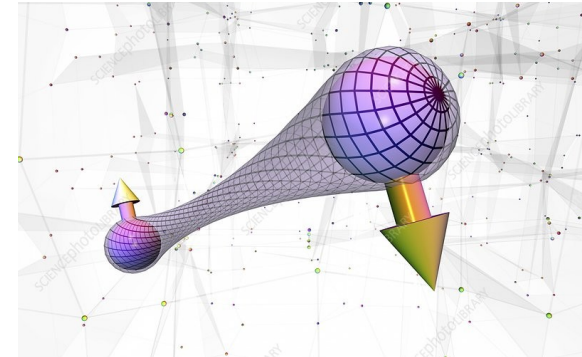


Quantum Entanglement and Bell inequalities of scalar/pseudoscalar decays in two spin-1 particles

Emidio Gabrielli

University of Trieste, Italy

NICPB, Tallinn, Estonia



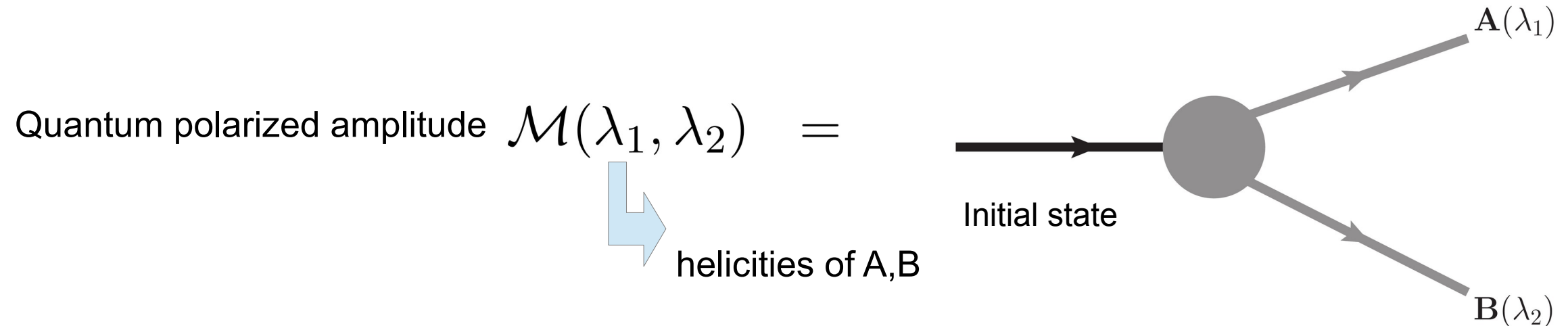
ATLAS meeting 30-Nov-2023

in collaboration with: Marco Fabbrichesi, Roberto Floreanini, and Luca Marzola

based on: 2304.02403 [hep-ph] and 2305.04982 [hep-ph]

Polarization density matrix: $S \rightarrow 2$ spin-1 particles

- Consider a scalar/pseudoscalar decaying into two **A,B** spin-1 particles



- polarization density matrix

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \frac{\mathcal{M}(\lambda_1, \lambda_2) \mathcal{M}^\dagger(\lambda'_1, \lambda'_2)}{|\overline{\mathcal{M}}|^2}$$

unpolarized

- Two methods available for computing or reconstructing density matrix of two qutrits
 - Gell-Mann matrices decomposition (or spherical harmonics basis) $\rightarrow$ more general
 - using **helicity amplitudes** $\rightarrow$ applies only to pure states

Gell-Mann matrices decomposition of density matrix for qutrits

Decompose **density matrix** ρ on the basis of tensor products of **Gell-Mann matrices**

T^a

$$[A \otimes B]_{ii'jj'} = A_{ii'} B_{jj'}$$

9x9 matrix

Unpolarized

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \left(\frac{1}{9} [\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [\mathbb{1} \otimes T^a] + \sum_a g_a [T^a \otimes \mathbb{1}] + \sum_{ab} h_{ab} [T^a \otimes T^b] \right)_{\lambda_1 \lambda'_1, \lambda_2 \lambda'_2}$$

helicities ↓

$$f_a = \frac{1}{6} \text{Tr} [\rho (\mathbb{1} \otimes T^a)]$$

Spin polarization of particle 1

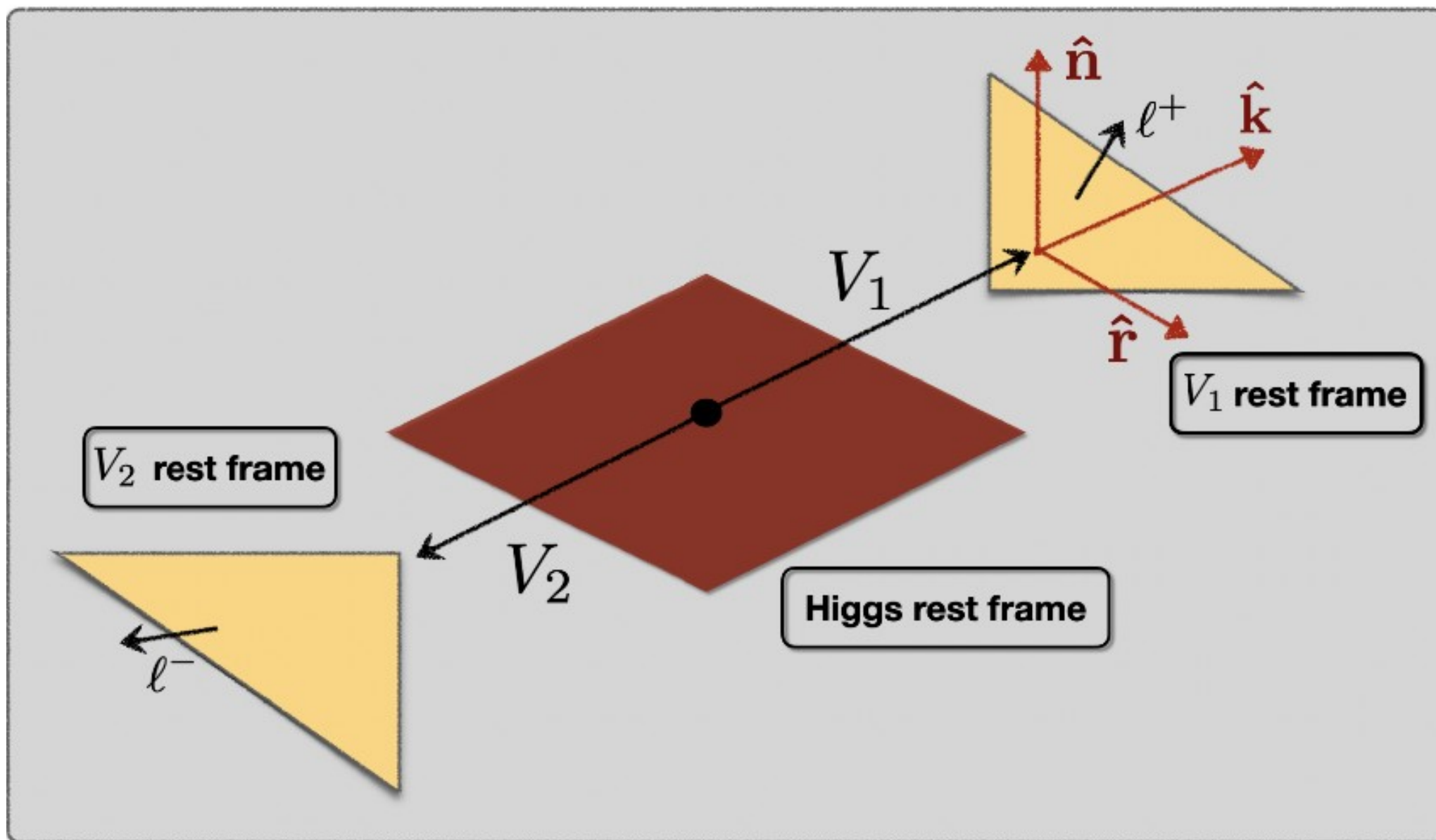
$$g_a = \frac{1}{6} \text{Tr} [\rho (T^a \otimes \mathbb{1})]$$

Spin polarization of particle 2

$$h_{ab} = \frac{1}{4} \text{Tr} [\rho (T^a \otimes T^b)]$$

Spin **correlations** of particles 1 and 2

● extract **f, g, h** coefficients from data using the angular distributions of their decay products



Extracting density matrix for **Two-Qutrits** from data

Differential cross section

Rahaman, Singh, NPB 984 (2022), 2109.09345 [hep-ph]

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega^+ d\Omega^-} = \left(\frac{3}{4\pi} \right)^2 \text{Tr} \left[\rho_{V_1 V_2} (\Gamma_+ \otimes \Gamma_-) \right]$$


 Kronecker product

$$d\Omega^\pm = \sin \theta^\pm d\theta^\pm d\phi^\pm \longrightarrow$$

phase space written in terms of the spherical coordinates (with independent polar axis) for the momenta of the final charged leptons in the respective rest frames of the decaying spin-1 particles

$\rho_{V_1 V_2}$ = density matrix of $V_1 V_2$

$\Gamma_\pm \longrightarrow$ Density matrices that describe the polarization of the two spin-1 massive vectors into final leptons (the charged ones assumed to be massless)

(in the case of the W-bosons these are projectors because of their chiral couplings to leptons)

The above example is for W+ and W- final states, but the method is general

can be computed by **rotating** to an arbitrary polar axis the spin ± 1 states of gauge bosons from the ones given in the **k-direction** quantization axis

$$\Gamma_{\pm} = \frac{1}{3} \mathbb{1} + \sum_{i=1}^8 q_{\pm}^a T^a$$

Example for the W+ W- final states

→ **Density matrices for W-bosons**

$q_{\pm}^a$ → **Winer q-symbols** (see backup slides) are functions of the solid angles $\Omega^{\pm}$

↓
set of polynomials
of spherical
coorindates
(see backup slide)

$$\begin{aligned} h_{ab} &= \frac{1}{\sigma} \int \int \frac{d\sigma}{d\Omega^+ d\Omega^-} p_+^a p_-^b d\Omega^+ d\Omega^- \\ f_a &= \frac{1}{\sigma} \int \frac{d\sigma}{d\Omega^+} p_+^a d\Omega^+ \\ g_a &= \frac{1}{\sigma} \int \frac{d\sigma}{d\Omega^-} p_-^a d\Omega^- \end{aligned}$$

$p_{\pm}^n$ a particular set of orthogonal functions (see backup slides) $\Rightarrow \left(\frac{3}{4\pi}\right) \int p_{\pm}^n q_{\pm}^m d\Omega^{\pm} = \delta^{nm}$

For **ZZ** case, the set of functions are linear combinations of $q_{\pm}^a$ (see backup slides)

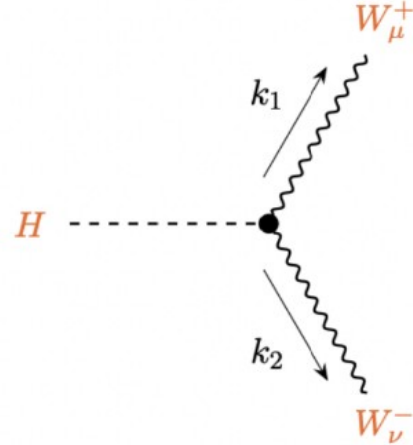
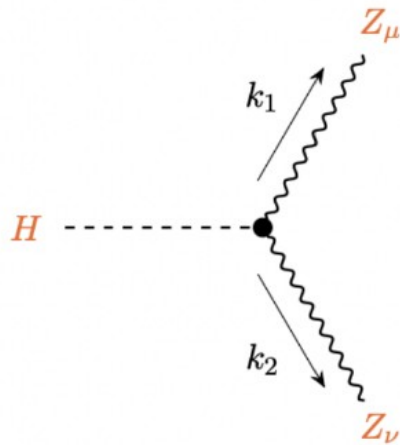
Reconstructing density matrix of $H \rightarrow VV^*$ from helicity amplitudes

- Consider as example the most general decay amplitude for the $H \rightarrow VV^*$ including anomalous couplings
- Fix the invariant mass of the virtual state of V^* (same as a massive spin-1)
- Method here can be applied to any scalar/pseudoscalar decay into two spin-1 particles with different masses
- Keep in mind that the following results hold only if the final state of $V_1 V_2$ is a pure state!

$$H \rightarrow V(k_1, \lambda_1) V^*(k_2, \lambda_2)$$

$$\lambda_{1,2} = \{+1, 0, -1\}$$

helicities



mass of V^*

$$M_V^* = f M_V$$

$$\xi_W = 1$$

$$\xi_Z = 1/(2c_W)$$

Polarized amplitude

$$\mathcal{M}(\lambda_1, \lambda_2) = M_{\mu\nu} \varepsilon^{\mu*}(k_1, \lambda_1) \varepsilon^{\nu*}(k_2, \lambda_2)$$

$$M^{\mu\nu} = g M_V \xi_V g^{\mu\nu} - \frac{g}{M_W} \left[a_V (k_1^\nu k_2^\mu - g^{\mu\nu} k_1 \cdot k_2) + \tilde{a}_V \epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} \right]$$

CP-odd

$$\rho_H = \frac{\mathcal{M}_{\mu\nu} \mathcal{M}_{\mu'\nu'}^\dagger}{|\overline{\mathcal{M}}|^2} \left[\underbrace{\mathcal{P}^{\mu\mu'}(k_1) \otimes \mathcal{P}^{\nu\nu'}(k_2)} \right]$$

Anomalous couplings

$$\text{SM} \rightarrow a_V = \tilde{a}_V = 0$$

- Remarkably the ZZ^* or WW^* with anomalous couplings is still a **pure state**

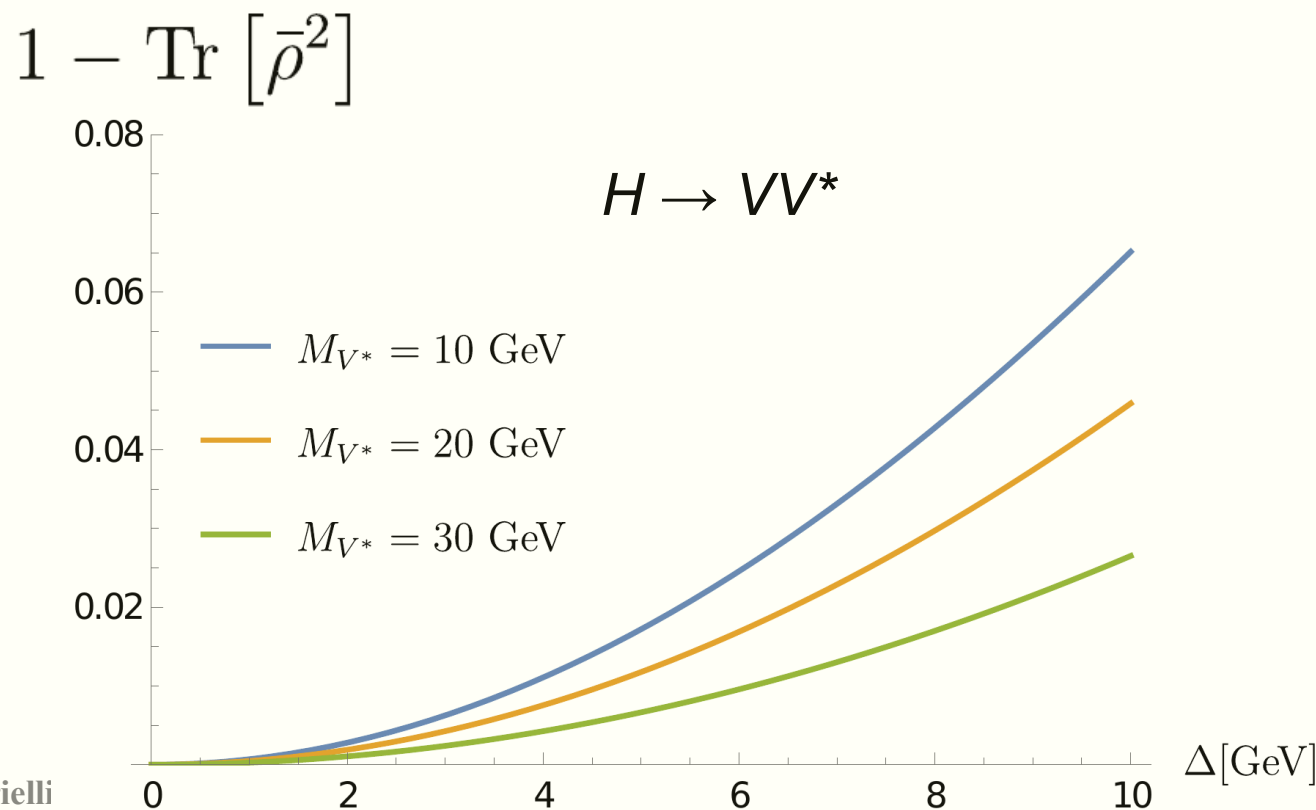
- Satisfying the relation $\rho_H^2 = \rho_H$ which holds only **at fixed mass** for V^*

- But we need to **average** density matrix in some bin Δ around the off-shell mass M_{V^*}

↓

$$\bar{\rho}(M_{V^*}, \Delta) = \frac{1}{2\Delta} \int_{M_{V^*}-\Delta}^{M_{V^*}+\Delta} dM_{V^*} \rho(M_{V^*})$$

- To quantifies the departure from pure state (as a function of Δ) we can use $1 - \text{Tr} [\bar{\rho}^2]$



- Effect is small, max of order of **5-6%** for large bins of order $\Delta \sim 10 \text{ GeV}$

- So we can safely use **Entropy** to quantify entanglement in $H \rightarrow ZZ^*$

Helicity amplitudes parametrization of ρ for qutrits

- Due to the fact that $H \rightarrow VV^*$ amplitude with anomalous couplings is the most general relativistic one, same conclusion holds for any $S/P \rightarrow V_1 V_2$
- At this purpose it is useful to define helicity amplitudes for $H \rightarrow V V^*$ decay

$$h_\lambda = \langle V(\lambda) V^*(-\lambda) | -\mathcal{L} | H \rangle \quad \text{with} \quad \lambda = 0, \pm$$

► Lagrangian

- Density matrix of the pure state is $\rho = |\Psi\rangle\langle\Psi|$

$$|\Psi\rangle = \frac{1}{|\mathcal{M}|} \left[h_+ |V(+)V^*(-)\rangle + h_0 |V(0)V^*(0)\rangle + h_- |V(-)V^*(+)\rangle \right]$$

Density matrix for a pure $V_1 V_2$ state can be written as

Master formula

Fabbrichesi, Floreanini, EG, Marzola
2304.02403 [hep-ph]
2305.04982 [hep-ph]

in helicity basis

$$\rho = \frac{1}{|H|^2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+ h_+^* & 0 & h_+ h_0^* & 0 & h_+ h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0 h_+^* & 0 & h_0 h_0^* & 0 & h_0 h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_- h_+^* & 0 & h_- h_0^* & 0 & h_- h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

It is a general formula valid for any scalar/pseudoscalar decay $S/P \rightarrow V_1 V_2$

Proved also using most general Lorentz invariant structure for the amplitude decay

- Alternatively, one can decompose the decay amplitude into three independent components, corresponding to **linear polarization** states of the spin-1 particles which are **either longitudinal (0) or transverse** to their directions of motion and **parallel (||) or perpendicular** to one another ($\perp$)

$$\frac{h_0}{|H|} = A_0, \quad \frac{h_+}{|H|} = \frac{A_{||} + A_{\perp}}{\sqrt{2}} \quad \text{and} \quad \frac{h_-}{|H|} = \frac{A_{||} - A_{\perp}}{\sqrt{2}}$$

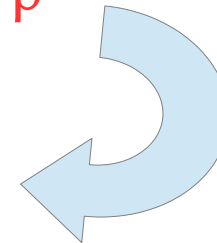
Polarization amplitudes directly measured (for example in vector meson decays of B (see next))

We need to substitute relations above into the expression of ρ

Master formula

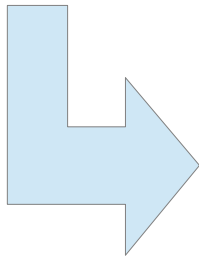
in helicity basis

$$\rho = \frac{1}{|H|^2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+ h_+^* & 0 & h_+ h_0^* & 0 & h_+ h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0 h_+^* & 0 & h_0 h_0^* & 0 & h_0 h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_- h_+^* & 0 & h_- h_0^* & 0 & h_- h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

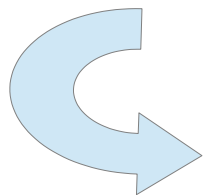


In the SM Higgs decay $H \rightarrow VV^*$ the state $|\Psi\rangle$ simplifies

$$|\Psi\rangle = \frac{1}{\sqrt{2 + \chi^2}} \left[|V(+)V^*(-) \rangle - \chi |V(0)V^*(0) \rangle + |V(-)V^*(+) \rangle \right]$$



$$\begin{aligned} \frac{h_+}{|\bar{\mathcal{M}}|} &= \frac{h_-}{|\bar{\mathcal{M}}|} = \frac{1}{\sqrt{2 + \chi^2}} \\ \frac{h_0}{|\bar{\mathcal{M}}|} &= -\frac{\chi}{\sqrt{2 + \chi^2}} \end{aligned}$$



$$\chi = 1 + (m_H^2 - (1 + f)^2 M_V^2) / (2f M_V^2)$$

= 1 at the threshold $\rightarrow$ (VV^* both at rest) $\rightarrow$ maximum entanglement

- Require 4 independent parameters: 3 amplitudes modulus and 2 relative phases

Polarization amplitudes

$$\begin{aligned} A_{\parallel} &= |A_{\parallel}| e^{i\delta_{\parallel}} \\ A_{\perp} &= |A_{\perp}| e^{i\delta_{\perp}} \\ A_0 &= |A_0| e^{i\delta_0} \end{aligned}$$

phases are due to final state interactions for $H \rightarrow VV^*$ (due to EW rescattering)
 $\rightarrow$ relative signs reabsorbed in the phases

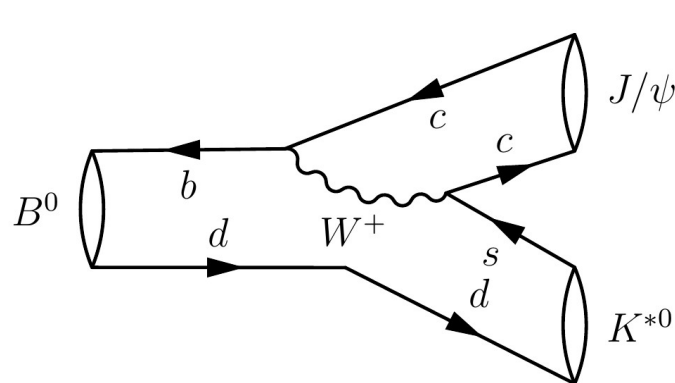
in the **Gell-Mann basis** decomposition for $H \rightarrow VV^*$

$$\rho_H = 2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{16} & 0 & 2h_{33} & 0 & h_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

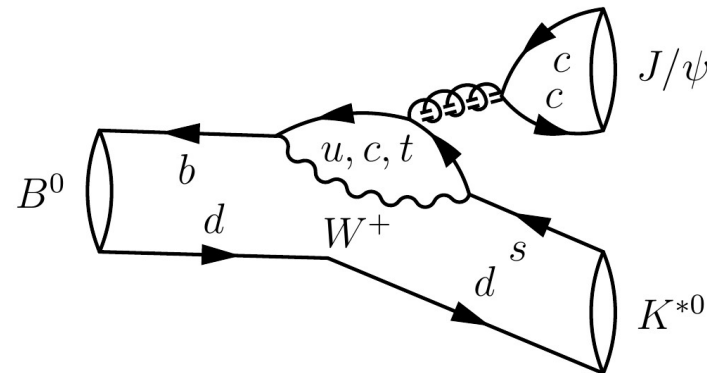
relation of coefficients in **Gell-Mann** and **helicity-amplitude** decompositions

$$\begin{aligned} h_{44} &= h_+ h_+^* = h_+ h_-^* \\ h_{16} &= h_+ h_0^* = h_0 h_+^* = h_- h_0^* = h_0 h_-^* \\ h_{33} &= h_+ h_+^* = h_0 h_0^* \end{aligned}$$

Example of B meson decay $B \rightarrow J/\psi K^*$



(a) Tree



(b) Penguin

R.Aaij et al [LHCb], PRD, 88 (2013) 052002, 1307.2782 [hep-exp]

From each decay

$$J/\psi \rightarrow \mu^+ \mu^-$$

$$K^* \rightarrow K^+ \pi^-$$

→ We can measure polarization amplitudes

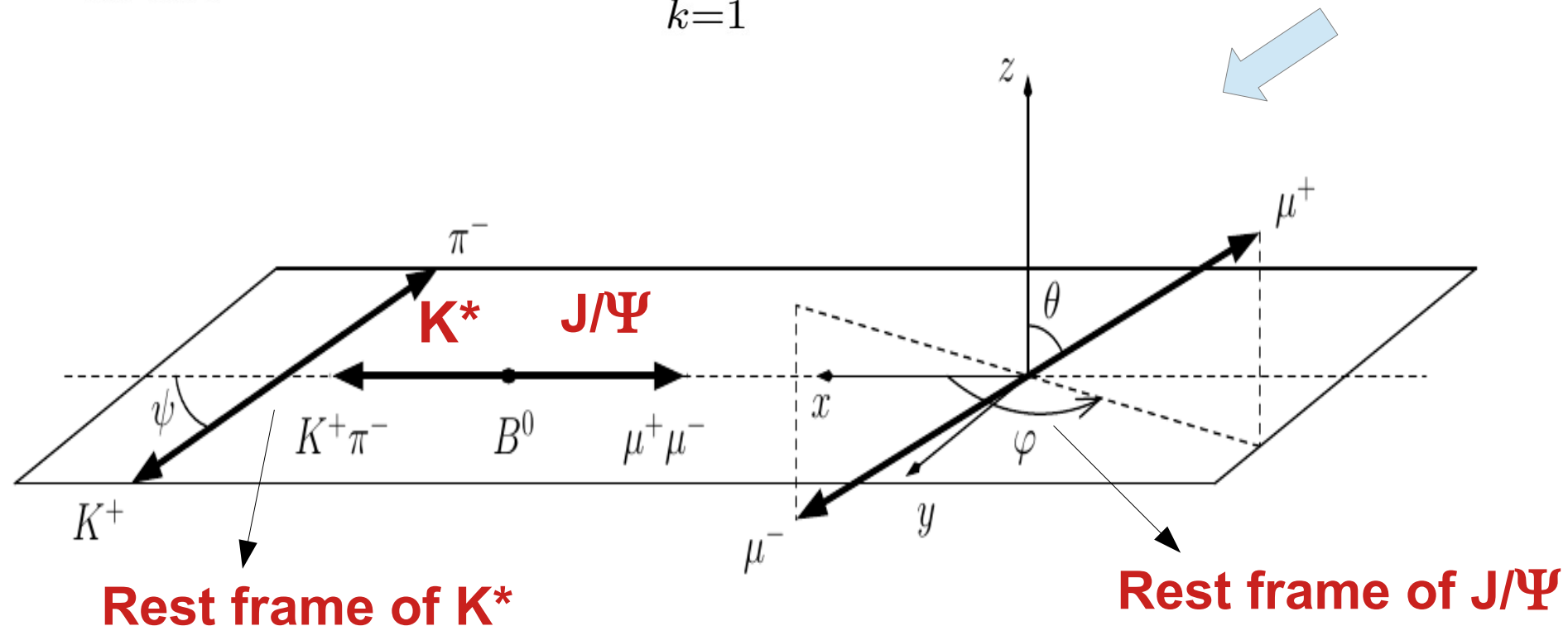
- from polarization amplitudes we can reconstruct the density matrix and compute the $\mathcal{I}_3$ observable for qutrits using the master formula above

- Same method can in principle be used also in $H \rightarrow Z Z^*$ for $Z / Z^* \rightarrow e^+ e^-$ or $\mu^+ \mu^-$

From angular distribution one can extract the polarization amplitudes and phases by a fit of coefficients h_k (see next slide)

$$\frac{d^4\Gamma(B^0 \rightarrow J/\psi K^{*0})}{dt d\Omega} \propto e^{-\Gamma_d t} \sum_{k=1}^{10} h_k f_k(\Omega)$$

$$\Omega = \{\cos \theta, \cos \psi, \varphi\}$$



FIT of coefficients h_k

$$\Omega = \{\cos \theta, \cos \psi, \varphi\}$$

1307.2782 [hep-exp]

Dighe, Dunietz, Lipkin,
Rosner, PLB 369 144
(1996), **9511363 [hep-ph]**

k	h_k	$f_k(\Omega)$
1	$ A_0 ^2$	$\frac{9}{32\pi} 2 \cos^2 \psi (1 - \sin^2 \theta \cos^2 \varphi)$
2	$ A_{\parallel} ^2$	$\frac{9}{32\pi} \sin^2 \psi (1 - \sin^2 \theta \sin^2 \varphi)$
3	$ A_{\perp} ^2$	$\frac{9}{32\pi} \sin^2 \psi \sin^2 \theta$
4	$ A_{\parallel} A_{\perp} \sin(\delta_{\perp} - \delta_{\parallel})$	$-\frac{9}{32\pi} \sin^2 \psi \sin 2\theta \sin \varphi$
5	$ A_0 A_{\parallel} \cos(\delta_{\parallel})$	$\frac{9}{32\pi\sqrt{2}} \sin 2\psi \sin^2 \theta \sin 2\varphi$
6	$ A_0 A_{\perp} \sin(\delta_{\perp})$	$\frac{9}{32\pi\sqrt{2}} \sin 2\psi \sin 2\theta \cos \varphi$
7	$ A_S ^2$	$\frac{3}{32\pi} 2(1 - \sin^2 \theta \cos^2 \varphi)$
8	$ A_{\parallel} A_S \cos(\delta_{\parallel} - \delta_S)$	$\frac{3}{32\pi} \sqrt{6} \sin \psi \sin^2 \theta \sin 2\varphi$
9	$ A_{\perp} A_S \sin(\delta_{\perp} - \delta_S)$	$\frac{3}{32\pi} \sqrt{6} \sin \psi \sin 2\theta \cos \varphi$
10	$ A_0 A_S \cos(\delta_S)$	$\frac{3}{32\pi} 4\sqrt{3} \cos \psi (1 - \sin^2 \theta \cos^2 \varphi)$

similar to method used to extract
correlation coefficients from
Gell-Mann basis decomposition

phases are due to strong interaction
rescattering of final states

(A_S contribution from
non-resonant J/Y K^* amplitude)

Conclusions

- Helicity amplitude method offers a powerful tool for reconstructing polarization density matrix in scalar/pseudoscalar decays in two spin-1 particles → Valid only for pure states
- In the case of SM Higgs → VV^* the density matrix of VV^* simplifies w.r.t to the most general case of scalar/pseudoscalar decay
- To extract correlation and polarization coefficients (f, g, h) from data one has to project the differential angular distributions of leptons (related to V and V^* decays in their respective rest frames) on the Wigner polynomials for the Gell-Mann decomposition, OR on different kind of polynomials by using helicity-amplitude method
- No particular advantages in using one method over the other
- We estimate that the condition of pure state $\rho_H^2 = \rho_H$ is broken by max 5-6 % effect in the reconstruction of density matrix for $H \rightarrow VV^*$ once we average over M_{V^*} → Entropy of Entanglement can be safely used to quantify entanglement in this case !

Backup slides

Wigner's Q symbols

$$\begin{aligned}
 q_{\pm}^1 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(\cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 q_{\pm}^2 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(\cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 q_{\pm}^3 &= \frac{1}{8} \left(1 \pm 4 \cos \theta^{\pm} + 3 \cos 2\theta^{\pm} \right), \\
 q_{\pm}^4 &= \frac{1}{2} \sin^2 \theta^{\pm} \cos 2\phi^{\pm}, \\
 q_{\pm}^5 &= \frac{1}{2} \sin^2 \theta^{\pm} \sin 2\phi^{\pm}, \\
 q_{\pm}^6 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(-\cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 q_{\pm}^7 &= \frac{1}{\sqrt{2}} \sin \theta^{\pm} \left(-\cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 q_{\pm}^8 &= \frac{1}{8\sqrt{3}} \left(-1 \pm 12 \cos \theta^{\pm} - 3 \cos 2\theta^{\pm} \right),
 \end{aligned}$$

$$\begin{aligned}
 p_{\pm}^1 &= \sqrt{2} \sin \theta^{\pm} \left(5 \cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 p_{\pm}^2 &= \sqrt{2} \sin \theta^{\pm} \left(5 \cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 p_{\pm}^3 &= \frac{1}{4} \left(5 \pm 4 \cos \theta^{\pm} + 15 \cos 2\theta^{\pm} \right), \\
 p_{\pm}^4 &= 5 \sin^2 \theta^{\pm} \cos 2\phi^{\pm}, \\
 p_{\pm}^5 &= 5 \sin^2 \theta^{\pm} \sin 2\phi^{\pm}, \\
 p_{\pm}^6 &= \sqrt{2} \sin \theta^{\pm} \left(-5 \cos \theta^{\pm} \pm 1 \right) \cos \phi^{\pm}, \\
 p_{\pm}^7 &= \sqrt{2} \sin \theta^{\pm} \left(-5 \cos \theta^{\pm} \pm 1 \right) \sin \phi^{\pm}, \\
 p_{\pm}^8 &= \frac{1}{4\sqrt{3}} \left(-5 \pm 12 \cos \theta^{\pm} - 15 \cos 2\theta^{\pm} \right).
 \end{aligned}$$

For the **ZZ production** the density matrices $\Gamma_{\pm}$ are not projector due to the Z boson coupling

$$\mathcal{L} \supset -i \frac{g}{\cos \theta_W} \left[g_L (1 - \gamma^5) \gamma_\mu + g_R (1 + \gamma^5) \gamma_\mu \right] Z^\mu$$

$$\tilde{\mathbf{q}}^n = \frac{1}{g_R^2 + g_L^2} \left[g_R^2 \mathbf{q}_+^n + g_L^2 \mathbf{q}_-^n \right]$$

$$\tilde{\mathbf{p}}^n = \sum_m \mathbf{a}_m^n \mathbf{p}_+^m$$

$$\mathbf{a}_m^n = \frac{1}{g_L^2 - g_R^2} \begin{pmatrix} g_R^2 & 0 & 0 & 0 & 0 & g_L^2 & 0 & 0 \\ 0 & g_R^2 & 0 & 0 & 0 & 0 & g_L^2 & 0 \\ 0 & 0 & g_R^2 - \frac{1}{2} g_L^2 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} g_L^2 \\ 0 & 0 & 0 & g_R^2 - g_L^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_R^2 - g_L^2 & 0 & 0 & 0 \\ g_L^2 & 0 & 0 & 0 & 0 & g_R^2 & 0 & 0 \\ 0 & g_L^2 & 0 & 0 & 0 & 0 & g_R^2 & 0 \\ 0 & 0 & \frac{\sqrt{3}}{2} g_L^2 & 0 & 0 & 0 & 0 & \frac{1}{2} g_L^2 - g_R^2 \end{pmatrix}.$$

quantifying entanglement in pure states

2302.00683 [hep-ph]

- If the bipartite (A,B) system is a pure state it is possible to quantify its entanglement via

Entropy of entanglement

$$\mathcal{E}[\rho] = -\text{Tr} [\rho_A \log \rho_A] = -\text{Tr} [\rho_B \log \rho_B]$$

density matrix of reduced A or B system

equality holds if and only if the bipartite is separable

$$0 \leq \mathcal{E}[\rho] \leq \ln 3$$

(Von Neumann Entropy)

↓
separable states

↓
Maximum entanglement

- Very useful in the case of bipartite (A,B) system of qutrit for which there is not a solution for the general case

Bell inequalities for Two-QUTRITS

- take a bi-partite system with components A,B
- perform two measurements $\longrightarrow (A_1, A_2) \quad (B_1, B_2)$ each can take values $\rightarrow \{0, 1, 2\}$ on A,B
- consider the following correlator $\mathcal{I}_3$ for probability measurements

$$\begin{aligned} \mathcal{I}_3 = & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ & - P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1) \end{aligned}$$

$\mathcal{I}_3$ can be expressed as

$$\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}]$$

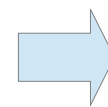
with $\mathcal{B}$ a suitable Bell operator

Collins, Gisin, Linden,
Massar, Popescu, PRL 88 (2002)

Bell inequalities for two-qutrits

For deterministic local models

$$\mathcal{I}_3 \leq 2$$



QM for qutrits can violate this inequality with upper bound = 4

- problem of finding optimal choice has been found

for the maximal entangled state

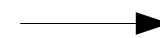
$$|\Psi_+\rangle = \frac{1}{\sqrt{3}} \sum_{i=1}^3 |i\rangle \otimes |i\rangle$$

$$\rho = |\Psi_+\rangle \langle \Psi_+|$$

given by the Bell operator $\mathcal{B}$ (see backup slides)

- Still freedom from Bob and Alice to choose directions where to measure the spin in such a way to maximize $\mathcal{I}_3$
- this can be achieved by performing unitary transformations on the Bell operator as

$$\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$



U, V are unitary 3x3 matrices
(depend on the kinematic of the process)

Quantum entanglement and Bell violation in B decays

2305.04982 [hep-ph]

$0 < \text{von Neumann entropy} < \ln[3] \sim 1.1$

$$\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}]$$

	$\mathcal{E}$	$\mathcal{I}_3$	
• $B^0 \rightarrow J/\psi K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015	$\rightarrow 36\sigma$
• $B^0 \rightarrow \phi K^*(892)^0$ [20]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$	$\rightarrow 1.1\sigma$
• $B^0 \rightarrow \rho K^*(892)^0$ [21]	$0.450 \pm 0.067^*$	$2.208 \pm 0.129^*$	$\rightarrow 1.6\sigma$
• $B_s \rightarrow \phi \phi$ [22]	$0.734 \pm 0.050^*$	$2.525 \pm 0.084^*$	$\rightarrow 6.2\sigma$
• $B_s \rightarrow J/\psi \phi$ [23]	0.731 ± 0.032	2.462 ± 0.080	$\rightarrow 5.8\sigma$

● all these decays show a high level of entanglement

● values with * refer to lower bound on errors (because correlation matrix is not available)

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