

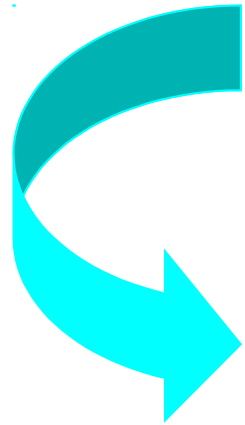
Physics of Extra-Dimensions

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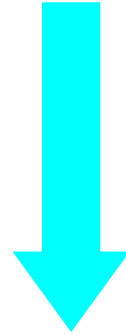
Extra-Dimensions



mm

ADD model:
compact ED on a
circle of large radius

only **gravity** can
propagates in the bulk



TeV^{-1}

- SM matter fields fixed on the brane:
only **gauge fields** propagate in the bulk
- universal ED:
all SM fields propagate in the bulk



Warped $\rightarrow$ AdS_5

RS Model
5-d non factorizable
Geometry

SM fields on TeV brane
only **gravity** propagates
in the bulk

Gravity



$$M_{Pl} = 10^{19} \text{ GeV}$$

The hierarchy problem

quantum
corrections

problem of
quadratic
divergencies

large fine tuning



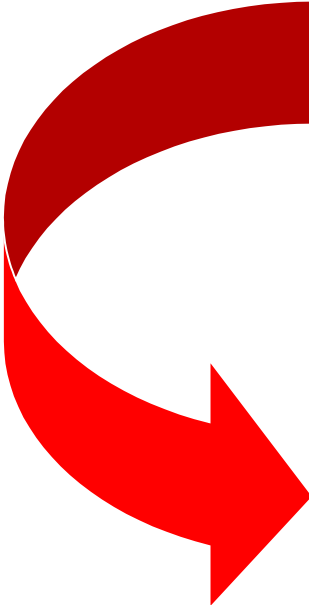
EW theory



$$M_W = 81 \text{ GeV}$$

ADD model

- EW scale is a fund. scale
- Planck mass is a derived effect from geometry

- 
- Compact ExtraDim on a circle of large radius
 - confinement of matter on subspace

Gravity propagating in

$$D = 4 + \delta$$

compactification
of each ED on a
circle

$$G_N^D$$

dim

$$G_N = \frac{G_N^D}{V_{D-4}}$$

fundamental constant: Newton constant
in D space : strong gravity

usual gravity is weak due to large compact
ED space

Newton law modified at small distances

- $r > R$

$$F_N(r) = G_N \frac{m_1 m_2}{r^2}$$

- $r < R$

$$F_N(r) = G_N R^\delta \left(\frac{m_1 m_2}{r^{2+\delta}} \right)$$

the potential

$$V(r) = G_N \frac{m_1 m_2}{r} \left(1 + C e^{-\frac{r}{\lambda}} \right)$$

relation between Plank masses

$$G_N = \frac{1}{M_{\text{pl}}^2}$$

$$G_N^D = \frac{1}{M_D^{D-2}}$$

$$\delta \geq 2$$

$$M_{\text{pl}} = \frac{1}{\sqrt{8\pi}} M_D (\mathbf{R} M_D)^{\frac{D-4}{2}}$$

$$M_D = 1 \text{ TeV}$$

$$R \simeq 10^{\frac{30}{\delta}-17} \text{ cm}$$

- $\delta = 1$ $R = (5 \times 10^{-4} \text{ eV})^{-1} \simeq 10^8 \text{ km}$

ruled out

- $\delta = 2$ $R = (5 \times 10^{-4} \text{ eV})^{-1} \simeq 0.4 \text{ mm}$

allowed

Kaluza-Klein excitations

- Massive KK Gravitons

$$\mathcal{L}_{\text{int}}(\mathbf{x}) \sim \frac{1}{M_{\text{pl}}} \sum_{\mathbf{n}} \mathbf{G}_{\mu\nu}^{(\mathbf{n})}(\mathbf{x}) T^{\mu\nu}(\mathbf{x})$$

- Massive KK Graviton Scalars

$$\mathcal{L}_{\text{int}}(\mathbf{x}) \sim \frac{1}{M_{\text{pl}}} \sum_{\mathbf{n}} \mathbf{S}^{(\mathbf{n})}(\mathbf{x}) T_{\mu}^{\mu}(\mathbf{x})$$

$T_{\mu\nu}(\mathbf{x})$ Energy Momentum Tensor

Masses of KK excitations $M_{\mathbf{n}}^2 \sim \frac{n^2}{R^2}$

Direct KK graviton production

- Mass splitting

$$\Delta m \simeq \frac{1}{R} = M_D \left(\frac{M_D}{M_{\text{pl}}} \right)^{2/\delta}$$

$$\Delta m = 20\text{KeV} \quad (\delta = 4)$$

$$\Delta m = 7\text{MeV} \quad (\delta = 6)$$

**almost a continuous
distribution of masses**

For small $\delta \leq 6$:

Large number of accessible KK modes.

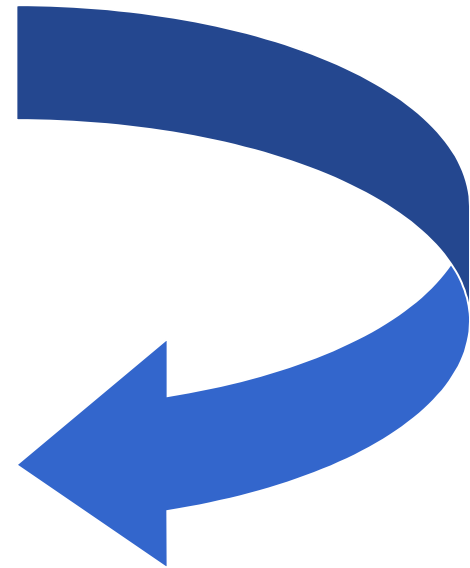
continuum: $\sum_n \longrightarrow \int dN$

$$dN = A_{\delta-1} \left(\frac{M_{Pl}^2}{M_D^{2+\delta}} \right) m_G^{\delta-1} dm_G$$

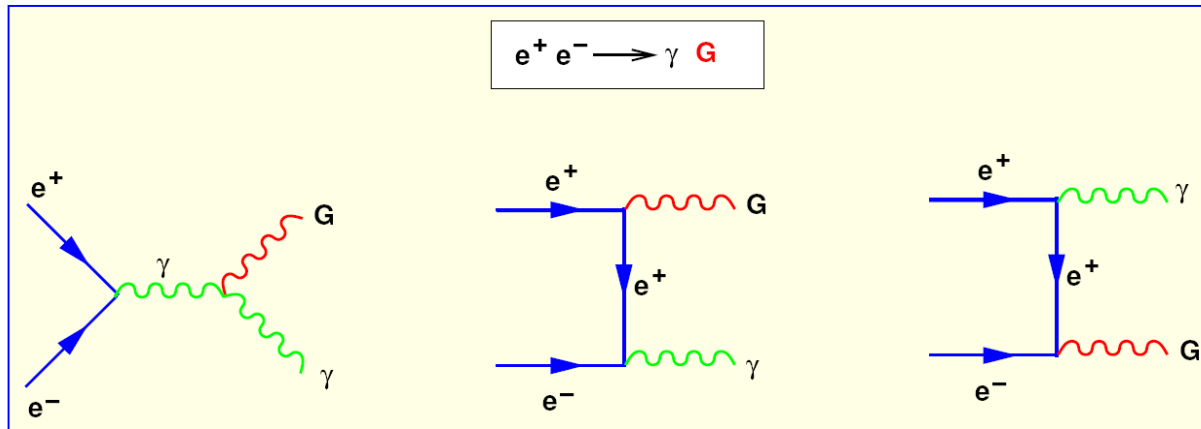
$A_{\delta-1}$ area of δ -sphere, m_G graviton mass

Inclusive production of **KK Gravitons**

$$(E/M_{Pl})^2 \Rightarrow (E/M_D)^{2+\delta}$$



KK Gravitons production mechanism at e^+e^- colliders



Hadron colliders (elementary processes)

$$q\bar{q} \rightarrow g + G$$

$$q g \rightarrow g + G$$

$$g g \rightarrow g + G$$

- Massive gravitons G decay outside the detector
- G detected as missing energy

G. Giudice, R. Rattazzi, and J.D. Wells ('98)

discovery requirement $(\sigma_{\text{Signal}} > 5\sqrt{\sigma_B \mathcal{L}}/\mathcal{L})$

$$e^+e^- \rightarrow \gamma + G$$

G as missing energy

- **LEP200**, $\mathcal{L} = 4 \times 500 \text{ fb}^{-1}$ ($\sqrt{s}=200 \text{ GeV}$)

$$\delta = 2 \quad M_D > 1.3 \text{ TeV}$$

Searches at LC and LHC

95% CL sensitivity

Hewett 2004

$e^+e^- \rightarrow \gamma + G_n$		2	4	6
LC	$P_{-,+} = 0$	5.9	3.5	2.5
LC	$P_- = 0.8$	8.3	4.4	2.9
LC	$P_- = 0.8, P_+ = 0.6$	10.4	5.1	3.3
$pp \rightarrow g + G_n$		2	3	4
LHC		4 – 8.9	4.5 – 6.8	5.0 – 5.8

M_D (TeV)

Polarized beam allows
high sensitivity on MD

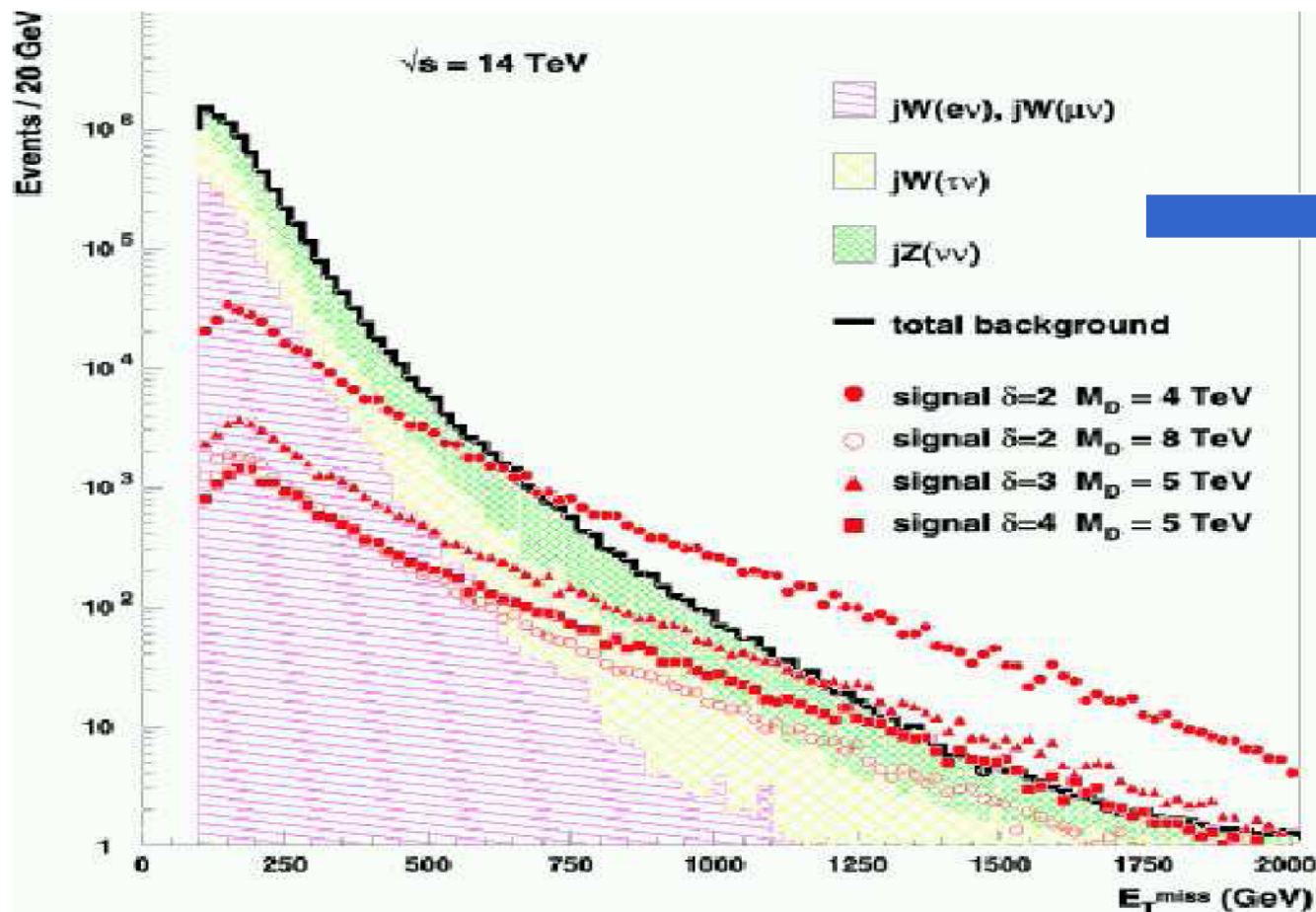
$\sqrt{s} = 800 \text{ GeV and } 1 \text{ ab}^{-1}$

100 fb^{-1}

LC

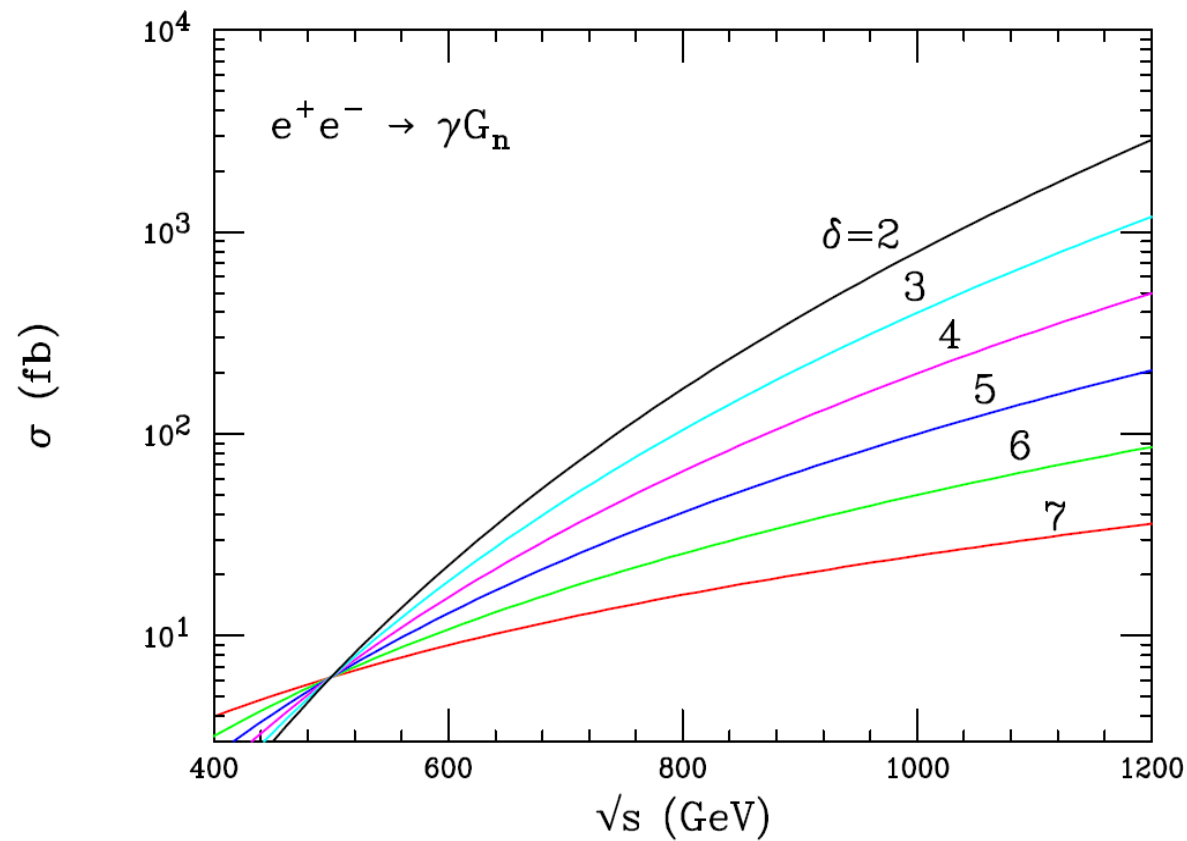
LHC

uncertainties in overall
normalization
i.e. parton densities



main
SM bckg

distribution versus missing energy
for $L = 100/\text{fb}$



MD=5 TeV $\delta=2$

LC would allow a precise determination of the fundamental parameters

main message

- inputs from the cross section measurements at LC would help to determine the overall normalization at LHC
- determination of fundamental parameters from LHC data would be improved

but don't forget...bounds from Astrophysics + Cosmology

SuperNova SN1987A collapse

$$M_D > 30 \text{ TeV } (\delta = 2)$$

- Nucleon-Nucleon Brehmstrahlung ($T \simeq 30 \text{ MeV}$)

Cosmic γ -Rays

$$G \rightarrow \gamma\gamma \quad \tau \simeq 3 \times 10^9 \text{ yrs } \left(\frac{100 \text{ MeV}}{m_G} \right)^3$$

Gravitons sources

- Thermal production after nucleos :

$$M_D > 110 \text{ TeV } (\delta = 2)$$

$$M_D > 5 \text{ TeV } (\delta = 3)$$

- Neutron stars :

$$M_D > 450 \text{ TeV } (\delta = 2)$$

$$M_D > 30 \text{ TeV } (\delta = 3)$$

KK graviton emission from heavy SM particle decays

the case of W , Z , top and H
for $M_D=1$ TeV and $\delta=2$

$$BR(Z, W) \rightarrow 10^{-8}$$

$$BR(H) \rightarrow 10^{-5}$$

$$\text{BR}(\text{Z} \rightarrow \sum_f f\bar{f} + \text{G}_X) = 8.2 \times 10^{-8} \left(\frac{\text{TeV}}{\text{M}_D} \right)^4 \quad (\delta = 2)$$

$$\text{BR}(\text{W} \rightarrow \sum_{f',f} f'\bar{f} + \text{G}_X) = 5 \times 10^{-8} \left(\frac{\text{TeV}}{\text{M}_D} \right)^4 \quad (\delta = 2)$$

$$\text{BR}(\text{t} \rightarrow \text{W b} + \text{G}_X) = 1.8 \times 10^{-7} \left(\frac{\text{TeV}}{\text{M}_D} \right)^4 \quad (\delta = 2)$$

High statistic
required for
Z and W

not realistic
for top

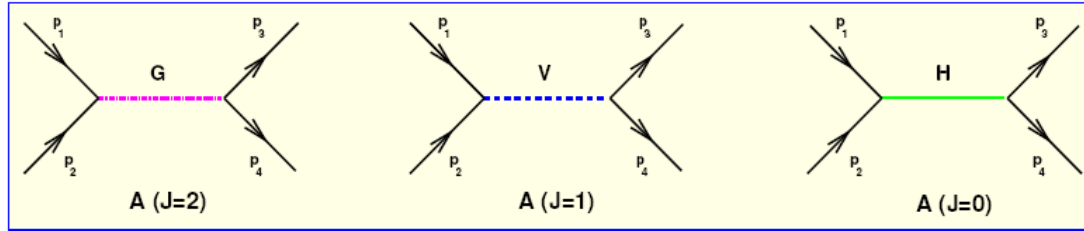
Heavy Higgs
can prove TeV scale
High statistic
required

$$\text{BR}(\text{H} \rightarrow \bar{\text{b}}\text{b} + \text{G}_X) = 2.2 \times 10^{-7} \left(\frac{\text{TeV}}{\text{M}_D} \right)^4 \quad 120 \text{ GeV}$$

$$\text{BR}(\text{H} \rightarrow \text{WW} + \text{G}_X) = 2.1 \times 10^{-5} \left(\frac{\text{TeV}}{\text{M}_D} \right)^4 \quad 500 \text{ GeV}$$

Virtual KK graviton exchanges

Virtual KK Gravitons exchange affect the SM processes



$e^-e^+ \rightarrow f\bar{f}$, $e^-e^+ \rightarrow \gamma\gamma$, $W^+W^- \rightarrow W^+W^-$, etc.

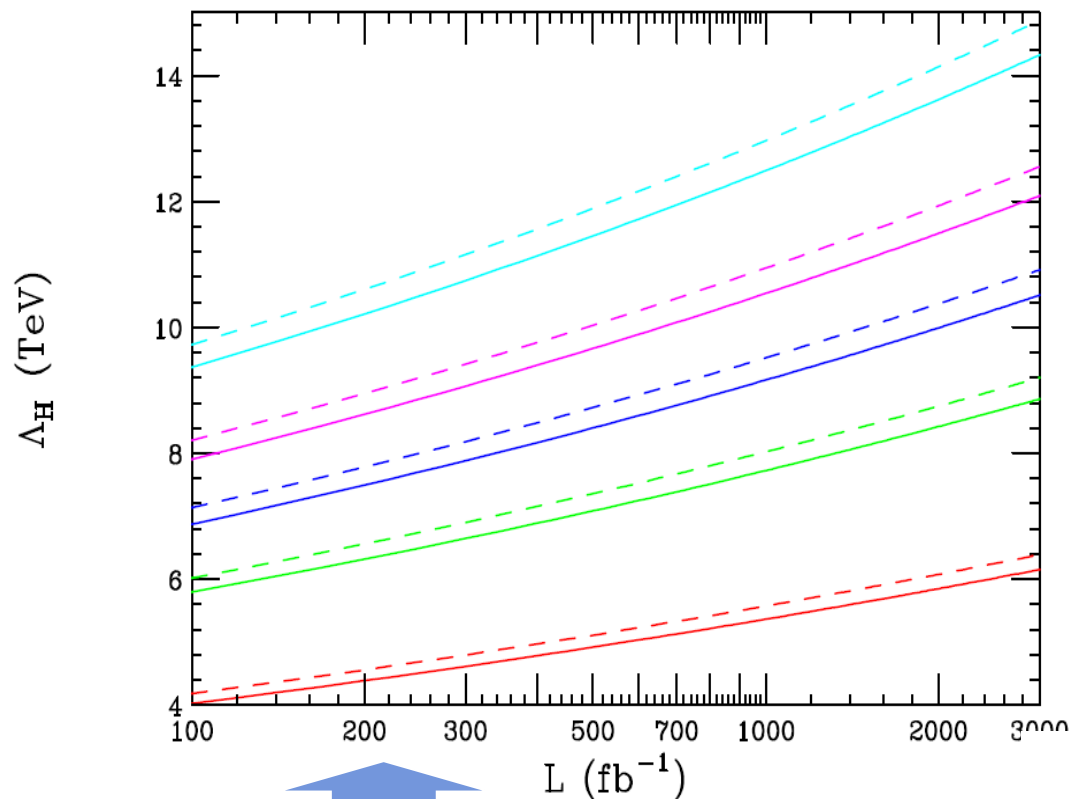
- Real part of amplitude $\text{Re}[A]$ is divergent.
Predictivity in terms of M_D is lost. $\longrightarrow$
- Imaginary part $\text{Im}[A]$ is finite (due to branch-cut singularity of real gravitons emission)

for $\delta > 2$
infinite sum
over KK tower

$$\mathcal{L} = i \frac{4\lambda}{\Lambda_H^2} T^{\mu\nu} T_{\mu\nu}$$

process governed by an
effective Lagrangian

$$e^+e^- \rightarrow f\bar{f}$$



95% CL search
each

solid (dashed)
corresponds
0(60)% positron
polarization

$$\sqrt{s} = 0.5, 0.8, 1.0, 1.2, \text{ and } 1.5 \text{ TeV}$$

$$\sqrt{s} = 0.5$$

	$e^+e^- \rightarrow \gamma\gamma$	
	"Realistic"	Optimistic
Luminosity [fb^{-1}]	Λ_H [TeV]	Λ_H [TeV]
1	2.0	2.0
10	2.6	2.6
100	3.0	3.4
1000	3.1	4.1
$\mathcal{L}^{QED}$ 1000	1.2	1.6

Rizzo (2003)

Higher sensitivity from the study of azimuthal Asymmetries

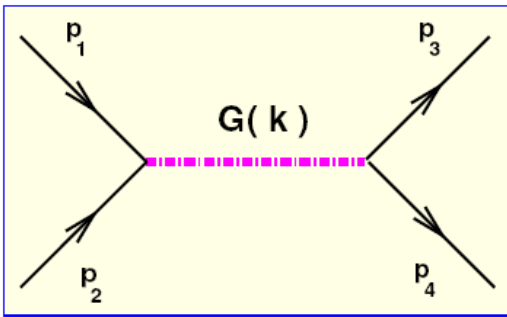
$\sqrt{s}$ (GeV)	Search Reach Λ_H (TeV)	ID Reach (TeV)
500	10.2	5.4
800	17.0	8.8
1000	21.5	11.1
1200	26.0	13.3
1500	32.7	16.7

If deviations due to virtual graviton exchange were observed at LHC, a LC with positron polarization could identify the spin-2 nature of the exchange for the entire LHC search region

Virtual Graviton Exchange at the Z-pole in Large Extra-Dimensions

A.Datta, E.G., B. Mele (2003)

- Resonant SM processes can have interference with $\text{Im}[\text{Amplitude}]$ of the graviton exchange
- finite results only predicted in terms of MD and number of Extra-Dim.



$$\text{---} (k^2=s) \text{---} = \sum_n \frac{-i}{s - m_n^2}$$

$T_{\mu\nu}$ does not depend on $n \rightarrow$ Factorization

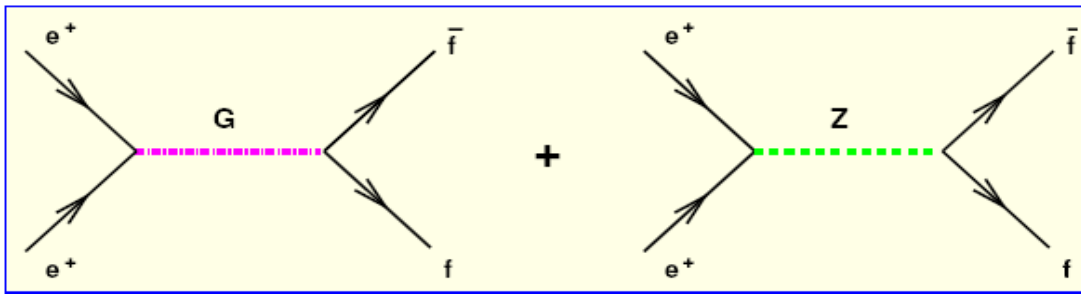
$$\mathcal{A} = \mathcal{S}(s) \left(T_{\mu\nu} T^{\mu\nu} - \frac{1}{\delta + 2} (T^\mu_\mu)^2 \right)$$

$$\mathcal{S}(s) = \frac{1}{M_{\text{pl}}^2} \sum_n \frac{1}{s - m_n^2}$$

continuum $\sum_n \rightarrow \int$

**Im[Amplitude]
is finite**

$$\text{Im}[\mathcal{S}(s)] = \frac{A_{\delta-1}}{2} \frac{s^{\frac{\delta-2}{2}}}{M_D^{2+\delta}}$$



Shift in the position of the peak

$$\Delta M_Z = - \left(\frac{\Gamma_Z^2}{8M_Z} \right) (\Delta_1 \delta) \left(\frac{I_2}{I_1} \right)$$

$$\Delta M_Z = -1.4 \text{ KeV}$$

final leptons

$$\frac{d\sigma_f}{d\cos\theta} = \frac{9\pi}{2M_Z^2} \left(\frac{\Gamma_e \Gamma_f}{\Gamma_Z^2} \right) S_f(\cos\theta)$$

$$S_f(x) = 1 + x^2 - \Delta_1 (1 - 3x^2) + 2A_e A_f (x + \Delta_2 x^3)$$

$$\Delta_{1,2} \propto \left(\frac{M_Z}{M_D} \right)^{2+\delta}$$

- **New contribution:** **vanishes** when integrated over the full range of $\cos\theta = (-1, 1)$
- $\delta = 2$, $M_D = 1 \text{ TeV} \Rightarrow \Delta_{1,2} \simeq \mathcal{O}(10^{-4})$
too small for present **EXP.** sensitivity

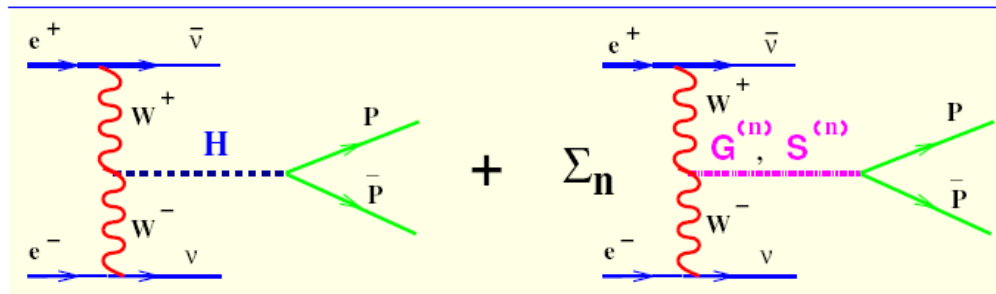
$$A_{\text{FB}}^f = \frac{3}{4} A_e A_f \left(1 + \frac{\Delta_2}{2} \right)$$

Virtual KK Gravitons in Higgs production

A. Datta, E.G, and B.Mele, JHEP 0310:003,2003

Heavy Higgs

- e^+e^- Colliders: $WW \rightarrow H \rightarrow WW$ Tree!
- Large interference $Im[A(WW \rightarrow G, S \rightarrow WW)]$
since $T_\mu^\mu \propto M_W^2$
- $\mu^+\mu^-$ Colliders: $\mu^+\mu^- \rightarrow H \rightarrow (WW), (\bar{t}t)$



$P = W, Z, t$

SM + Interf. with KK Gravitons exchange

$$\frac{d\sigma_{\lambda}^P}{d\cos\theta} = \frac{\bar{\sigma}_{\lambda}^P}{2} \left\{ 1 + \Delta_0^P + \Delta_{2,\lambda}^P (1 - 3\cos^2\theta) \right\}$$

($x = \cos\theta$) Δ_0^P = Gravisalars $\Delta_{2,\lambda}^P$ = Gravitons

$\lambda = T, L$ are the W polarizations

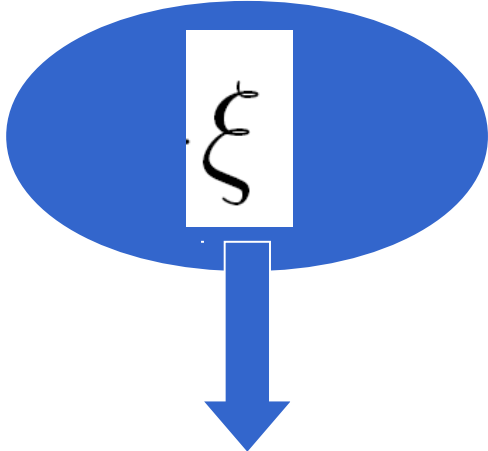
- The Graviton interference weighted by $(1 - 3\cos^2\theta)$ vanishes
- Gravisalar interference effects survive
- To pin down Gravitons effects $\rightarrow$ angular cuts, new asymmetries
- $m_H = 500$ GeV Grav-Interf/SM $\simeq 0.5$ %
- $m_H = 800$ GeV Grav-Interf/SM $\simeq \mathcal{O}(\%)$

Graviscalar effects in Higgs production

$$S = -\xi \int d^4x \sqrt{g_{ind}} R(g_{ind}) H^\dagger H$$

after the usual shift on the VEV
a mixing term between the Higgs
field **h** and graviscalar **s** arises

$$\mathcal{L}_{\text{mix}} = -\frac{2\sqrt{2}}{M_{Pl}} \xi v m_h^2 \sqrt{\frac{3(\delta-1)}{\delta+2}} h \sum_{\vec{n}>0} s_{\vec{n}}$$



*New fundamental
parameter of $O(1)$*

this will give rise to an invisible Higgs decay
in graviscalar affecting all standard Higgs decays

Graviscalar effects in Higgs production

LC will be able to improve the determination of model parameters considerably with respect to LHC alone

$$\text{TeV}^{-1}$$

Extra Dimensions

Gauge fields in the bulk

naturally arising in braneworld theories

- fermions confined on the brane
- two kind of models:
 - all fermions on one brane
 - quarks and leptons on opposite branes

$$M_n^2 = (nM_c)^2 + M_0^2$$

 **masses of KK**

$$M_c = 1/R$$

**radius of
compactification** 

EW precision data constrain
 $M_c > 4-5 \text{ TeV}$

at LC, masses above that range are
easily observable by virtual exchange.

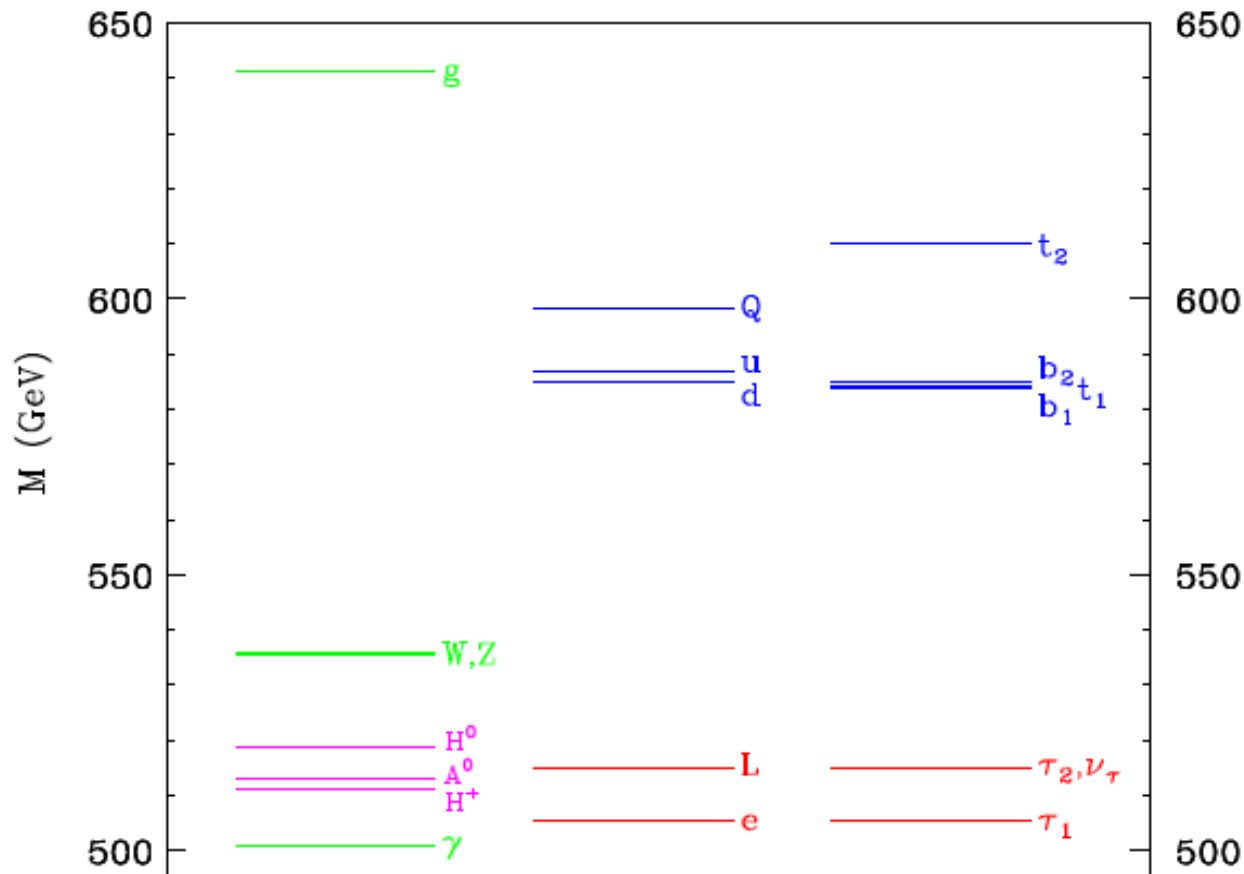
first KK excit. of gauge bosons
high degenerate

degeneracy still resolvable at LC
but NOT at LHC

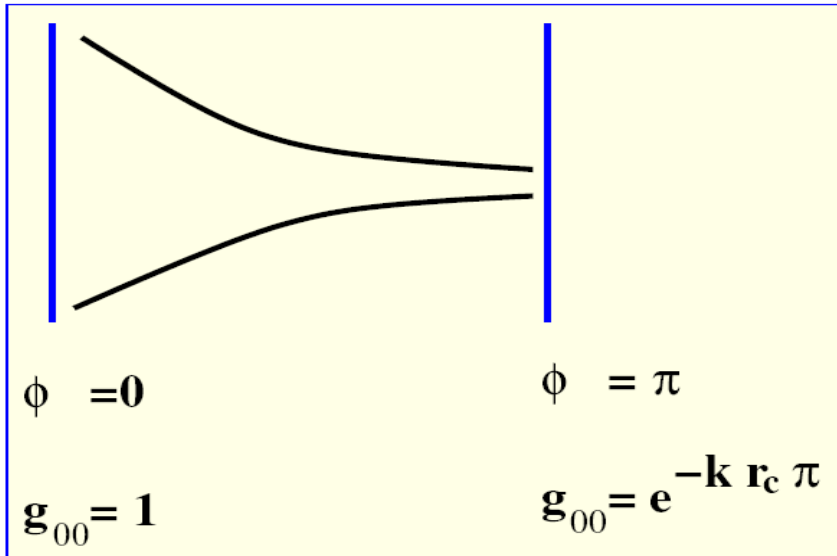
Universal Extra Dimensions

- all fields propagate in the bulk
- high predictivity in the Spectrum
- every SM field carries a KK partner
 - KK carries conserved QN
- similar to SUSY with R parity
 - BUT same spin than SM particles
- cleanest way to identify UED is the observation of second KK level
- spin can be determined at LHC

- could be confused with SUSY particles
- LC could be able to disentangle between SUSY and UED



Randall-Sundrum Model



- Masses related by

$$\frac{m_\pi}{m_0} = e^{-k r_c \pi}$$

- The Einstein Eqs. in 5 dimension, admits a solution for a **non-factorizable metric**

$$ds^2 = e^{-2kr_c|\varphi|} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\varphi^2$$

where $-\pi < \varphi < \pi$

- if Planck masses in 5-dim are $O(\text{TeV})$

$$M_{\text{pl}}^2 = \frac{M_5^3}{k} \left[e^{2kr_c\pi} - 1 \right] \gg M_5^2$$

- hierarchy problem naturally explained for $kr=O(10)$
- graviton KK have large mass gap $O(\text{TeV})$ and are strongly coupled

- KK masses = $x(n) \Lambda k/M_{Pl}$ Λ in TeV

	$k/\overline{M}_{Pl}$		
	0.01	0.1	1.0
LC $\sqrt{s} = 0.5$ TeV	20.0	5.0	1.5
LC $\sqrt{s} = 1.0$ TeV	40.0	10.0	3.0
LHC	20.0	7.0	3.0

- LC $\rightarrow L=500 \text{ fb}^{-1}$
- LHC $\rightarrow L=100 \text{ fb}^{-1}$

Conclusions

Large Extra-Dim scenario:

KK production

LC and LHC have comparable search ;
LC can determine absolute normalization of
 σ -sec and numb. of ED

KK exchange

If positron pol. is available LC can probe scale
up to 21 TeV for $\sqrt{s}=1$ TeV

Data from both LHC and LC will improve the
accuracy of the determination of model param.

TeV^{-1} Extra Dim scenario:

gauge field in the bulk:

LHC may discover KK in mass range $M_c = 4-6 \text{ TeV}$
Indirect effects available at LC for $M_c < 20 \text{ TeV}$

all SM fields in the bulk (universal ED):

can cause confusion with SUSY particles
LC would be able to disentangle it.

warped Extra-Dim scenario:

spin-2 resonances of $m < O(\text{TeV})$ can be both
observed at LC and LHC-spectacular signatures.