the distribution is assumed to be Gaussian

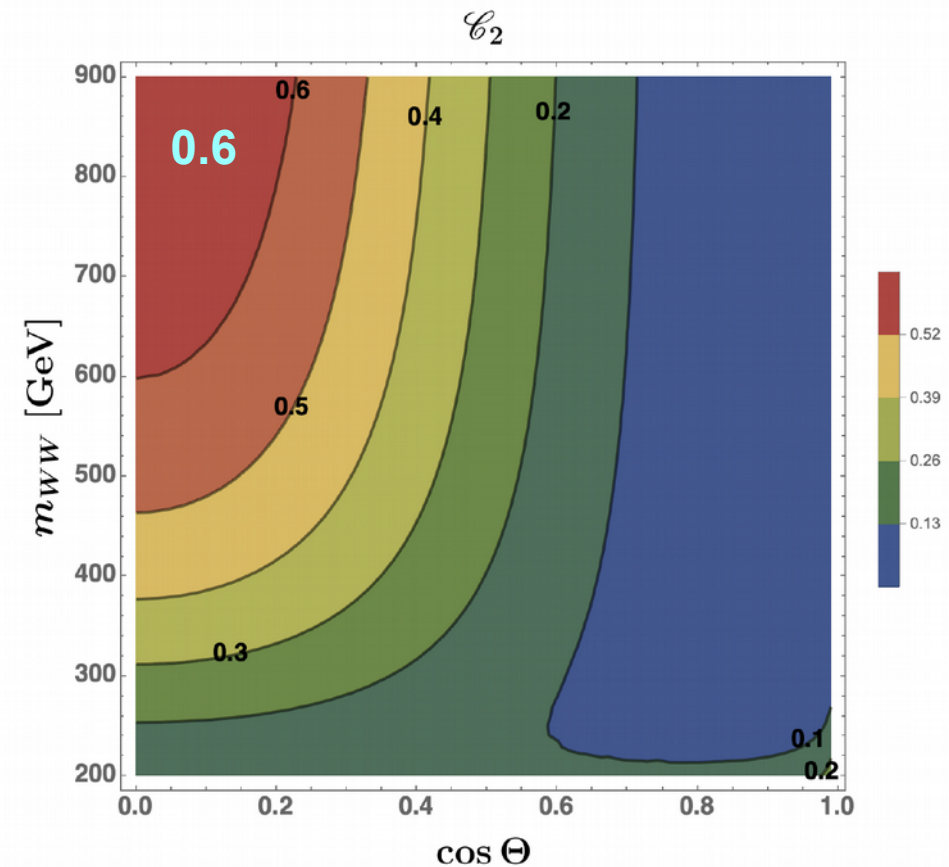
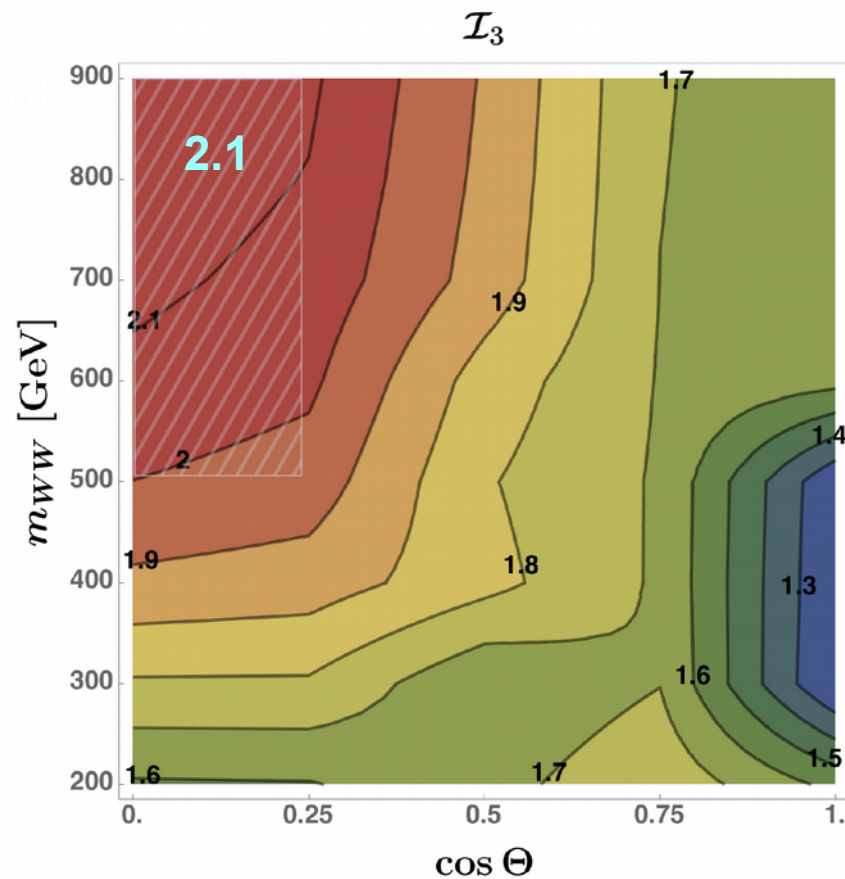
$$pp \rightarrow W^+W^-$$



optimization for **maximum value** of $\mathcal{I}_3$
of Bell's inequality violation is employed point
by point in the Θ m_{WW} space

$$\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

(see backup slides for their expressions in hatched area)



hatched area in the left-plot for $\mathcal{I}_3 > 2$ indicates bin used as reference for our estimation of the significance
(see next slides)

Events and sensitivity

$$p p \rightarrow W^+ W^-$$

(run2) $\mathcal{L} = 140 \text{ fb}^{-1}$

(Hi-Lumi) $\mathcal{L} = 3 \text{ ab}^{-1}$

events

288

6145

- estimated by using MADGRAPH5 @ LO for cross sections, corrected by the k-factors at the NNLO

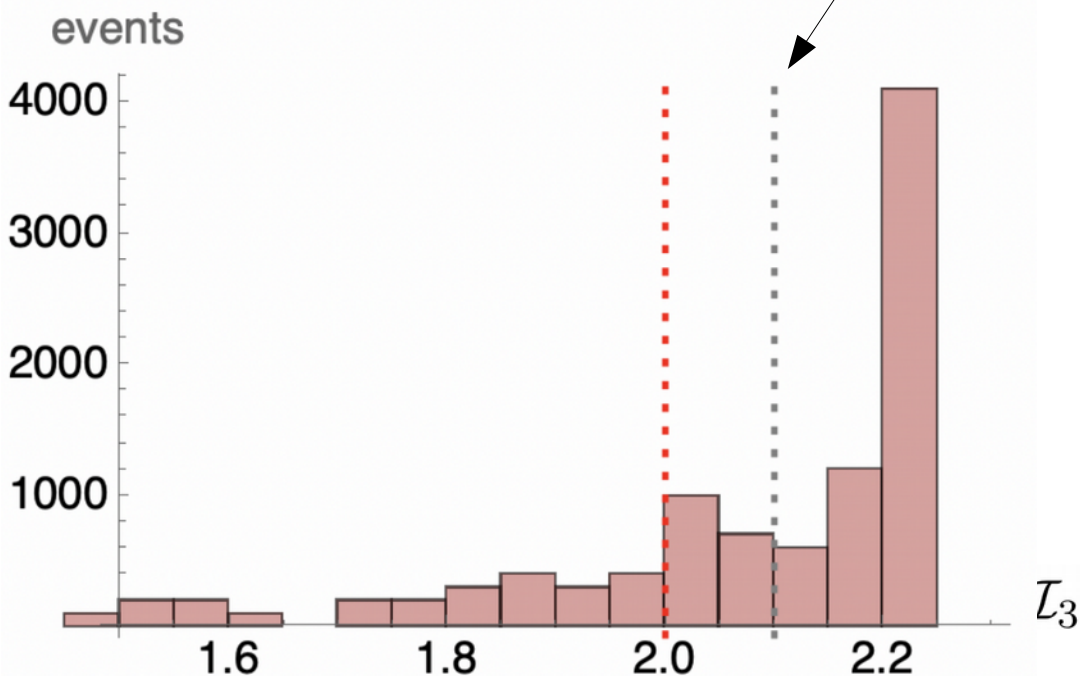
- 70% efficiency in identification of each lepton
→ number of events reduced accordingly

$m_{WW} > 500 \text{ GeV}$ and $\cos \Theta < 0.25$

- model uncertainty as a Gaussian dispersion in determination of WW invariant mass and scattering angle controlled by the N. of events
- add a systematic error with benchmark value of 5 GeV that takes into account significant uncertainty in reconstruction of neutrino momenta

Bell's inequality violation

mean value = 2.1



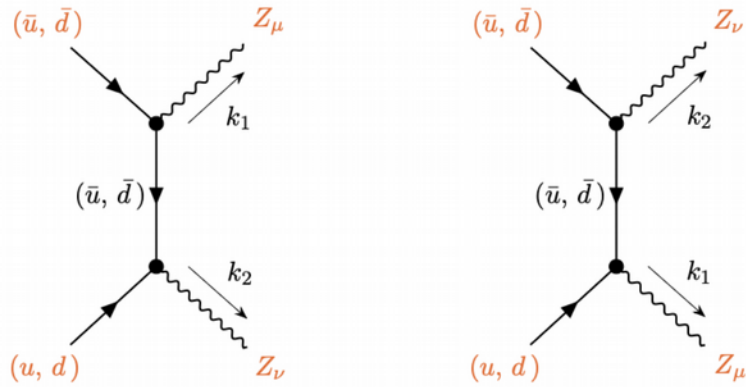
10^4 pseudo experiments

- similar plot for Hi-Lumi (statistical error dominate)

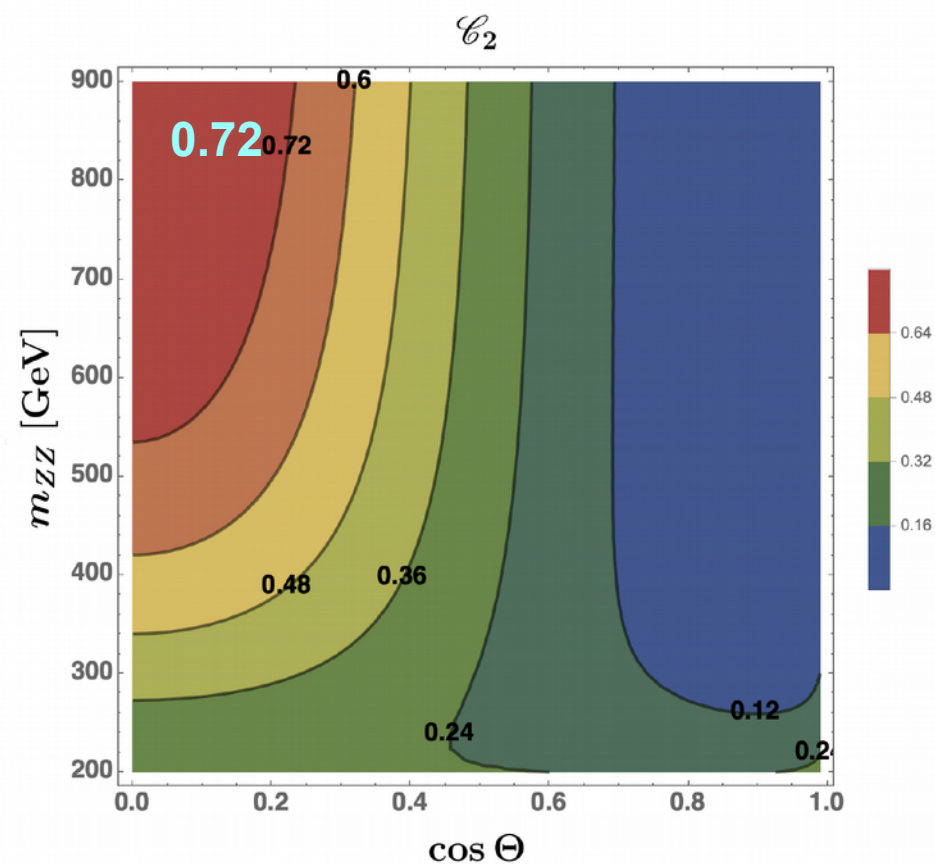
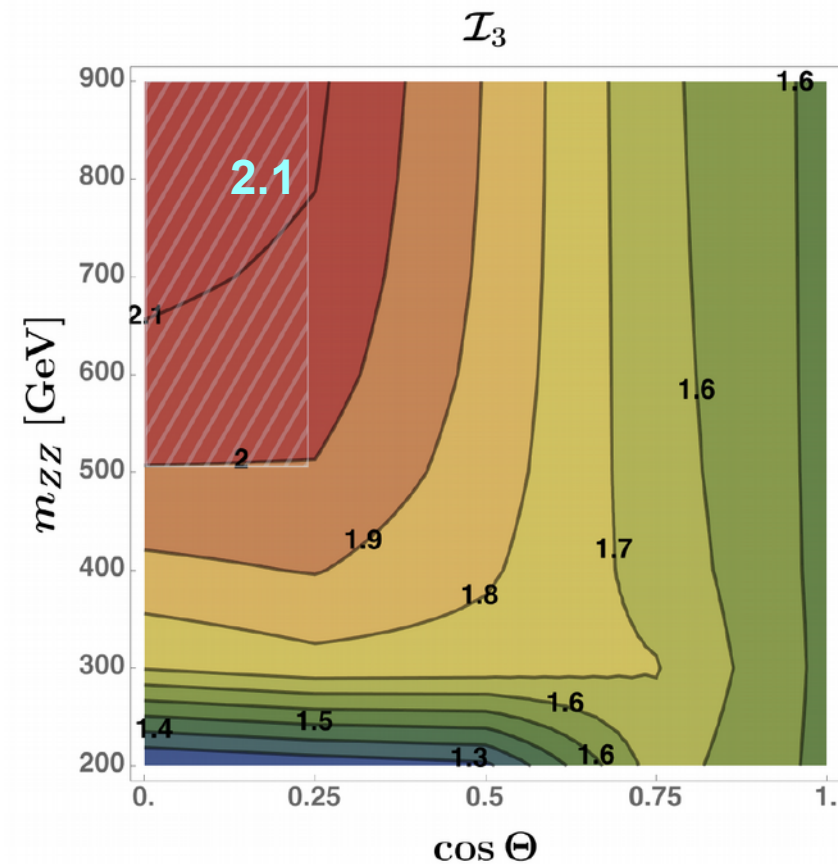
- Significance 0.8 at run 2 and similar at Hi-lumi to reject the null hypothesis $\mathcal{I}_3 \leq 2$

$$pp \rightarrow ZZ$$

arXiv: 2302.00683 [hep-ph]



- same strategy as in WW for optimization of maximum value of Bell's inequality violation
- reduced effect for Bell violations with respect to WW, but error is smaller being dominated by statistics



hatched area in the left-plot for $\mathcal{I}_3 > 2$ indicates bins used as reference for our estimation of the significance (see next slides)

Events and sensitivity

$$p p \rightarrow Z Z$$

(run2) $\mathcal{L} = 140 \text{ fb}^{-1}$ (Hi-Lumi) $\mathcal{L} = 3 \text{ ab}^{-1}$

events	4	77

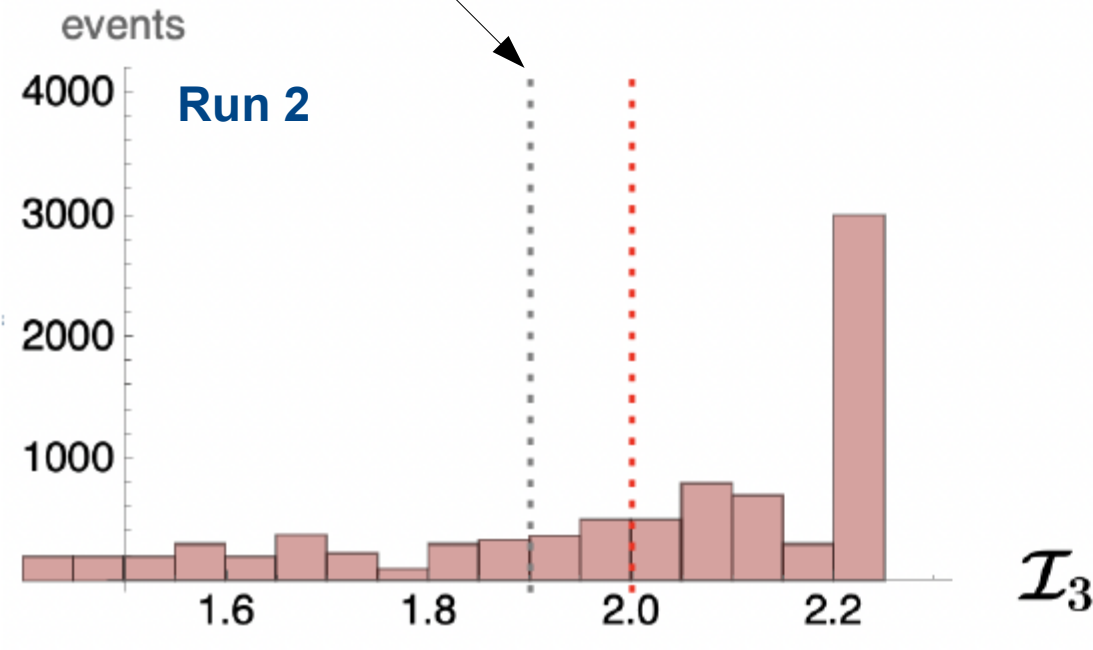
Less events than WW but statistical error dominates

$m_{ZZ} > 500 \text{ GeV}$ and $\cos \Theta < 0.25$

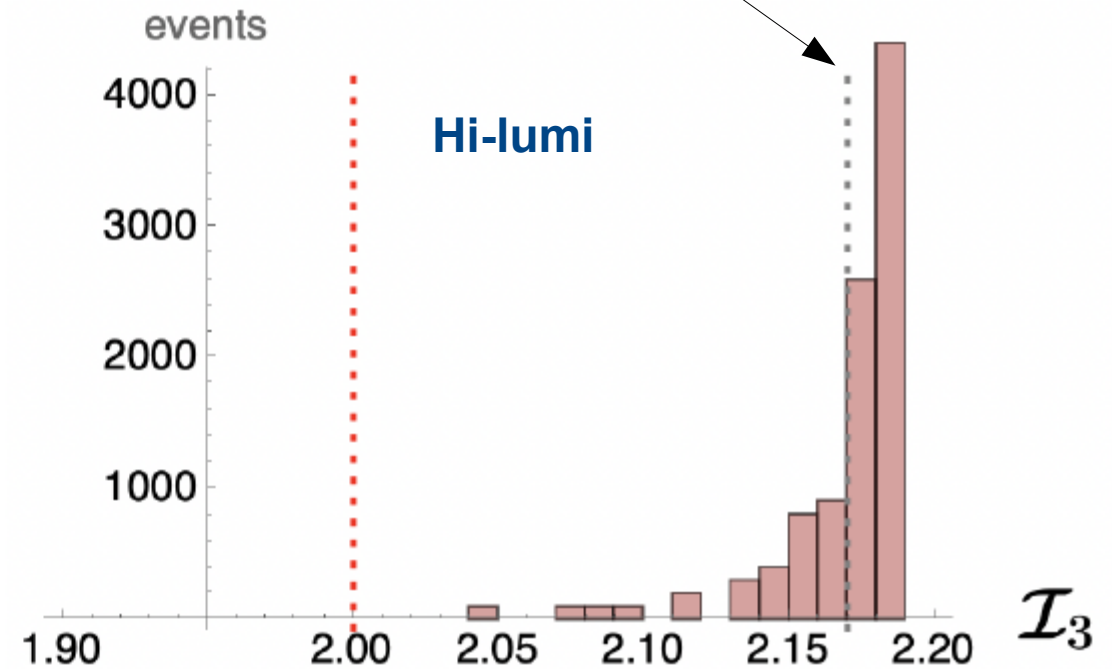
10^4 pseudo experiments

Bell's inequality violation

mean value = 1.9



mean value = 2.2



At Hi-lumi significance more than 2 to reject the null hypothesis $\mathcal{I}_3 \leq 2$

U,V matrices maximizing the Bell observable I_3 in $B \rightarrow J/\Psi K^*$

$$U = \begin{pmatrix} \frac{5}{14} - \frac{8i}{13} & \frac{1}{20} - \frac{4i}{7} & \frac{1}{4} - \frac{i}{3} \\ 0 & \frac{1}{18} - \frac{7i}{12} & -\frac{7}{15} + \frac{11i}{17} \\ -\frac{4}{11} + \frac{3i}{5} & \frac{1}{17} - \frac{4i}{7} & \frac{1}{4} - \frac{i}{13} \end{pmatrix}, \quad V = \begin{pmatrix} -\frac{5}{12} - \frac{i}{99} & -\frac{2}{9} + \frac{8i}{15} & \frac{1}{51} + \frac{5i}{7} \\ -\frac{4}{5} - \frac{i}{38} & \frac{3}{13} - \frac{6i}{11} & 0 \\ -\frac{5}{12} - \frac{i}{59} & -\frac{3}{13} + \frac{9i}{17} & -\frac{5i}{7} \end{pmatrix}$$

approximated matrices within 1% , unitary barring $O(10^{-2})$