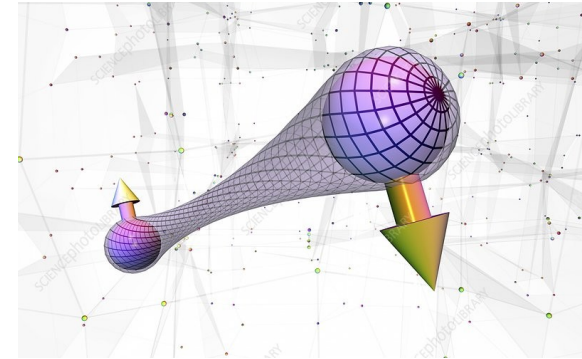


Violation of Bell Inequalities in B meson decays

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based on: 2305.04982 [hep-ph]

Take home message:

using results on analyses already published we show that it is possible to explore a new kind of physics, never explored before at high energy, which is based on **quantum entanglement**.

These results show that both **entanglement and Bell inequality violation** can be measured with significance **well above 5 sigma**.

- “**Entanglement**” between two systems is a pure quantum phenomena
- Expected to violates **Bell inequalities** → set of (classical) probability correlations
- Violations incompatible with classical physics based on causality and local realism (locality) (**EPR paradox, hidden variables theories**)
- Violation of Bell inequality observed at **low energy**

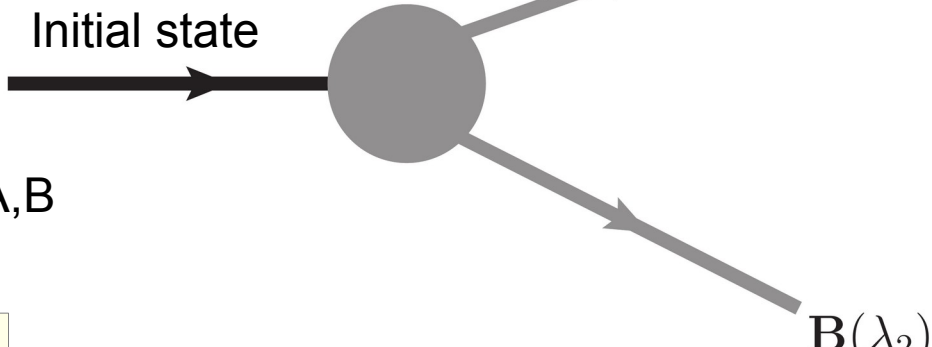
pair of photons Freedman-Clauser PRL 28 (1972), Aspect-Dalibard-Roger PRL 49 (1982),
Zelinger et al. , Nature 433, 230 (2005)

From LHCb analysis

- observation of **Entanglement** (as ATLAS results for tops, but larger significance)
- **first observation** of **Bell violation** at high energy in B meson decays and in the presence of **WEAK and STRONG** interactions !

Entanglement and Bell inequalities in spin systems

- Requires the knowledge of the **polarization density matrix** of two-particles **A,B** in production
- it can be fully reconstructed from the angular distributions of the single **A,B decay products**
- **but it can also be computed analytically**

Quantum polarized amplitude $\mathcal{M}(\lambda_1, \lambda_2) =$ 

 helicities of A,B

Polarization density matrix ρ

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \frac{\mathcal{M}(\lambda_1, \lambda_2) \mathcal{M}^\dagger(\lambda'_1, \lambda'_2)}{|\overline{\mathcal{M}}|^2}$$

 unpolarized

- knowledge of the full **polarization density matrix** allows to quantify (where possible) entanglement and Bell inequality violations

Quantifying entanglement

in general is a difficult task → simplifies in the case of **pure states**

✓ Pure states (or pure ensembles): the system is composed only by **one** quantum state $|\psi\rangle$

$$\text{Density matrix } \rho = |\psi\rangle\langle\psi| \quad \text{for pure states}$$

✓ in general we do not have pure states → but **mixture of states** $|\psi_n\rangle_{n=1,2,\dots}$

$$\rho = \sum_n p_n |\psi_n\rangle\langle\psi_n|, \quad \sum_n p_n = 1 \quad \Rightarrow \quad \text{Tr}[\rho] = 1$$

$\langle A \rangle = \text{Tr}[\rho A]$ → expectation value of operators (A) → can be computed in any base since Trace is base independent

✓ For pure states

$$\rho \doteq \begin{pmatrix} 0 & & & & & & 0 \\ & 0 & & & & & \\ & & \ddots & & & & \\ & & & 0 & & & \\ & & & & 1 & & \\ & & & & & 0 & \\ & & & & & & 0 \\ & & & & & & & \ddots & \\ & & & & & & & & 0 \end{pmatrix} \quad \text{(diagonal form)}$$

$$\rho^2 = \rho$$

$$\text{Tr}[\rho^2] = 1$$

quantifying entanglement in pure states

2302.00683 [hep-ph]

- If the bipartite (A,B) system is a pure state it is possible to quantify its entanglement via

Entropy of entanglement

R. Horodecki, P. Horodecki, M. Horodecki and K. Horodecki, Rev. Mod. Phys. **81**, 865-942 (2009) [arXiv:quant-ph/0702225 [quant-ph]].

$$\mathcal{E}[\rho] = -\text{Tr} [\rho_A \log \rho_A] = -\text{Tr} [\rho_B \log \rho_B]$$

density matrix of reduced A or B system

equality holds if and only if the bipartite is separable

$$0 \leq \mathcal{E} \leq \log 3 \quad (\text{Von Neumann Entropy}) \quad (\log \rightarrow \text{natural log.})$$

classical state (separability)

Maximum entanglement in QM

- useful in the case of bipartite (A,B) system of two qutrits for which there is not a solution for the general case

Two-QUTRITS systems

consider two massive spin-1 particles

Useful to decompose **density matrix** on the basis of tensor products of **Gell-Mann matrices** T^a

9x9 matrix

Unpolarized

$$[A \otimes B]_{ii'jj'} = A_{ii'} B_{jj'}$$

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \left(\frac{1}{9} [\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [\mathbb{1} \otimes T^a] + \sum_a g_a [T^a \otimes \mathbb{1}] + \sum_{ab} h_{ab} [T^a \otimes T^b] \right)_{\lambda_1 \lambda'_1, \lambda_2 \lambda'_2}$$

helicities ↓

$$f_a = \frac{1}{6} \text{Tr} [\rho (\mathbb{1} \otimes T^a)]$$

Spin polarization of particle 1

$$g_a = \frac{1}{6} \text{Tr} [\rho (T^a \otimes \mathbb{1})]$$

Spin polarization of particle 2

$$h_{ab} = \frac{1}{4} \text{Tr} [\rho (T^a \otimes T^b)]$$

Spin **correlations** of particles 1 and 2

● we can also **extract them from data**, using the angular distributions of their decay products

Bell inequalities for Two-QUTRITS

- take a bi-partite system with components A,B
- perform two measurements $\longrightarrow (A_1, A_2) \quad (B_1, B_2)$ each can take values $\rightarrow \{0, 1, 2\}$ on A,B
- consider the following correlator $\mathcal{I}_3$ for probability measurements

$$\begin{aligned} \mathcal{I}_3 = & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ & - P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1) \end{aligned}$$

$\mathcal{I}_3$ can be expressed as

$$\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}]$$

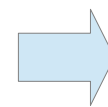
with $\mathcal{B}$ a suitable Bell operator

Collins, Gisin, Linden,
Massar, Popescu, PRL 88 (2002)

Bell inequalities for two-qutrits

For deterministic local models

$$\mathcal{I}_3 \leq 2$$

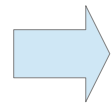


QM for qutrits can violate this
inequality with upper bound = 4

- The explicit form of $\mathcal{B}$ depends on the choice of measured operators $\hat{A}_i, \hat{B}_i, i \in \{1, 2\}$ given the two-qutrit state ρ it is possible to enhance $\mathcal{I}_3$ through a specific choice of these operators

- problem of finding optimal choice for these operators has been found

for maximal entangled state



$$|\Psi_+\rangle = \frac{1}{\sqrt{3}} \sum_{i=1}^3 |i\rangle \otimes |i\rangle$$

Pure state

$$\rho = |\Psi_+\rangle \langle \Psi_+|$$

given by the Bell operator $\mathcal{B}$ (see backup slides)

- Still freedom from Bob and Alice to choose directions where to measure the spin in such a way to maximize $\mathcal{I}_3$
- this can be achieved by performing unitary transformations on the Bell operator as

$$\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$



U, V are unitary 3x3 matrices
(depend on the kinematic of the process)

scalar/pseudoscalar decay in two vectors $S \rightarrow V_1 V_2$

- $V_1 V_2$ comes out as a pure state

Fabbrichesi, Floreanini, EG, Marzola
2304.02403 [hep-ph]
2305.04982 [hep-ph]

Approach based on helicity amplitudes

helicity amplitude $\rightarrow$

$$h_\lambda = \langle V_1(\lambda) V_2(-\lambda) | \mathcal{H} | B \rangle \quad \text{with} \quad \lambda = (+, 0, -)$$

generic scalar meson (for instance B meson)

There are 3 helicity amplitude combination

helicities defined with respect to
z-direction (spin quantization axis)
in rest frame of one of the two Vectors

$$|\Psi\rangle = \frac{1}{\sqrt{|H|^2}} \left[h_+ |V_1(+) V_2(-)\rangle + h_0 |V_1(0) V_2(0)\rangle + h_- |V_1(-) V_2(+)\rangle \right]$$

$\downarrow$

$$|H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2$$

Relative weight of $|0\ 0\rangle$ with respect to $|+\ -\rangle$ or $|-\ +\rangle$ controlled by angular momentum conservation

Density matrix for a pure $V_1 V_2$ state can be written as

$$\rho = |\Psi\rangle\langle\Psi|$$

Fabbrichesi, Floreanini, EG, Marzola
2304.02403 [hep-ph]
2305.04982 [hep-ph]

Master formula

in helicity basis

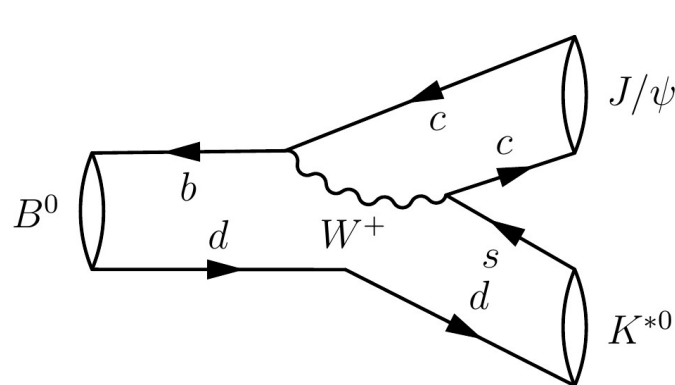
$$\rho = \frac{1}{|H|^2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+ h_+^* & 0 & h_+ h_0^* & 0 & h_+ h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0 h_+^* & 0 & h_0 h_0^* & 0 & h_0 h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_- h_+^* & 0 & h_- h_0^* & 0 & h_- h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

proved also using most general Lorentz invariant structure for amplitude

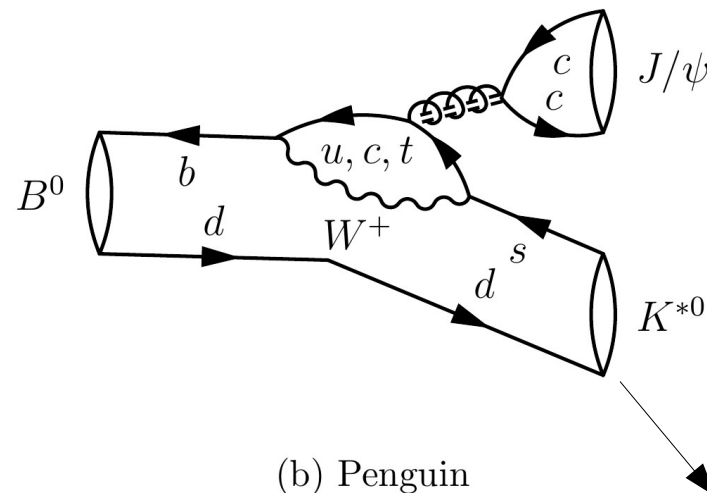
$$\frac{h_0}{|H|} = A_0, \quad \frac{h_+}{|H|} = \frac{A_{\parallel} + A_{\perp}}{\sqrt{2}} \quad \text{and} \quad \frac{h_-}{|H|} = \frac{A_{\parallel} - A_{\perp}}{\sqrt{2}}$$

Polarization amplitudes directly measured in experimental analysis of B meson decays !

Bell inequality violation in B meson decay $B \rightarrow J/\psi K^*$



(a) Tree



(b) Penguin

Smaller contribution, not relevant for Bell inequality, but crucial for FCNC, CP violation tests

Measured by several collaborations, most recent one

R.Aaij et al [LHCb], PRD, 88 (2013) 052002, 1307.2782 [hep-exp]

From each decay

$$J/\psi \rightarrow \mu^+ \mu^-$$

$$K^* \rightarrow K^+ \pi^-$$



We can measure polarization amplitudes

- from polarization amplitudes we can compute the $\mathcal{I}_3$ observable for qutrits
- This technique can be applied to ALL type of two spin-1 decays of B mesons

- 4 independent parameters: 3 amplitudes modulus and 2 relative phases

Polarization amplitudes

$$A_{\parallel} = |A_{\parallel}| e^{i\delta_{\parallel}}$$

We choose $\delta_0 = 0$

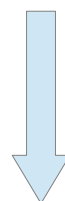
$$A_{\perp} = |A_{\perp}| e^{i\delta_{\perp}}$$

Phases due to final state interactions

$$A_0 = |A_0| e^{i\delta_0}$$

induced by strong interactions due to rescattering

Distance when both have decayed
can be estimated to be $\sim 1100 \text{ fm}$



Decays take place outside the range of strong interaction at the time of their production ($3 \times 10^{-5} \text{ fm}$ gluon exchange) as well as of final state interactions (1.5 fm exchange of pions)

Using experimental analysis from LHCb for $B \rightarrow J/\psi K^*$

$|A_{\parallel}|^2 = 0.227 \pm 0.004 \text{ (stat)} \pm 0.011 \text{ (sys)}$ $|A_{\perp}|^2 = 0.201 \pm 0.004 \text{ (stat)} \pm 0.008 \text{ (sys)}$ $\delta_{\parallel} \text{ [rad]} = -2.94 \pm 0.02 \text{ (stat)} \pm 0.03 \text{ (sys)}$ $\delta_{\perp} \text{ [rad]} = 2.94 \pm 0.02 \text{ (stat)} \pm 0.02 \text{ (sys)},$

1307.2782 [hep-exp]
+ error matrix correlation (included)

$$|A_0|^2 + |A_{\perp}|^2 + |A_{\parallel}|^2 = 1$$

$\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}]$

maximized using $\mathcal{B} \rightarrow (U \otimes V)^{\dagger} \cdot \mathcal{B} \cdot (U \otimes V)$

(see backup slides for U,V)

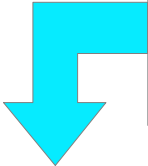
classical locality requires $\mathcal{I}_3 < 2$

2305.04982 [hep-ph]

Bell inequality $\rightarrow$

$\mathcal{I}_3 = 2.548 \pm 0.015$

$\Rightarrow B^0 \rightarrow J/\psi K^*(892)^0$

 $\mathcal{I}_3 < 2$ is violated with a significance of 36σ

First time violation of Bell inequalities in qutrits and in the presence of weak and strong interactions

Possibility to measure also entanglement in B decays

- Due to the fact that B is a pseudo-scalar particle we can quantify **entanglement** in the **two qutrits system** of its two spin-1 particle decays
- by using **Entropy of Entanglement**: expressed in terms of helicity amplitudes h_i

$$\mathcal{E} = - \sum_i h_i^* h_i \log (h_i^* h_i) \quad i = +, -, 0$$

- and substituting relation of helicity amplitudes with polarization amplitudes

$$\mathcal{E} = 0.756 \pm 0.0009 \quad \Rightarrow \quad B^0 \rightarrow J/\psi K^*(892)^0$$
$$\mathcal{E}^{\max} = \log 3 \simeq 1.1$$

2305.04982 [hep-ph]

86 σ level from null hypothesis $\mathcal{E} = 0$

Quantum entanglement and Bell violation in other B decays

2305.04982 [hep-ph]

$0 < \text{von Neumann entropy} < \log(3) \sim 1.1$

$$\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}]$$

	$\mathcal{E}$	$\mathcal{I}_3$	
• $B^0 \rightarrow J/\psi K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015	
• $B^0 \rightarrow \phi K^*(892)^0$ [20]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$	$\rightarrow 1.1\sigma$
• $B^0 \rightarrow \rho K^*(892)^0$ [21]	$0.450 \pm 0.067^*$	$2.208 \pm 0.129^*$	$\rightarrow 1.6\sigma$
• $B_s \rightarrow \phi\phi$ [22]	$0.734 \pm 0.050^*$	$2.525 \pm 0.084^*$	$\rightarrow 6.2\sigma$
• $B_s \rightarrow J/\psi\phi$ [23]	0.731 ± 0.032	2.462 ± 0.080	$\rightarrow 5.8\sigma$

● all these decays show a high level of entanglement

● values with * refer to lower bound on errors (because correlation matrix is not available)

significance on Bell inequality should increase after including error correlations

- [20] K. F. Chen *et al.* [Belle], Phys. Rev. Lett. **94**, 221804 (2005) [arXiv:hep-ex/0503013 [hep-ex]].
- [5] R. Aaij *et al.* [LHCb], Phys. Rev. D **88**, 052002 (2013) [arXiv:1307.2782 [hep-ex]].
- [21] R. Aaij *et al.* [LHCb], JHEP **05**, 026 (2019) [arXiv:1812.07008 [hep-ex]].
- [22] R. Aaij *et al.* [LHCb], [arXiv:2304.06198 [hep-ex]].
- [23] G. Aad *et al.* [ATLAS], Eur. Phys. J. C **81**, no.4, 342 (2021) [arXiv:2001.07115 [hep-ex]].

Summary

- **Bell inequality is violated in B meson decay** $B \rightarrow J/\psi K^*$
 - ▶ **First time observation** (with **36σ** level significance) in **Weak** and **Strong** interactions and at **high energies** ($\sim \text{GeV}$) and in **quark** systems
- **Observation of Entanglement** (same as ATLAS) with **85σ** level significance
- Entanglement and Bell violation observed also in other B mesons decays (with less significance)
- $B \rightarrow \phi \phi$ violates Bell inequality at **6σ level** and it is also **locality loophole free**
→ higher significance could be achieved if error correlations are included
- Final remark: LHCb experiment **was not designed** to test the Bell inequality and quantum entanglement

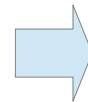
Backup slides

Closing the **locality loophole**

Bell locality, even if satisfied, could be bypassed if it is possible for the entangled states to communicate by means of a local interaction. This is the case for entangled states that end up in each other future cone, either because decay at different times or because they do not move apart fast enough

- One must consider the case where the decay products are the same particles (same width)
- $B \rightarrow \phi \phi$ decay offers this possibility

events satisfying the space-like condition



$$\frac{|t_1 - t_2| c}{(t_1 + t_2) v} < 1$$

$t_{1,2}$ → time of decays

Using the fact that decay times follow the PDF distribution in time (t) → $P(t) \sim \exp[-t \Gamma / \gamma]$

Γ → the particle width at rest
 γ → the Lorentz factor

- we find that almost 90% of events satisfies this condition
- Locality loophole can be closed

- Bell operator on the basis of S_3 spin operator

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Maximization of $\mathbf{I}_3$ achieved by $\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$

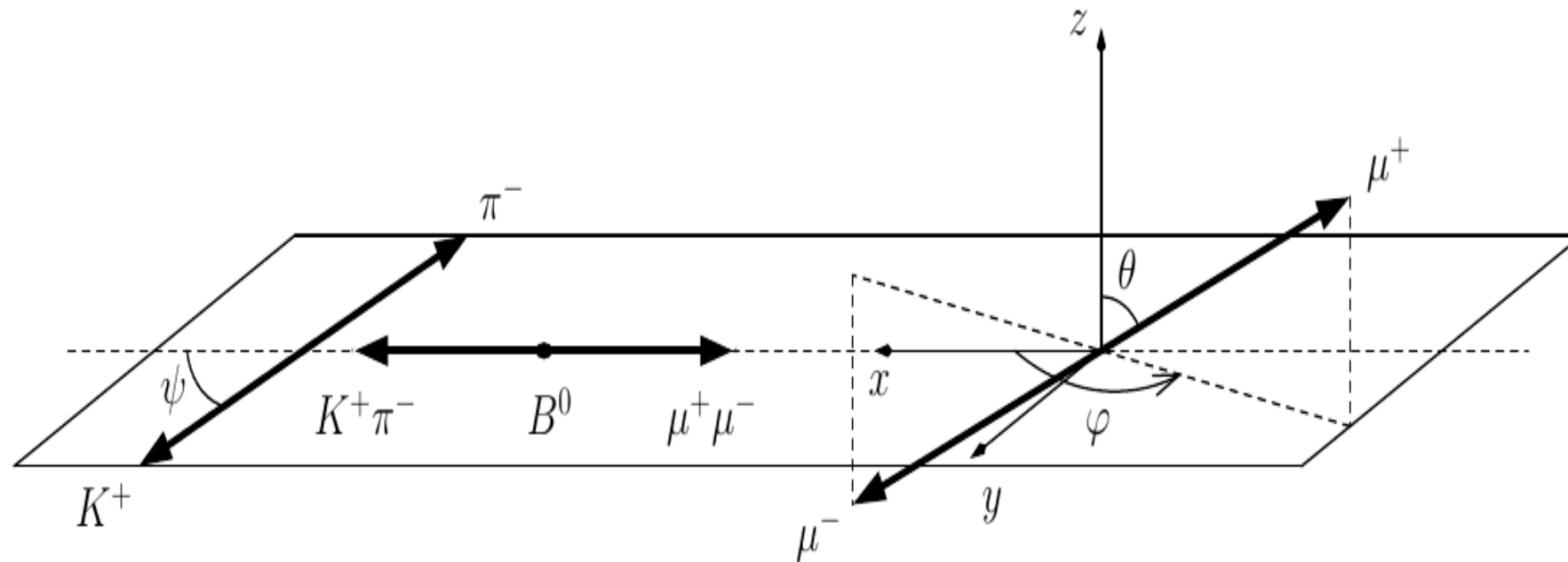
- approximated matrices up to 1% for the central value of $\mathbf{I}_3$ for $B^0 \rightarrow J/\psi K^*$

$$U = \begin{pmatrix} \frac{5}{14} - \frac{8i}{13} & \frac{1}{20} - \frac{4i}{7} & \frac{1}{4} - \frac{i}{3} \\ 0 & \frac{1}{18} - \frac{7i}{12} & -\frac{7}{15} + \frac{11i}{17} \\ -\frac{4}{11} + \frac{3i}{5} & \frac{1}{17} - \frac{4i}{7} & \frac{1}{4} - \frac{i}{13} \end{pmatrix}, \quad V = \begin{pmatrix} -\frac{5}{12} - \frac{i}{99} & -\frac{2}{9} + \frac{8i}{15} & \frac{1}{51} + \frac{5i}{7} \\ -\frac{4}{5} - \frac{i}{38} & \frac{3}{13} - \frac{6i}{11} & 0 \\ -\frac{5}{12} - \frac{i}{59} & -\frac{3}{13} + \frac{9i}{17} & -\frac{5i}{7} \end{pmatrix}$$

- We have available U,V these matrices for all B decay processes analyzed

From angular distribution one can extract the polarization amplitudes and phases by a fit of coefficients h_k (see next slide)

$$\frac{d^4\Gamma(B^0 \rightarrow J/\psi K^{*0})}{dt d\Omega} \propto e^{-\Gamma_d t} \sum_{k=1}^{10} h_k f_k(\Omega) \quad \Omega = \{\cos \theta, \cos \psi, \varphi\}$$



FIT of coefficients h_k

$$\Omega = \{\cos \theta, \cos \psi, \varphi\}$$

1307.2782 [hep-exp]

$$\frac{d^4\Gamma(B^0 \rightarrow J/\psi K^{*0})}{dt d\Omega} \propto e^{-\Gamma_d t} \sum_{k=1}^{10} h_k f_k(\Omega)$$

k	h_k	$f_k(\Omega)$
1	$ A_0 ^2$	$\frac{9}{32\pi} 2 \cos^2 \psi (1 - \sin^2 \theta \cos^2 \varphi)$
2	$ A_{\parallel} ^2$	$\frac{9}{32\pi} \sin^2 \psi (1 - \sin^2 \theta \sin^2 \varphi)$
3	$ A_{\perp} ^2$	$\frac{9}{32\pi} \sin^2 \psi \sin^2 \theta$
4	$ A_{\parallel} A_{\perp} \sin(\delta_{\perp} - \delta_{\parallel})$	$-\frac{9}{32\pi} \sin^2 \psi \sin 2\theta \sin \varphi$
5	$ A_0 A_{\parallel} \cos(\delta_{\parallel})$	$\frac{9}{32\pi\sqrt{2}} \sin 2\psi \sin^2 \theta \sin 2\varphi$
6	$ A_0 A_{\perp} \sin(\delta_{\perp})$	$\frac{9}{32\pi\sqrt{2}} \sin 2\psi \sin 2\theta \cos \varphi$
7	$ A_S ^2$	$\frac{3}{32\pi} 2(1 - \sin^2 \theta \cos^2 \varphi)$
8	$ A_{\parallel} A_S \cos(\delta_{\parallel} - \delta_S)$	$\frac{3}{32\pi} \sqrt{6} \sin \psi \sin^2 \theta \sin 2\varphi$
9	$ A_{\perp} A_S \sin(\delta_{\perp} - \delta_S)$	$\frac{3}{32\pi} \sqrt{6} \sin \psi \sin 2\theta \cos \varphi$
10	$ A_0 A_S \cos(\delta_S)$	$\frac{3}{32\pi} 4\sqrt{3} \cos \psi (1 - \sin^2 \theta \cos^2 \varphi)$

(A_S contribution from
non-resonant $J/\psi K^*$ amplitude)