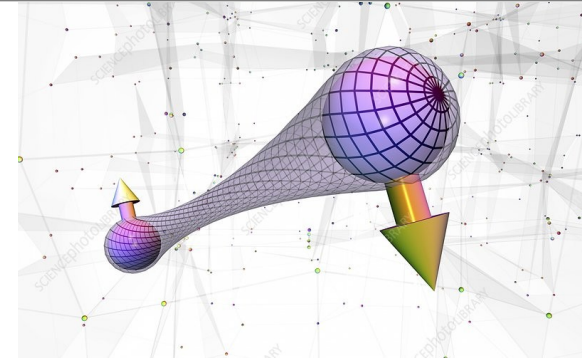


Quantum Entanglement in Diboson production

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Effective Field Theory in Multiboson production

Padova, June 10 – 11, 2024

in collaboration with: Marco Fabbrichesi, Roberto Floreanini, and Luca Marzola

based on: **JHEP 09** (2023) 195; arXiv:2304.02403; **EPJC 83** (2023) 9, 823; arXiv:2302.00683;
PRD 109 (2024) 3, L031104; arXiv:2305.04982

- “**Entanglement**” between two systems is a pure quantum phenomena
- Expected to violates **Bell inequalities** → set of (classical) probability correlations
- Violations incompatible with classical physics based on causality and local realism (locality) (**EPR paradox, hidden variables theories**)
- Violation of Bell inequality observed at **low energy**

pair of photons Freedman-Clauser PRL 28 (1972), Aspect-Dalibard-Roger PRL 49 (1982),
Zelinger et al. , Nature 433, 230 (2005)

At high energy (B physics):

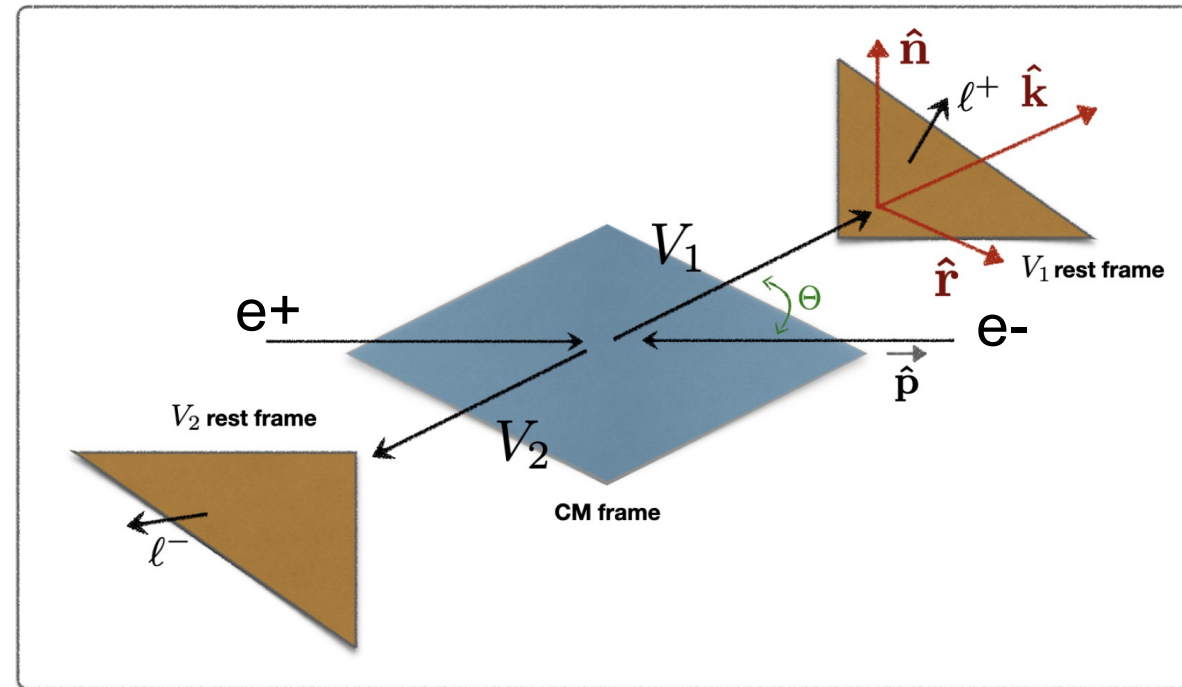
- **first observation** of **Bell violation** at high energy in B meson decays and in the presence of **WEAK** and **STRONG** interactions ! (large significance $\gg 5\sigma$)
- observation of **Entanglement** (as ATLAS [arXiv:2311.07288] and CMS [CMS-PAS-TOP-23001] results for tops)

Fabbrichesi, Floreanini, EG, Marzola, PRD 109 (2024) 3, L031104 ; arXiv: 2305.04982

Entanglement & Bell inequalities in spin-1 systems (qutrits)

- Requires the knowledge of the **Polarization Density Matrix (PDM)** of spin-1 $V_1 V_2$ production
- PDM can be fully reconstructed from the angular distributions of the single $V_1 V_2$ **decay products**

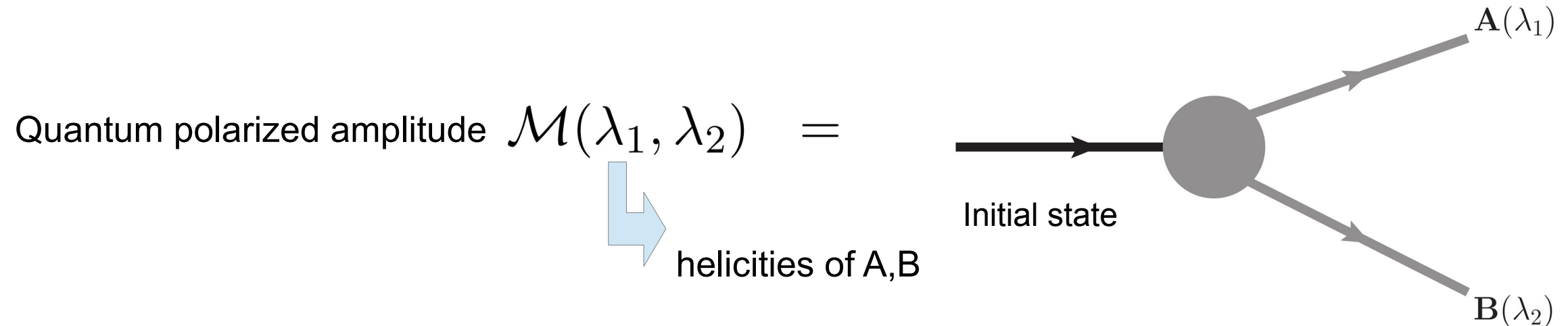
PDM depends only on invariant mass of $V_1 V_2$ system and on scattering angle θ in the $V_1 V_2$ c.m. frame



- but it can also be computed analytically
- knowledge of full **PDM** allows to quantify (where possible) entanglement and Bell inequality violation

Polarization density matrix: two final spin-1 particles

- Consider an initial quantum state going into two final **A,B** spin-1 particles



- polarization density matrix

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \frac{\mathcal{M}(\lambda_1, \lambda_2) \mathcal{M}^\dagger(\lambda'_1, \lambda'_2)}{|\overline{\mathcal{M}}|^2}$$

9x9 matrix

unpolarized

- Two methods available for computing or reconstructing **PDM** of two qutrits
 - Gell-Mann matrices decomposition (or spherical harmonics basis) → more general
 - using **helicity amplitudes** → suitable for pure states

Decomposition of Polarization Density Matrix for qutrits

PDM ρ can be decomposed on the basis of tensor products of **Gell-Mann matrices** T^a

9x9 matrix

Unpolarized

$$[A \otimes B]_{ii'jj'} = A_{ii'} B_{jj'}$$

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \left(\frac{1}{9} [\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [\mathbb{1} \otimes T^a] + \sum_a g_a [T^a \otimes \mathbb{1}] + \sum_{ab} h_{ab} [T^a \otimes T^b] \right)_{\lambda_1 \lambda'_1, \lambda_2 \lambda'_2}$$

helicities ↓

$$f_a = \frac{1}{6} \text{Tr} [\rho (\mathbb{1} \otimes T^a)]$$

Spin polarization of particle 1

$$g_a = \frac{1}{6} \text{Tr} [\rho (T^a \otimes \mathbb{1})]$$

Spin polarization of particle 2

$$h_{ab} = \frac{1}{4} \text{Tr} [\rho (T^a \otimes T^b)]$$

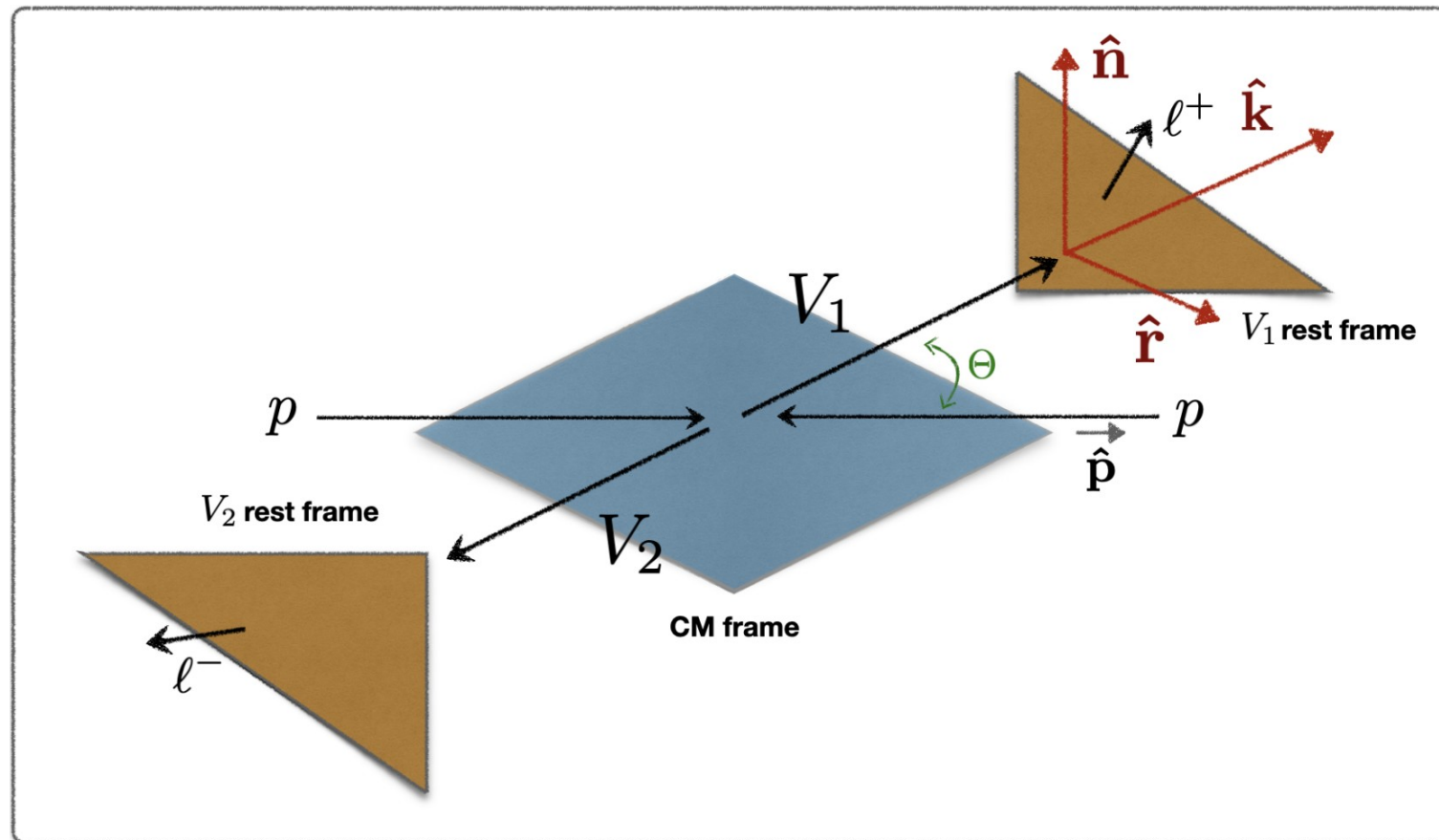
Spin **correlations** of particles 1 and 2

● extract **f, g, h** coefficients from data using the angular distributions of their decay products

Right-handed basis $\{\hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}\}$:

$$\hat{\mathbf{p}} \cdot \hat{\mathbf{k}} = \cos \Theta.$$

$$\hat{\mathbf{n}} = \frac{1}{\sin \Theta} (\hat{\mathbf{p}} \times \hat{\mathbf{k}}), \quad \hat{\mathbf{r}} = \frac{1}{\sin \Theta} (\hat{\mathbf{p}} - \cos \Theta \hat{\mathbf{k}})$$



$$d\Omega^\pm = \sin \theta^\pm d\theta^\pm d\phi^\pm$$

phase space written in terms of the spherical coordinates
(with independent polar axis) for the momenta
of the final charged leptons in the respective rest frames
of the decaying spin-1 particles

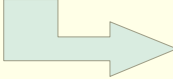
Extracting PDM for Two-Qutrits from data

The following example is for W⁺ and W⁻ final states, but the method is general

Rahaman, Singh, NPB 984 (2022), arXiv:2109.09345

Differential cross section

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega^+ d\Omega^-} = \left(\frac{3}{4\pi} \right)^2 \text{Tr} \left[\rho_{V_1 V_2} (\Gamma_+ \otimes \Gamma_-) \right]$$

 Kronecker product

$$d\Omega^\pm = \sin \theta^\pm d\theta^\pm d\phi^\pm \longrightarrow$$

phase space written in terms of the spherical coordinates (with independent polar axis) for the momenta of the final charged leptons in the respective rest frames of the decaying spin-1 particles

$$\rho_{V_1 V_2} = \text{density matrix of } \mathbf{V}_1 \mathbf{V}_2$$

$\Gamma_\pm \longrightarrow$ Density matrices that describe the polarization of the two spin-1 massive vectors into final leptons (the charged ones assumed to be massless)

(in the case of the W-bosons these are projectors because of their chiral couplings to leptons)

can be computed by **rotating** to an arbitrary polar axis the spin ± 1 states of gauge bosons from the ones given in the **k-direction** quantization axis

$$\Gamma_{\pm} = \frac{1}{3} \mathbb{1} + \sum_{i=1}^8 q_{\pm}^a T^a$$

Example for the W⁺ W⁻ final states

→ **Density matrices for W-bosons**

$q_{\pm}^a$ → **Wigner q-symbols** (see backup slides) are functions of the solid angle: $\Omega^{\pm}$

↓
set of polynomials
of spherical
coordinates
(see backup slide)

$$h_{ab} = \frac{1}{\sigma} \int \int \frac{d\sigma}{d\Omega^+ d\Omega^-} p_+^a p_-^b d\Omega^+ d\Omega^-$$

$$f_a = \frac{1}{\sigma} \int \frac{d\sigma}{d\Omega^+} p_+^a d\Omega^+$$

$$g_a = \frac{1}{\sigma} \int \frac{d\sigma}{d\Omega^-} p_-^a d\Omega^-$$

$p_{\pm}^n$ a particular set of orthogonal functions (see backup slides) $\Rightarrow \left(\frac{3}{4\pi}\right) \int p_{\pm}^n q_{\pm}^m d\Omega^{\pm} = \delta^{nm}$

For **ZZ** case, the set of functions are linear combinations of $q_{\pm}^a$ (see backup slides)

Quantify entanglement

For pure states $\rho = |\psi\rangle\langle\psi| \Rightarrow \rho^2 = \rho \Rightarrow \text{Tr}[\rho^2] = 1$

CONCURRENCE

$$\mathcal{C}[|\Psi\rangle] = \sqrt{2(1 - \text{Tr}[(\rho_r)^2])}$$

$r = A \text{ or } B$

Hill, Wootters, PRL 78 (1997)

P. Rungta et al, PRA 64 (2001)

→ vanish for separable (not entangled) states

Trace performed in the subsystems $r = A \text{ or } B$

For mixed states $\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$, $p_i \geq 0$, $\sum_i p_i = 1$

$$\mathcal{C}[\rho] = \inf_{\{|\Psi\rangle\}} \sum_i p_i \mathcal{C}[|\Psi_i\rangle]$$

→ optimization problem
(analytical solution only for qubit)

analytical solution exists for the lower bound

$$(\mathcal{C}[\rho])^2 \geq \mathcal{C}_2[\rho]$$

Mintert, Buchleitner,
PRL 98 (2007)

(witness of entanglement)

$$\mathcal{C}_2[\rho] = 2 \max \left(0, \text{Tr} [\rho^2] - \text{Tr} [(\rho_A)^2], \text{Tr} [\rho^2] - \text{Tr} [(\rho_B)^2] \right)$$

● If non-vanishing unequivocally signal the presence of entanglement

● used as observable for entanglement in WW, ZZ and WZ productions

Quantify entanglement for pure states

Fabbrichesi, Floreanini, EG, Marzola

EPJC 83 (2023) 9, 823 ; arXiv: 2302.00683]

- If the bipartite (A,B) system is a pure state it is possible to quantify its entanglement via

Entropy of entanglement

$$\mathcal{E}[\rho] = -\text{Tr} [\rho_A \log \rho_A] = -\text{Tr} [\rho_B \log \rho_B]$$

Log is in natural basis

equality holds if and only if the bipartite is separable

$$0 \leq \mathcal{E}[\rho] \leq \log d \quad d = 3 \text{ for qutrits} \quad (\text{Von Neumann Entropy})$$

- Very useful in the case of Higgs decay into $H \rightarrow WW^*$ or ZZ^* (pure state)

Bell inequality for Two-QUTRITS

- take a bi-partite system with components A, B
- perform two measurements $\longrightarrow (A_1, A_2) \quad (B_1, B_2)$ each can take values $\rightarrow \{0, 1, 2\}$
- consider the following correlator $\mathcal{I}_3$ for probability measurements

$$\begin{aligned} \mathcal{I}_3 = & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ & - P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1) \end{aligned}$$

can be expressed as

$$\mathcal{I}_3 = \text{Tr}[\rho \mathcal{B}]$$

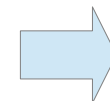
with $\mathcal{B}$ a suitable Bell operator

Collins, Gisin, Linden,
Massar, Popescu, PRL 88 (2002)

Bell inequalities for two-qutrits

For deterministic local models

$$\mathcal{I}_3 \leq 2$$



QM for qutrits can violate this
inequality with upper bound = 4

in order to maximize Bell's inequality

$$\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

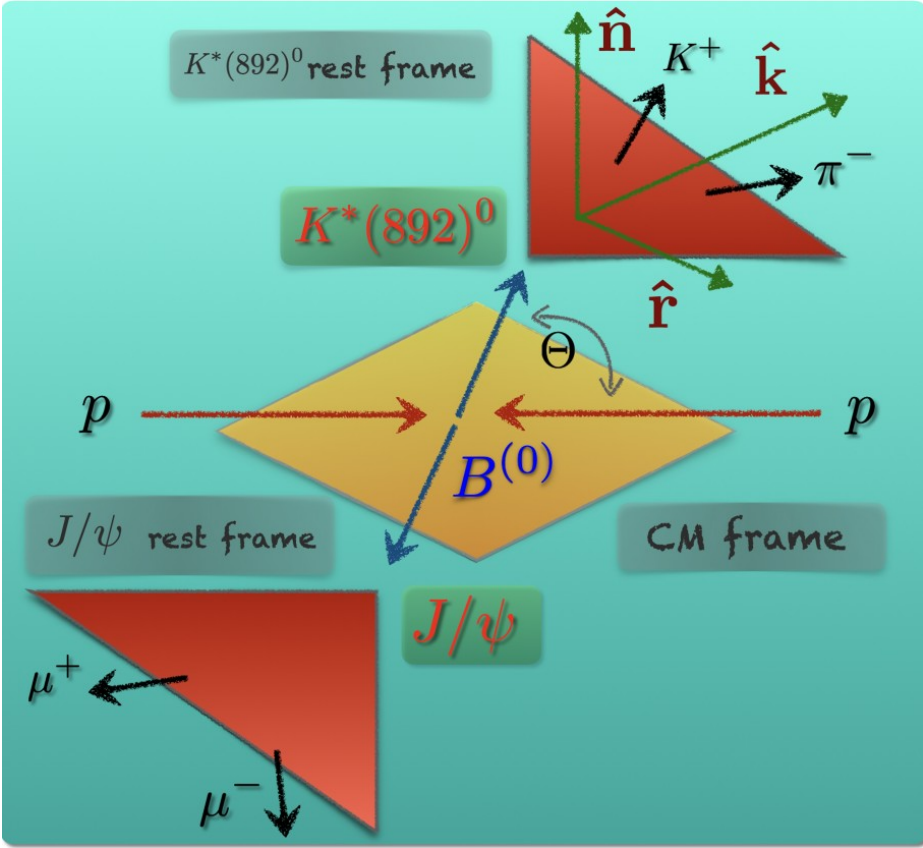
 U, V unitary 3x3 matrices

● **first observation of Bell violation** at high energy in B meson decays and in the presence of **WEAK** and **STRONG** interactions and for **qutrit systems** (spin-1) !

● work based on data from LHCb experiment at CERN [Phys. Rev. D 88, 052002 (2013)]

$$B^0 \rightarrow J/\psi K^*(892)^0$$

PHYSICAL REVIEW D 109, L031104 (2024)



	$\mathcal{E}$	$\mathcal{I}_3$
• $B^0 \rightarrow J/\psi K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015
• $B^0 \rightarrow \phi K^*(892)^0$ [18]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$
• $B^0 \rightarrow \rho K^*(892)^0$ [19]	$0.450 \pm 0.077^*$	$2.208 \pm 0.151^*$
• $B_s \rightarrow \phi\phi$ [20]	0.734 ± 0.037	2.525 ± 0.064
• $B_s \rightarrow J/\psi\phi$ [21]	0.731 ± 0.032	2.462 ± 0.080

CGLMP inequality $\mathcal{I}_3 < 2$
(for qutrits)

$$\mathcal{I}_3 = 2.548 \pm 0.015,$$

Significance (to reject the null hypothesis)

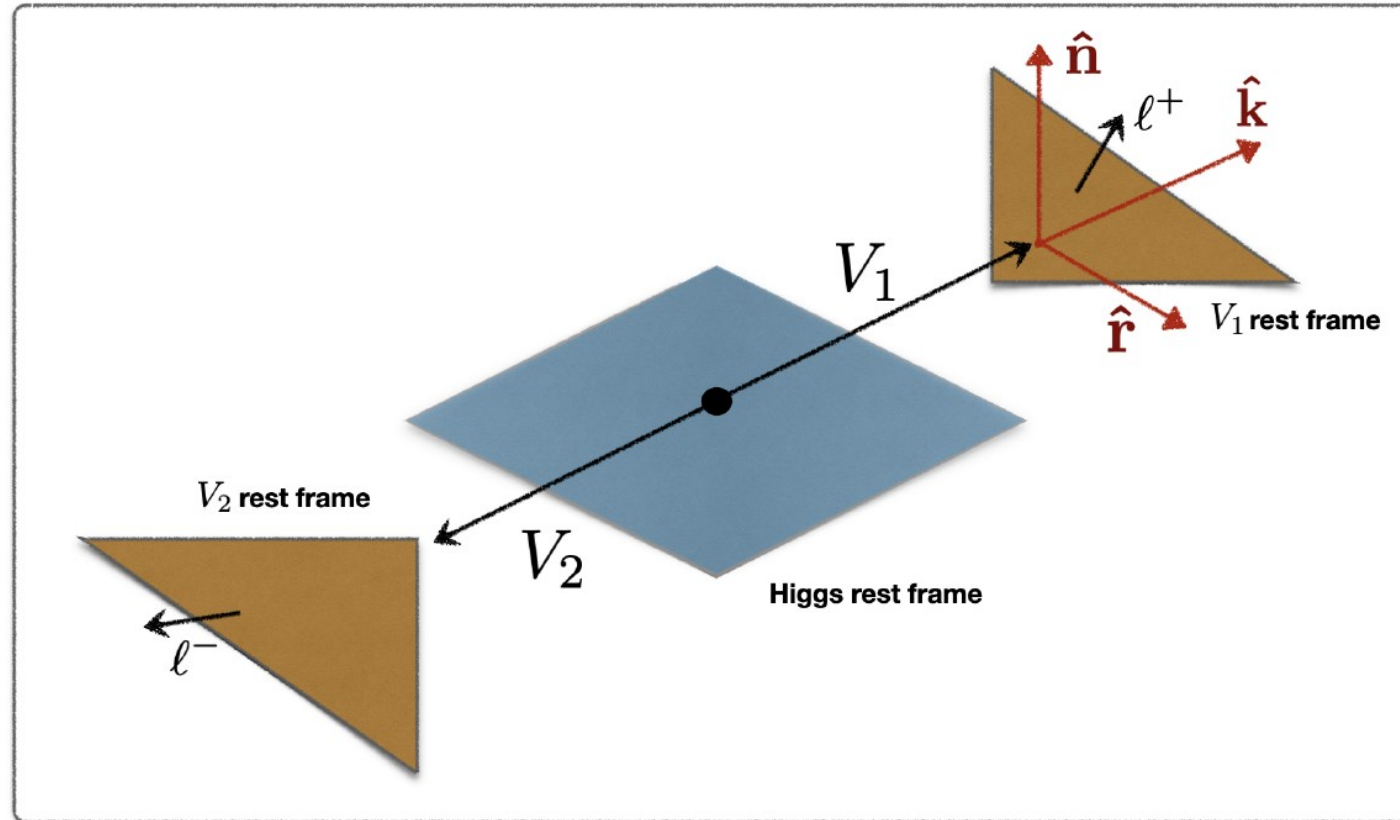
Entanglement (Entropy): **86 σ**

Bell inequality violation: **36 σ**

Di-boson production in Higgs boson decays

EPJC 83 (2023) 9, 823 ; arXiv: 2302.00683

$$H \rightarrow V(k_1, \lambda_1) V^*(k_2, \lambda_2)$$



In the Higgs boson rest frame the density matrix of the bipartite VV^* system does not depend on the scattering angle, but only by the Higgs mass, the V mass and the off-shell V^* mass

Di-boson production in Higgs boson decays

A. Barr, PLB 825 (2022) ;
2106.01377

Aguilar-Saavedra, Bernal,
Casas, Moreno
PRD 107 (2023) 1;
2209.13441

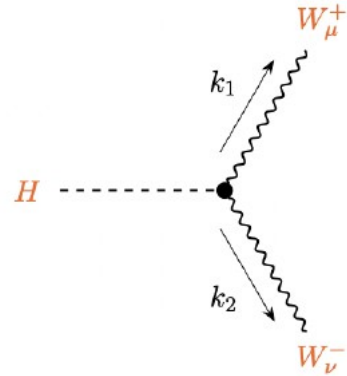
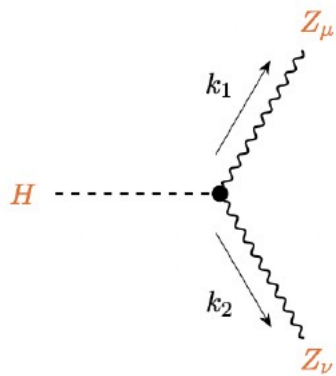
$$H \rightarrow V(k_1, \lambda_1) V^*(k_2, \lambda_2)$$

Results based on

EPJC 83 (2023) 9, 823 ;
2302.00683

analytic computation of PDM

PDM extracted from simulation at parton level



$$\left\{ \begin{array}{l} \xi_W = 1, \text{ and } \xi_Z = 1/(2c_W), \text{ with } c_W = \cos \theta_W \\ g \text{ is the weak coupling} \end{array} \right.$$

Weinberg angle

V^* regarded as an off-shell vector boson with mass
mass of V boson M_V

$$M_V^* = f M_V \quad 0 < f < 1$$

Quantum Amplitude

$$\mathcal{M}_H(\lambda_1, \lambda_2) = g M_V \xi_V g_{\mu\nu} \varepsilon^{\mu*}(k_1, \lambda_1) \varepsilon^{\nu*}(k_2, \lambda_2)$$

$$\mathcal{M}_H(\lambda_1, \lambda_2) \mathcal{M}_H(\lambda'_1, \lambda'_2)^\dagger = g^2 M_V^2 \xi_V^2 g_{\mu\nu} g_{\mu'\nu'} \mathcal{P}_{\lambda_1 \lambda'_1}^{\mu\mu'}(k_1) \mathcal{P}_{\lambda_2 \lambda'_2}^{\nu\nu'}(k_2)$$

By projecting into the Gell-Mann matrix basis for 2-qutrits one can extract the **f,g** and **h** coefficients

Inserting the **f,g** and **h** coefficients into the Gell-Mann basis for 2-qutrits

$$\rho_H = 2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{16} & 0 & 2h_{33} & 0 & h_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\text{Tr} [\rho_H] = 1$$

density matrix is idempotent

$$\rho_H^2 = \rho_H$$

Signaling that $H \rightarrow VV^*$ is a **pure state**

$$\rho_H = |\Psi_H\rangle\langle\Psi_H|$$

using the basis

$$|\lambda \lambda'\rangle = |\lambda\rangle \otimes |\lambda'\rangle \text{ with } \lambda, \lambda' \in \{+, 0, -\}$$

where the pure state is

EPJC 83 (2023) 9, 823 ; 2302.00683

$$|\Psi_H\rangle = \frac{1}{\sqrt{2 + \varkappa^2}} [|+-\rangle - \varkappa |00\rangle + |-+\rangle]$$

$$\varkappa = 1 + \frac{m_H^2 - (1 + f)^2 M_V^2}{2f M_V^2}$$

Aguilar-Saavedra, Bernal, Casas, Moreno
PRD 107 (2023) 1; 2209.13441

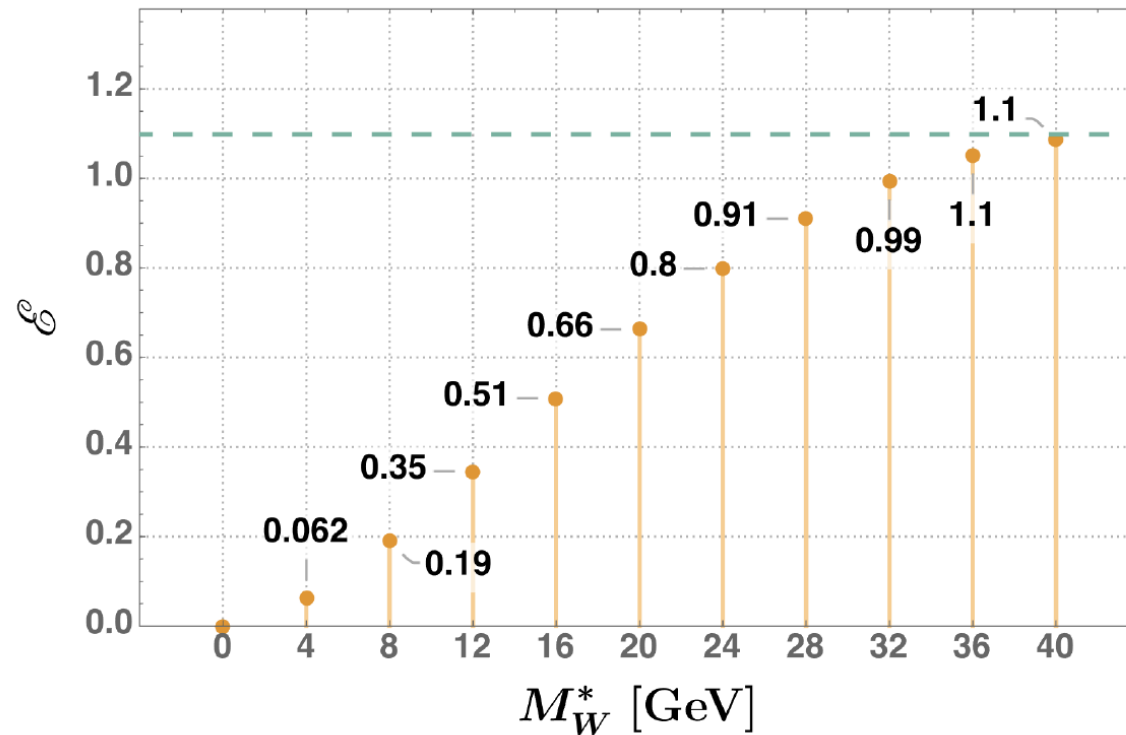
Entropy of entanglement for $H \rightarrow VV^*$

- Being a pure state we can **quantify** entanglement via **Entropy of entanglement** (Von Neumann Entropy)

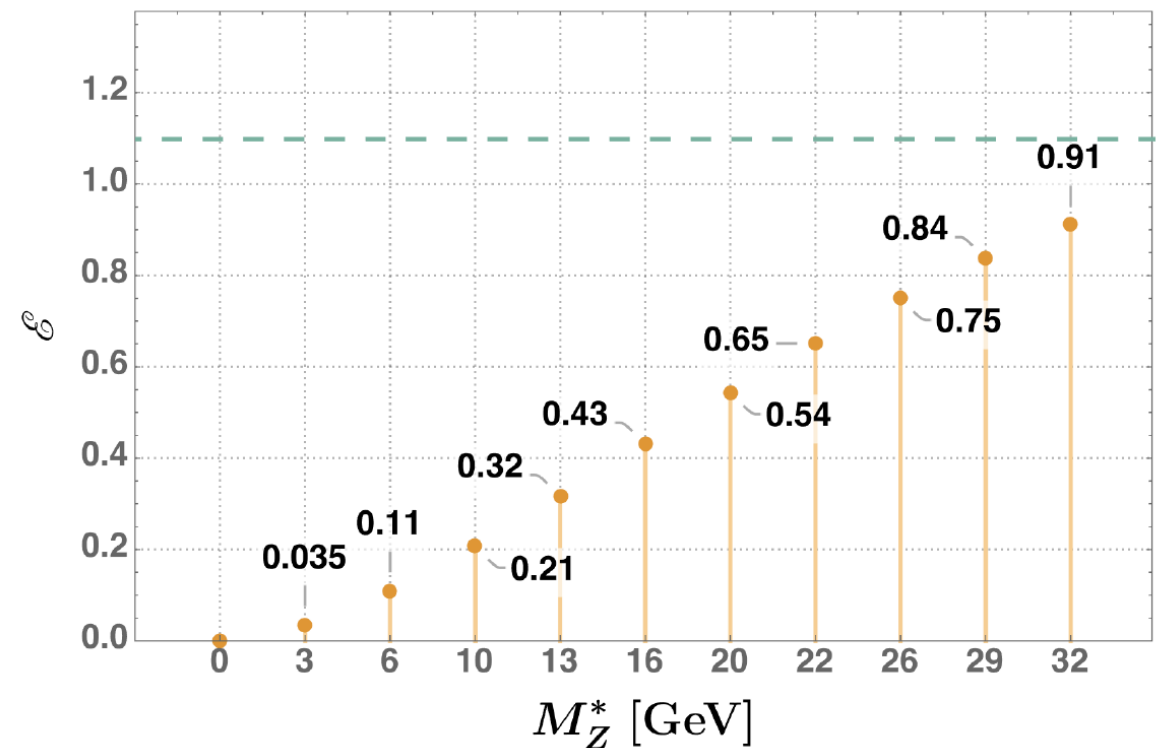
$$\mathcal{E}[\rho] = -\text{Tr} [\rho_A \log \rho_A] = -\text{Tr} [\rho_B \log \rho_B]$$

$$0 \leq \mathcal{E}[\rho] \leq \log 3 \sim 1.01 \quad \text{equality holds if and only if the bipartite is separable}$$

$$H \rightarrow WW^*$$



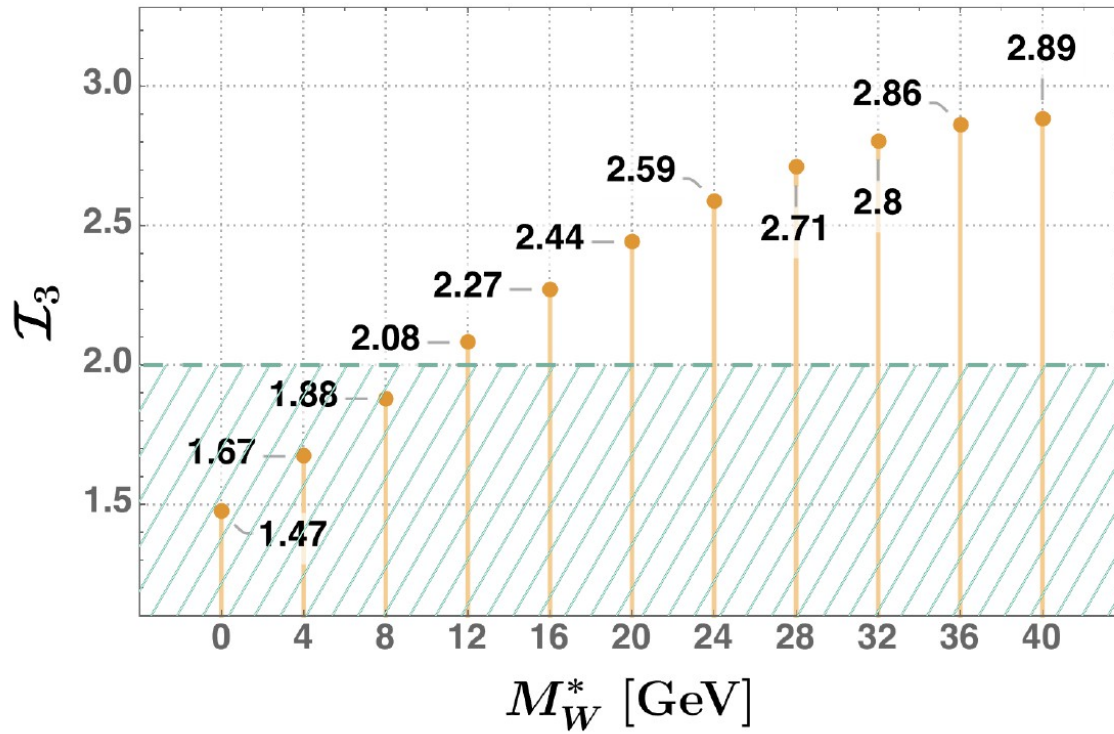
$$H \rightarrow ZZ^*$$



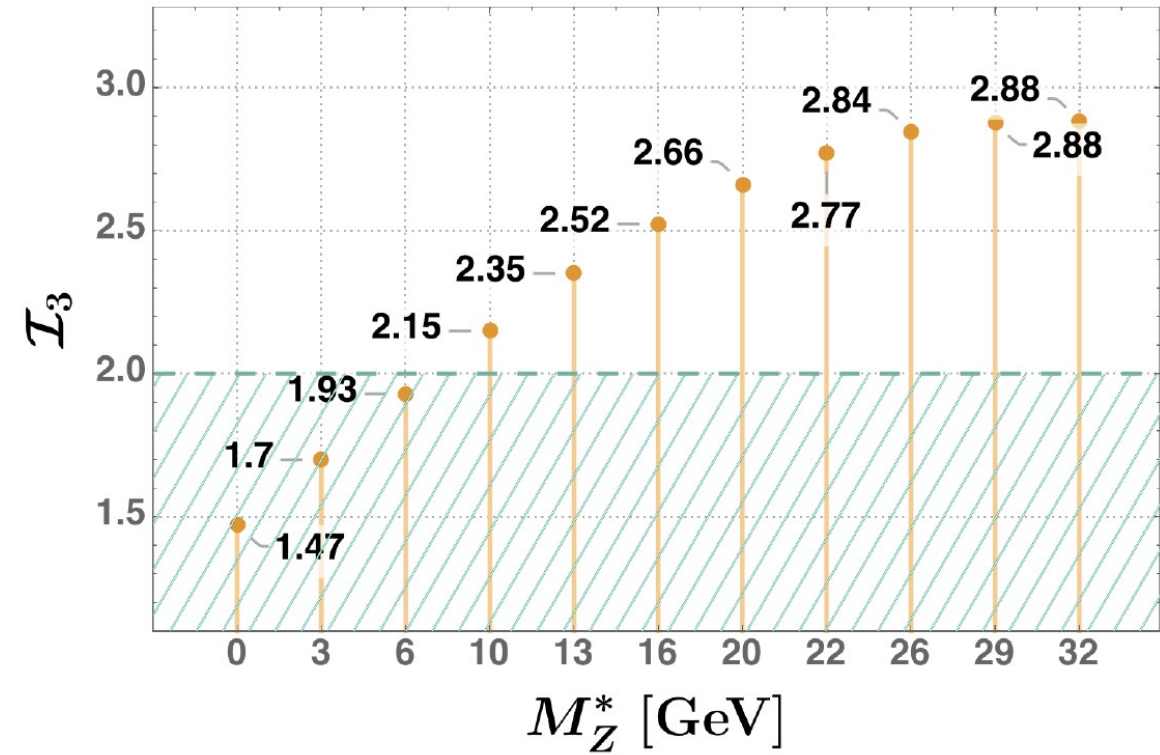
Bell inequality violation for $H \rightarrow VV^*$

$$\mathcal{I}_3 > 2$$

$$H \rightarrow WW^*$$



$$H \rightarrow ZZ^*$$



Maximization of $\mathcal{I}_3$ performed point by point as function of M_{V^*}

Summary of results for $H \rightarrow WW^*$ and $H \rightarrow ZZ^*$

significance S for rejecting the null hypothesis $\mathcal{I}_3 \leq 2$

1000 pseudo experiments

S (WW^*) $\rightarrow$ uncertainty dominated by the **systematic** error

S (ZZ^*) $\rightarrow$ uncertainty dominated by the **statistical** error

LHC Run 2

1

1.3

LHC Hi-Lumi

1

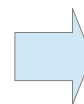
5.6

$\rightarrow$ larger due to smaller uncertainty and higher mean value (2.9)

ZZ^* promising channel at LHC Hi-Lumi

In real simulation a **reduced significance** is expected (systematic uncertainties due to unfolding, background, and detector were neglected or only partially modeled here)

Results are also consistent with analysis of



A. Barr, PLB 825 (2022) ; 2106.01377

Aguilar-Saavedra, Bernal, Casas, Moreno PRD 107 (2023) 1; 2209.13441 [hep-ph]

where simulations of events have been adopted for the reconstruction of the density matrix

Constraining **HWW** and **HZZ** anomalous couplings with Quantum Tomography

Fabbrichesi, Floreanini, EG, Marzola,
JHEP 09 (2023) 195; 2304.02403

We use **polarization density matrix** of the processes

$$H \rightarrow WW^*$$

$$H \rightarrow ZZ^*$$

to constrain anomalous Higgs couplings to WW and ZZ

 CP-even

 CP-odd

$V^{\mu\nu} \rightarrow$ Field strength , **V=W,Z**
 $\tilde{V}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} V_{\alpha\beta}$

Effective **Higgs-VV** Lagrangian (including SM)

$$\begin{aligned} \mathcal{L}_{HVV} = & g m_W W_\mu^+ W^{-\mu} H + \frac{g}{2 \cos \theta_W} m_Z Z_\mu Z^\mu H \\ & - \frac{g}{m_W} \left[\frac{\lambda_1^W}{2} W_{\mu\nu}^+ W^{-\mu\nu} + \lambda_2^W \left(W^{+\nu} \partial^\mu W_{\mu\nu}^- + \text{H.c.} \right) + \frac{\tilde{\lambda}_{CP}^W}{4} W_{\mu\nu}^+ \tilde{W}^{-\mu\nu} \right. \\ & \left. + \frac{\lambda_1^Z}{2} Z_{\mu\nu} Z^{\mu\nu} + \lambda_2^Z Z^\nu \partial^\mu Z_{\mu\nu} + \frac{\tilde{\lambda}_{CP}^Z}{4} Z_{\mu\nu} \tilde{Z}^{\mu\nu} \right] H , \end{aligned}$$

This state can be described in terms of helicity amplitudes h_i

$$|\Psi\rangle = \frac{1}{|\overline{\mathcal{M}}|} \left[h_+ |V(+)V^*(-) \rangle + h_0 |V(0)V^*(0) \rangle + h_- |V(-)V^*(+) \rangle \right]$$

$$h_\lambda = \langle V(\lambda)V^*(-\lambda) | -\mathcal{L} | H \rangle \quad \text{with} \quad \lambda = 0, \pm$$

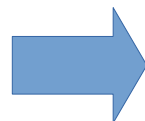
$$|\overline{\mathcal{M}}|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2,$$

$$\rho_H = \frac{1}{|\overline{\mathcal{M}}|^2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+ h_+^* & 0 & h_+ h_0^* & 0 & h_+ h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0 h_+^* & 0 & h_0 h_0^* & 0 & h_0 h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_- h_+^* & 0 & h_- h_0^* & 0 & h_- h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



see backup slide for
same density matrix in
Gell-Mann basis

$$\rho_H^2 = \rho_H$$



again a pure state!

observables to constrain anomalous couplings in $H \rightarrow VV^*$

Entropy of entanglement

$$\mathcal{E}_{ent} = -\text{Tr} [\rho_A \log \rho_A] = -\text{Tr} [\rho_B \log \rho_B] \quad \Rightarrow \quad \text{CP-even}$$

Lower bound on concurrence (entanglement)

$$\mathcal{C}_2 = 2 \max \left[0, -\frac{2}{9} - 12 \sum_a f_a^2 + 6 \sum_a g_a^2 + 4 \sum_{ab} h_{ab}^2, \right. \\ \left. -\frac{2}{9} - 12 \sum_a g_a^2 + 6 \sum_a f_a^2 + 4 \sum_{ab} h_{ab}^2 \right]. \quad \Rightarrow \quad \text{CP-even}$$

$$\mathcal{C}_{odd} = \frac{1}{2} \sum_{\substack{a,b \\ a < b}} |h_{ab} - h_{ba}| \quad \Rightarrow \quad \text{CP-odd}$$

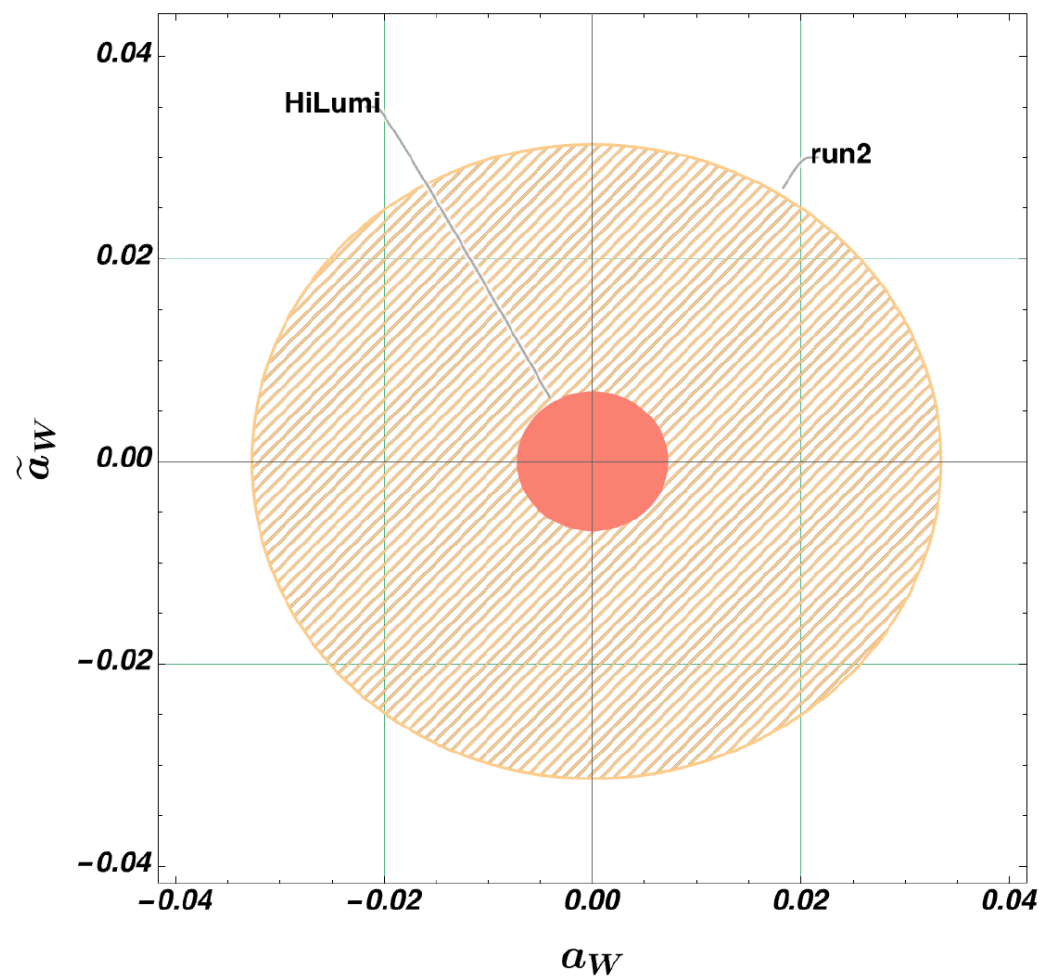
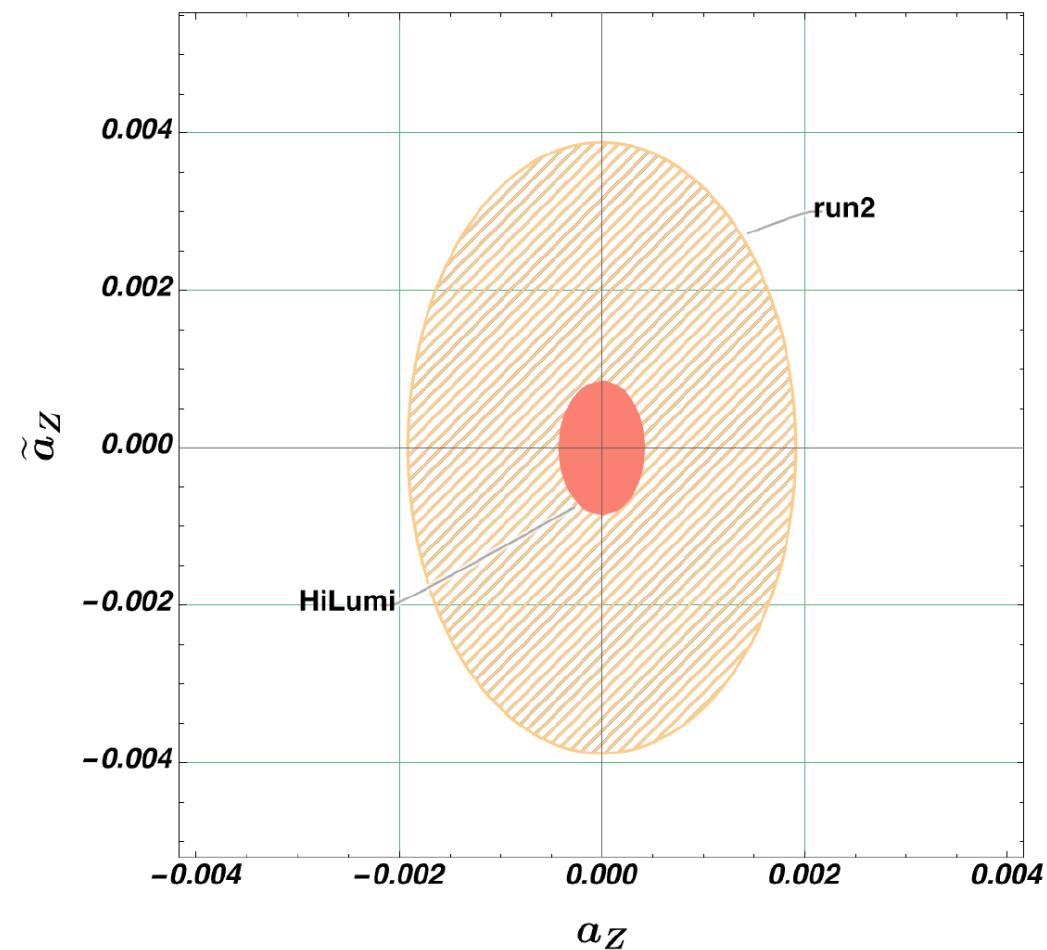
Constraining the anomalous couplings in $H \rightarrow VV^*$

$$\chi^2 = \sum_i \left[\frac{O_i(a_V, \tilde{a}_V) - O_i(0, 0)}{\sigma_i} \right]^2 \leq 5.991 \quad \begin{array}{l} 95\% \text{ CL} \\ i=1,2 \end{array}$$

Combinations of two O_i

$$m_H = 124 \pm 0.18 \pm 0.04$$

- **Main uncertainty** comes from the measurement of polarizations → originates in the reconstruction of the rest frame of the decaying Higgs boson → related to VV^* invariant mass
- We propagate uncertainty in the Higgs mass to the operators by means of a Montecarlo simul.
→ giving variances σ_i^2 associated to the operators O_i
- Works well for ZZ^*
- For WW we can determine only transverse mass → comes with an error of 5 GeV for fully leptonic decays, and we take half for semileptonic decays

$H \rightarrow WW^*$  $H \rightarrow ZZ^*$ 

HiLumi → rescaling run2 uncertainties by

→ ratio of Luminosities $\sigma_i \rightarrow \sigma_i \sqrt{\frac{14}{300}}$

including bckg effects weakens a bit the bounds (see case for ZZ in backup slides)

95% CL limits

	<i>LHC</i>	<i>run2</i>	<i>HiLumi</i>
No background		$ a_W \leq 0.033$	$ a_W \leq 0.0070$
		$ \tilde{a}_W \leq 0.031$	$ \tilde{a}_W \leq 0.0068$
		$ a_Z \leq 0.0019$	$ a_Z \leq 0.00040$
		$ \tilde{a}_Z \leq 0.0039$	$ \tilde{a}_Z \leq 0.00086$

- Comparison with other theoretical analysis based on polarizations (not entanglement)

1σ limits and for $L=1\text{ab}^{-1}$ $\Rightarrow a_Z = 6.88 \times 10^{-3}, \quad \tilde{a}_Z = 9.53 \times 10^{-3}$

K. Rao, S.D. Rindani, P. Sarmah
NPB 964 (2021) 115317; 2009.00980

- Comparison with CMS $f_{g2} = \frac{\sigma_2}{\sigma} |a_V|^2, \quad \text{and} \quad f_{g3} = \frac{\sigma_3}{\sigma} |\tilde{a}_V|^2$

Our 95% CL limits $f_{g2}^Z < 7.8 \times 10^{-6}, \quad f_{g3}^Z < 1.5 \times 10^{-5}$

CMS 95% CL limits $f_{g2}^V < 3.4 \times 10^{-3}, \quad f_{g3}^V < 1.4 \times 10^{-2}$

Di-boson production in pp collisions

$$p p \rightarrow V_1 V_2$$

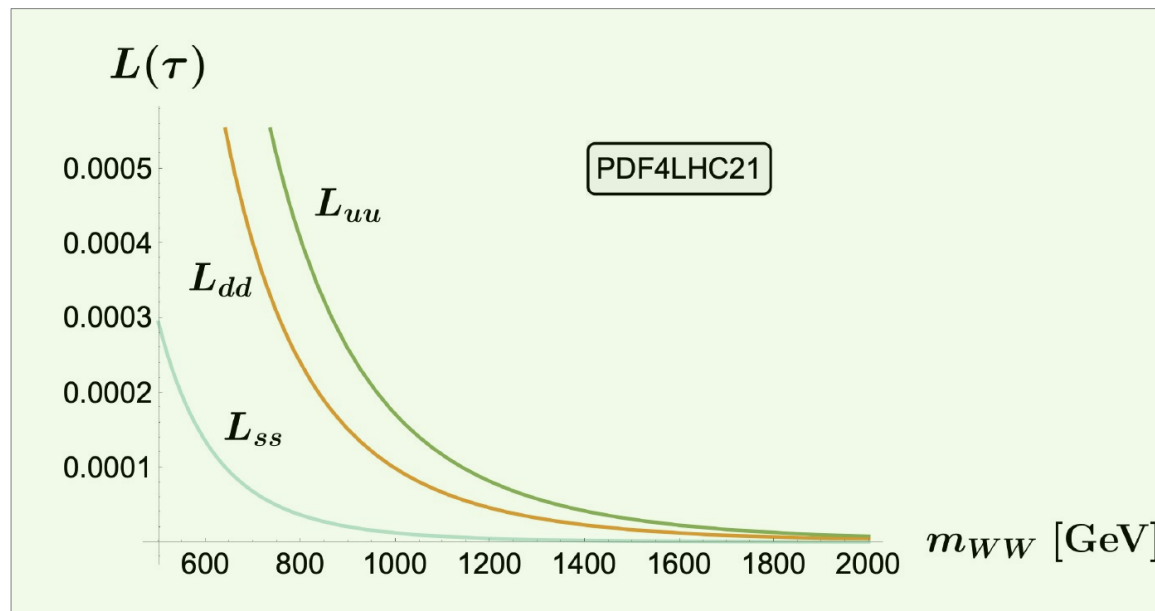
$$V_1, V_2 \in \{W, Z\}$$

Proceeds via the partonic quark-antiquark process

(away from the Higgs resonance)

$$q_1 \bar{q}_2 \rightarrow V_1 V_2$$

weighted by the parton luminosity $L^{q_1 \bar{q}_1}(\tau)$



parton luminosity of the initial $q_1 \bar{q}_2$ state

$$L^{q_1 \bar{q}_1}(\tau) = \frac{4\tau}{\sqrt{s}} \int_{\tau}^{1/\tau} \frac{dz}{z} q_{q_1}(\tau z) q_{\bar{q}_2}\left(\frac{\tau}{z}\right)$$

$\tau = m_{VV} / \sqrt{s}$ → pp c.o.m. energy
 invariant mass of $V_1 V_2$ system

Decomposing the polarization matrix density into the Gell-Mann matrix basis

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) = \left(\frac{1}{9} [\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [\mathbb{1} \otimes T^a] + \sum_a g_a [T^a \otimes \mathbb{1}] + \sum_{ab} h_{ab} [T^a \otimes T^b] \right)_{\lambda_1 \lambda'_1, \lambda_2 \lambda'_2}$$

we obtain for the **h** correlations coefficients in VV production $\longrightarrow$ depend also on **scattering angle** Θ

$$h_{ab}[m_{VV}, \Theta] = \frac{\sum_{q=u,d,s} L^{q\bar{q}}(\tau) \left(\tilde{h}_{ab}^{q\bar{q}}[m_{VV}, \Theta] + \tilde{h}_{ab}^{q\bar{q}}[m_{VV}, \Theta + \pi] \right)}{\sum_{q=u,d,s} L^{q\bar{q}}(\tau) \left(A^{q\bar{q}}[m_{VV}, \Theta] + A^{q\bar{q}}[m_{VV}, \Theta + \pi] \right)}$$

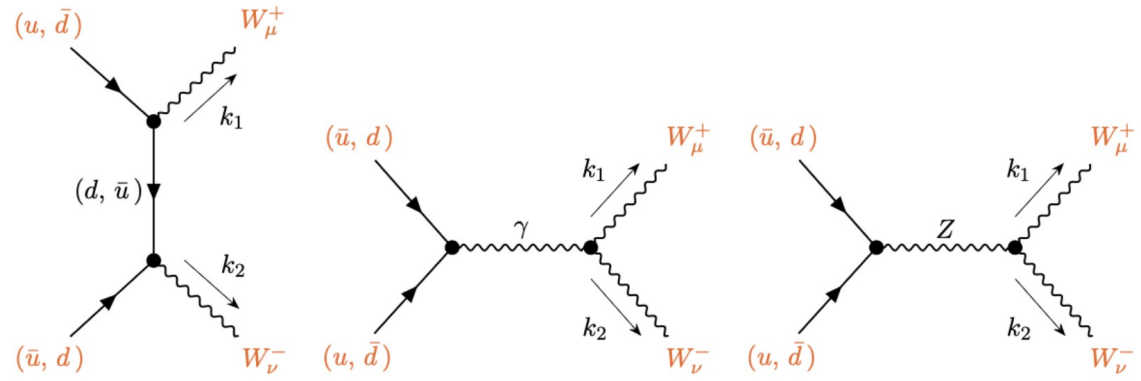
and analogously for the f_a and g_a correlation coefficients, where

$$\tilde{h}_{ab} = A^{q\bar{q}} h_{ab}$$

$A^{q\bar{q}}$ = unpolarized square amplitude of the partonic process

- main **uncertainty** on the correlation coefficients comes from the missing higher order **QCD corrections**
- giving approx a **10% uncertainty** on the main entanglement observables
- other theoretical uncertainties**, mainly from PDF and other SM inputs, is **negligible** $\longrightarrow$ of the order of **permille** effect

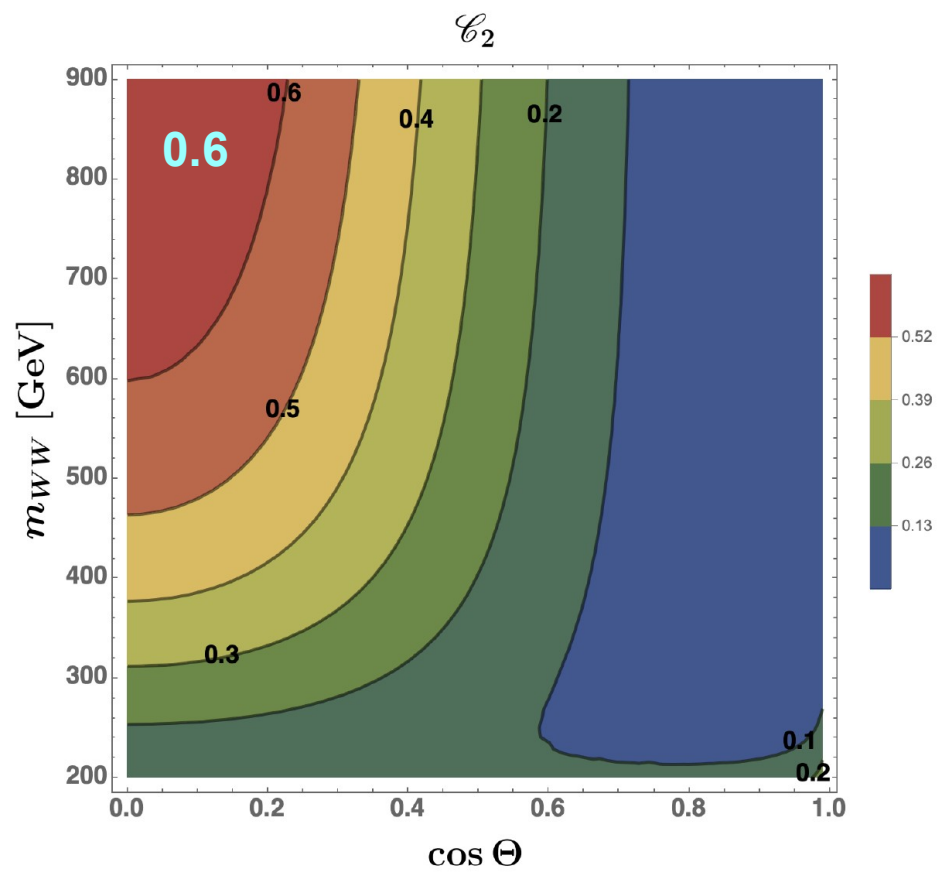
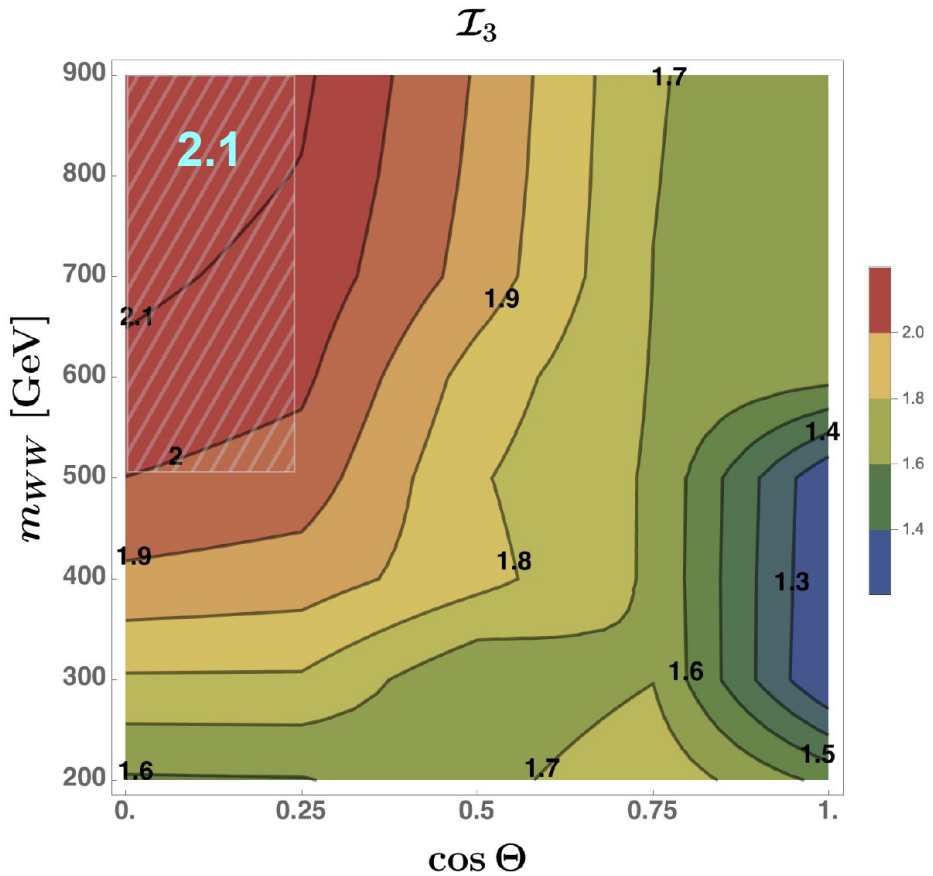
$pp \rightarrow W^+W^-$



optimization for **maximum value** of $\mathcal{I}_3$
of Bell's inequality violation is employed point
by point in the $\Theta - m_{WW}$ space

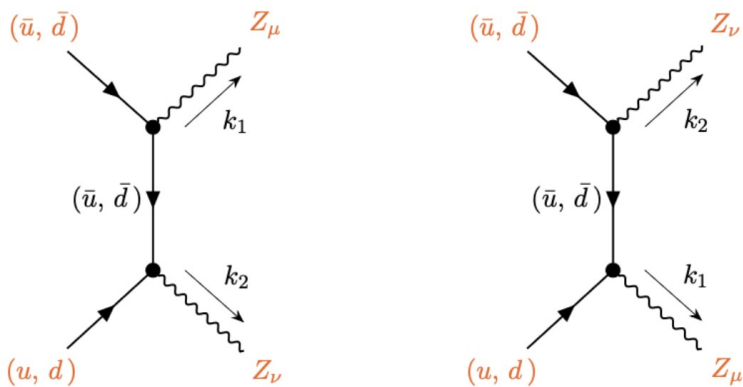
$$\mathcal{B} \rightarrow (U \otimes V)^\dagger \cdot \mathcal{B} \cdot (U \otimes V)$$

(see backup slides for their expressions in hatched area)

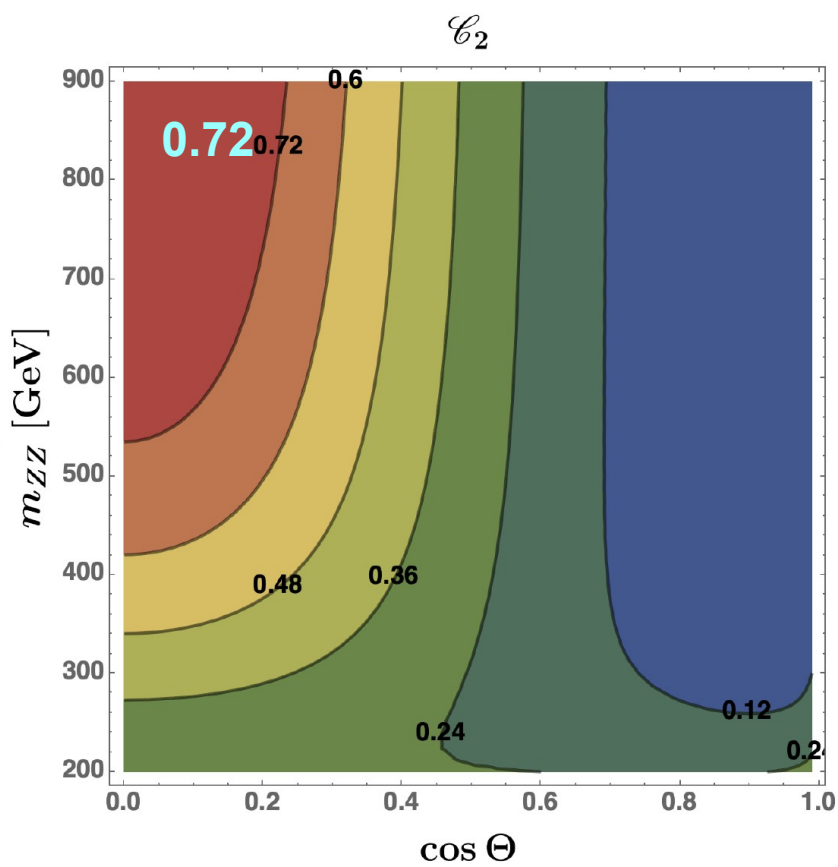
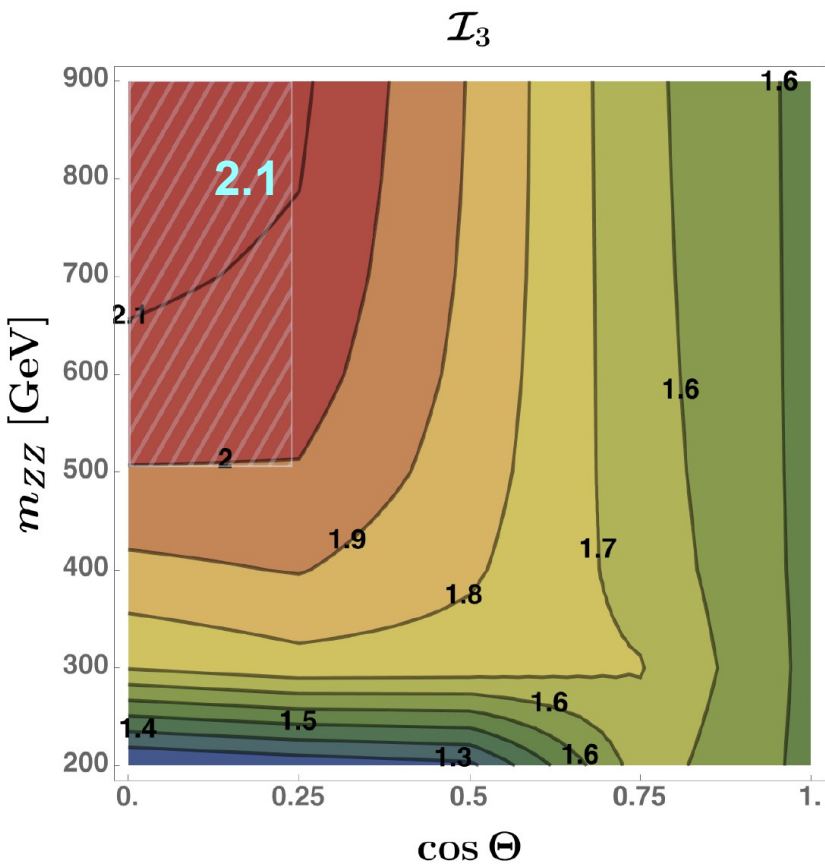


hatched area in the left-plot for $\mathcal{I}_3 > 2$ indicates bin used as reference for our estimation of the significance
(see next slides)

$$pp \rightarrow ZZ$$



- same strategy as in WW for optimization of maximum value of Bell's inequality violation
- reduced effect for Bell violations with respect to WW, but error is smaller being dominated by statistics



hatched area in the left-plot for $\mathcal{I}_3 > 2$ indicates bins used as reference for our estimation of the significance (see next slides)

Bell's inequalities in Di-boson production at LHC

EPJC 83 (2023) 9, 823 ; arXiv: 2302.00683

$$p p \rightarrow W^+ W^- \quad p p \rightarrow Z Z \quad p p \rightarrow W Z$$

(backup slides for details)

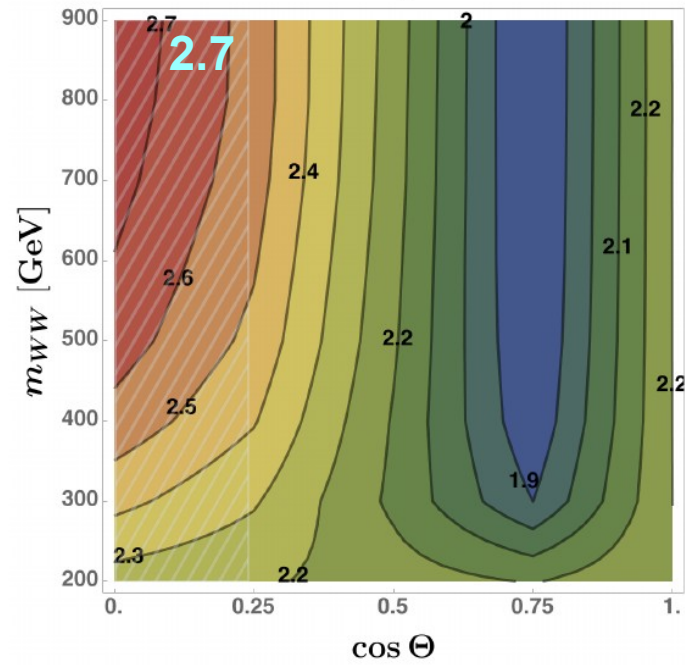
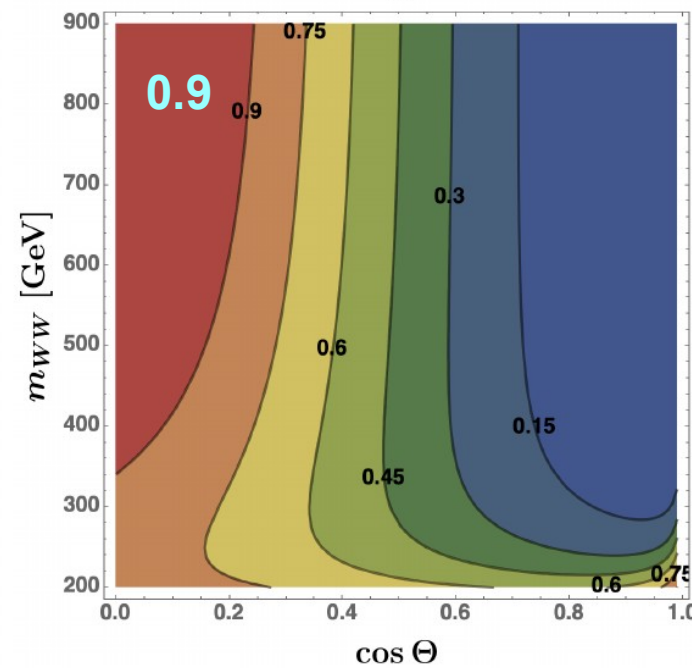
- not sufficient N. of events to test Bell's inequalities with large significance in the relevant kinematic region
- $pp \rightarrow WW$ largely affected by systematic error induced by the missing neutrino momentum reconstruction
- $pp \rightarrow ZZ$ is the most promising channel

At Hi-lumi **significance more than 2** to reject the null hypothesis

- estimates of significance expected to be reduced by a full simulation analysis

Lepton colliders

$$e^+ e^- \rightarrow W^+ W^-$$

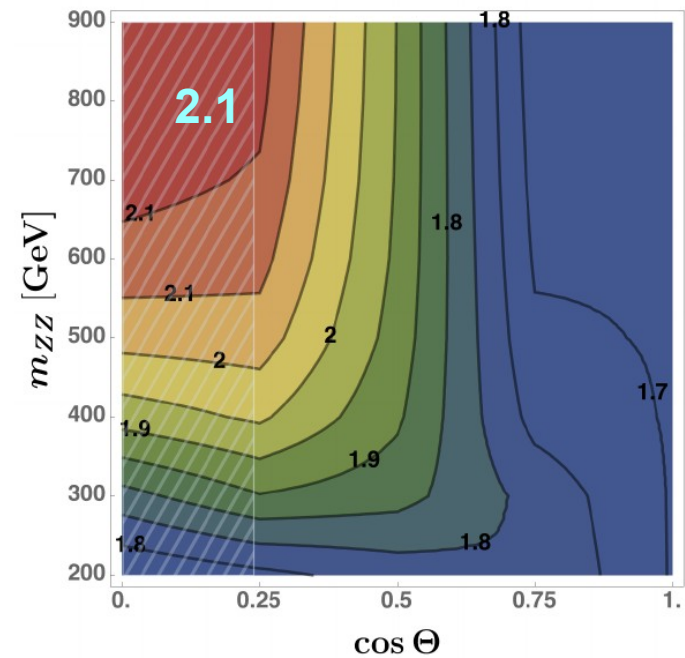
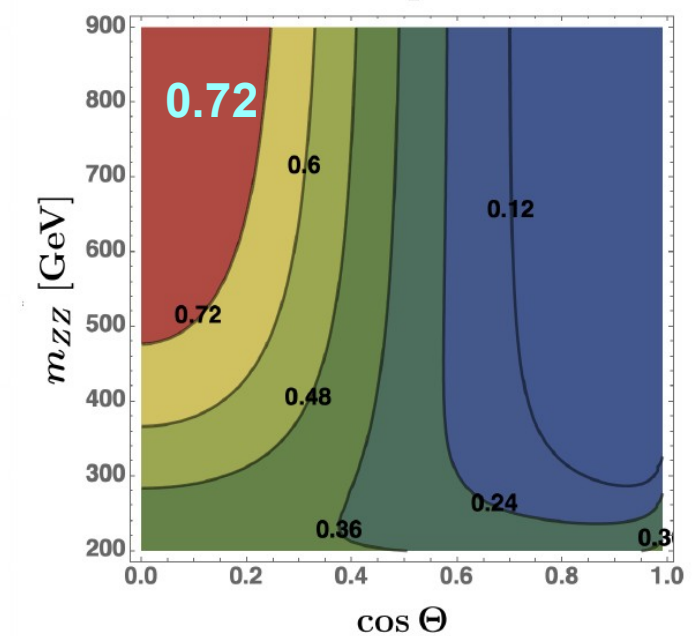
 $\mathcal{I}_3$

 $\mathcal{C}_2$


$$h_{ab}[m_{WW}, \Theta] = \frac{\tilde{h}_{ab}^{\ell\bar{\ell}}[m_{WW}, \Theta]}{A^{\ell\bar{\ell}}[m_{WW}, \Theta]},$$

$$f_a[m_{WW}, \Theta] = \frac{\tilde{f}_a^{\ell\bar{\ell}}[m_{WW}, \Theta]}{A^{\ell\bar{\ell}}[m_{WW}, \Theta]},$$

$$g_a[m_{WW}, \Theta] = \frac{\tilde{g}_a^{\ell\bar{\ell}}[m_{WW}, \Theta]}{A^{\ell\bar{\ell}}[m_{WW}, \Theta]},$$

absence of PDF increases entanglement and Bell violations in WW

 $\mathcal{I}_3$

 $\mathcal{C}_2$


$$e^+ e^- \rightarrow Z Z$$

ZZ channel not much different than LHC → due to the fact that PDF almost factorize (t,u channels)

Lepton colliders : events and sensitivity

$$e^+ e^- \rightarrow Z Z$$

promising channel

$$\sqrt{s} = 1 \text{ TeV}$$

$$\sqrt{s} = 368 \text{ GeV}$$

$$(\text{muon}) \mathcal{L} = 1 \text{ ab}^{-1}$$

$$(\text{FCC}) \mathcal{L} = 1.5 \text{ ab}^{-1}$$

events

44

748

$$m_{WW} > 200 \text{ GeV and } \cos \Theta < 0.25$$

Significance to reject null hypothesis:

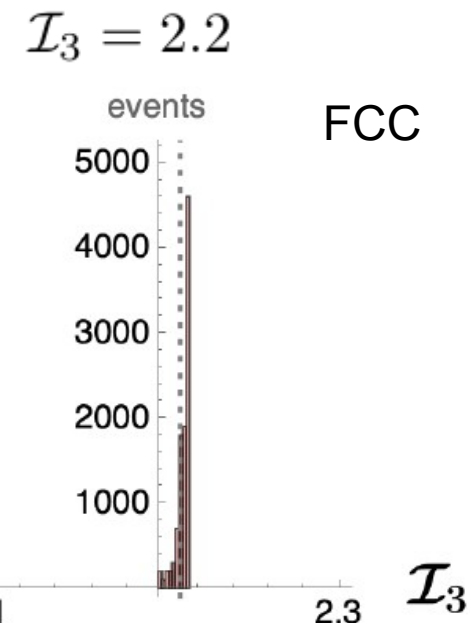
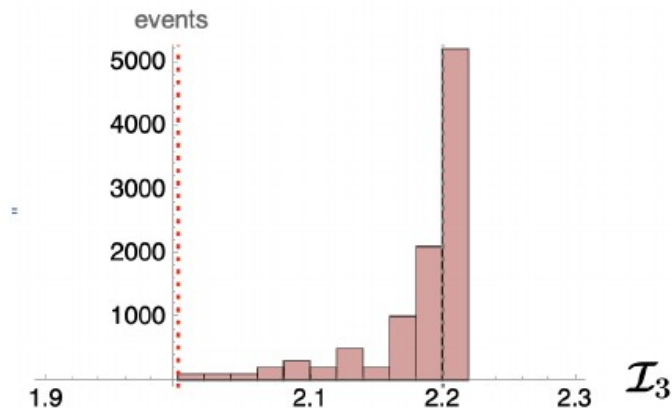
2 @ muon collider and > 4 @ FCC

- Events computed with MADGRAPH@LO
- Efficiency 70% in each lepton identification
→ n. of events reduced accordingly
- only statistic error assumed
- no background considered

FCC significance is larger than muon collider due to larger n. of events at FCC (statistical error reduced)

10^4 pseudo experiments

$\mathcal{I}_3 = 2.2$
Muon collider



Summary

- Quantum tomography in diboson production is a very powerful tool
- require the knowledge of **polarization density matrix**
- that can be fully reconstructed from decay products of W and Z
- **and allow to investigate quantum entanglement and Bell inequality violation**
- Entanglement in diboson production difficult to quantify in general case (mixtures)
(but we can use lower bound on concurrence as witness of entanglement)
- It is possible to quantify entanglement in pure states $H \rightarrow VV^*$ (using **Entropy**)
- Large significance ($> 5\sigma$) to observe entanglement and Bell inequality violation in $H \rightarrow ZZ^*$ expected at LHC Hi-Lumi
- Entanglement as powerful tool to constrain **anomalous couplings in $H \rightarrow VV^*$**

Backup slides

● **Matrix element of two-Vector Boson production** $\bar{q}(p_1) q(p_2) \rightarrow V_1(k_1, \lambda_1) V_2(k_2, \lambda_2)$

$$\mathcal{M}(\lambda_1, \lambda_2) = \mathcal{M}_{\mu\nu} \varepsilon^{\mu*}(k_1, \lambda_1) \varepsilon^{\nu*}(k_2, \lambda_2)$$

Density matrix

$$\rho(\lambda_1, \lambda'_1, \lambda_2, \lambda'_2) =$$

9 x 9 matrix

covariant expression

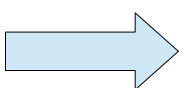
$$\frac{\mathcal{M}_{\mu\nu} \mathcal{M}_{\mu'\nu'}^\dagger \mathcal{P}_{\lambda_1\lambda'_1}^{\mu\mu'}(k_1) \mathcal{P}_{\lambda_2\lambda'_2}^{\nu\nu'}(k_2)}{|\overline{\mathcal{M}}|^2}$$

↓ unpolarized matrix element square

$$\mathcal{P}_{\lambda\lambda'}^{\mu\nu}(p) = \varepsilon^\mu(p, \lambda)^* \varepsilon^\nu(p, \lambda')$$

(spin-1 eigenstates **at rest**)

k=axis of spin-quantization



$$\psi_{\pm} = -\frac{1}{\sqrt{2}} (\pm \hat{\mathbf{n}} + i \hat{\mathbf{r}}) \quad \text{and} \quad \psi_0 = \hat{\mathbf{k}}$$

● How to related spin-1 eigenstates at rest to the boosted frame ? (see next slide)

covariant polarization vector of spin-1

rest frame limit

$$\varepsilon^\mu(p, \lambda) \xrightarrow{(\beta \rightarrow 0)} \psi_\pm, \psi_0$$

β is the velocity

helicity $\lambda = \pm 1, 0$

boosted base

$$n_1^\mu = (0, \hat{\mathbf{n}}), \quad n_2^\mu = (0, \hat{\mathbf{r}}), \quad n_3^\mu = \frac{E}{M}(\beta, \hat{\mathbf{k}})$$

Covariant Projector

$$\mathcal{P}_{\lambda\lambda'}^{\mu\nu}(p) = \varepsilon^\mu(p, \lambda)^* \varepsilon^\nu(p, \lambda')$$

master formula

$$\varepsilon^{0123} = 1$$

$$= \frac{1}{3} \left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2} \right) \delta_{\lambda\lambda'} - \frac{i}{2M} \epsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta^i (S_i)_{\lambda\lambda'} - \frac{1}{2} n_i^\mu n_j^\nu (S_{ij})_{\lambda\lambda'}$$

$S_i, i \in \{1, 2, 3\} \rightarrow$ rotation matrices for spin-1 particle

H.S. Song, *Lett. Nuovo Cim.* 25 (1979)
S.Y. Choi, T. Lee, H.S. Song, *PRD* 40 (1989)
Fabbrichesi, Floreanini, EG, Marzola,
2302.00683 [hep-ph]

$$S_{ij} = S_i S_j + S_j S_i - \frac{4}{3} \mathbb{1} \delta_{ij} \quad (\text{see backup slides})$$

basis correspondence for $(S_i)_{\lambda\lambda'}$ $|+\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad |-\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ corresponding to eigenevalues $\lambda = \pm 1, 0$

Sensitivity to the Bell violation

$$H \rightarrow WW^* \quad ZZ^*$$

		$\ell^+ \nu_\ell \ell^- \bar{\nu}_\ell$	$\ell^- \ell^+ \ell^- \ell^+$
<u>LHC run2</u>	$(\mathcal{L} = 140 \text{ fb}^{-1})$	4571	28
<u>Hi-Lumi</u>	$(\mathcal{L} = 3 \text{ ab}^{-1})$	9.8×10^3	589

cuts in V^* invariant mass: 40 GeV for WW^* and 30 GeV for ZZ^*

- we used MADGRAPH5 @ LO corrected by k factors at N3LO + N3LL
Bonvini, Marzani, Muselli, Rottoli, JHEP 08, 105 (2016) 1603.08000 [hep-ph]
- take into account efficiency (70%) in the identification of final leptons
- events reduced of 25% for ZZ^* and 50% for WW^*

- backgrounds can be efficiently reduced after considering topology of final states and use of suitable cuts
- we neglect all background effects in our assessment of significance
- We model uncertainty as a Gaussian dispersion in the determination of the mass of the off-shell weak boson controlled by the number of events
- For the WW^* we add a systematic error taking into account uncertainty in the reconstruction of W^* mass, taking as benchmark value = 5 GeV for the systematic error in reconstruction of neutrino momenta

Relations between helicity amplitudes h_i and h_{ab} , f_a , g_a , of Gell-Mann basis

$$\begin{aligned}
 \hat{h}_- \hat{h}_-^* &= \frac{1}{9} \left[1 + 3\sqrt{3} (f_8 - 2g_8 - 2h_{38}) + 9f_3 - 6h_{88} \right], \\
 \hat{h}_0 \hat{h}_-^* &= h_{16} + i (h_{17} - h_{26}) + h_{27}, \\
 \hat{h}_+ \hat{h}_-^* &= h_{44} + i (h_{45} - h_{54}) + h_{55}, \\
 \hat{h}_0 \hat{h}_0^* &= \frac{1}{9} \left[1 - 9(f_3 + g_3 - h_{33}) + 3\sqrt{3} (f_8 + g_8 - h_{38} - h_{83}) + 3h_{88} \right], \\
 \hat{h}_+ \hat{h}_0^* &= h_{61} + i (h_{62} - h_{71}) + h_{72}, \\
 \hat{h}_+ \hat{h}_+^* &= \frac{1}{9} \left[1 + 3\sqrt{3} (g_8 - 2f_8 - 2h_{83}) + 9g_3 - 6h_{88} \right],
 \end{aligned}$$

Including background for $H \rightarrow Z\ell^+\ell^-$

$$\rho_{ZZ} = \alpha \rho_{H \rightarrow ZZ} + (1 - \alpha) \rho_{\text{BCKG}} \quad 0 \leq \alpha \leq 1$$

Main EW background

$$pp \rightarrow \bar{Z}Z/Z\gamma \rightarrow 4\ell$$

$$S/(S+B)$$

We take $\alpha = 0.8$ corresponding to bckg 4 times smaller than signal

<i>LHC</i>	<i>run2</i>	<i>HiLumi</i>
	$ a_Z \leq 0.0028$	$ a_Z \leq 0.00062$
	$ \tilde{a}_Z \leq 0.0039$	$ \tilde{a}_Z \leq 0.00086$



bckg effects as expected weakens a bit the bounds

Lepton colliders : events and sensitivity

$$e^+ e^- \rightarrow W^+ W^-$$

$$\sqrt{s} = 1 \text{ TeV}$$

$$\sqrt{s} = 368 \text{ GeV}$$

$$(\text{muon}) \mathcal{L} = 1 \text{ ab}^{-1}$$

$$(\text{FCC}) \mathcal{L} = 1.5 \text{ ab}^{-1}$$

events

$$3.6 \times 10^3$$

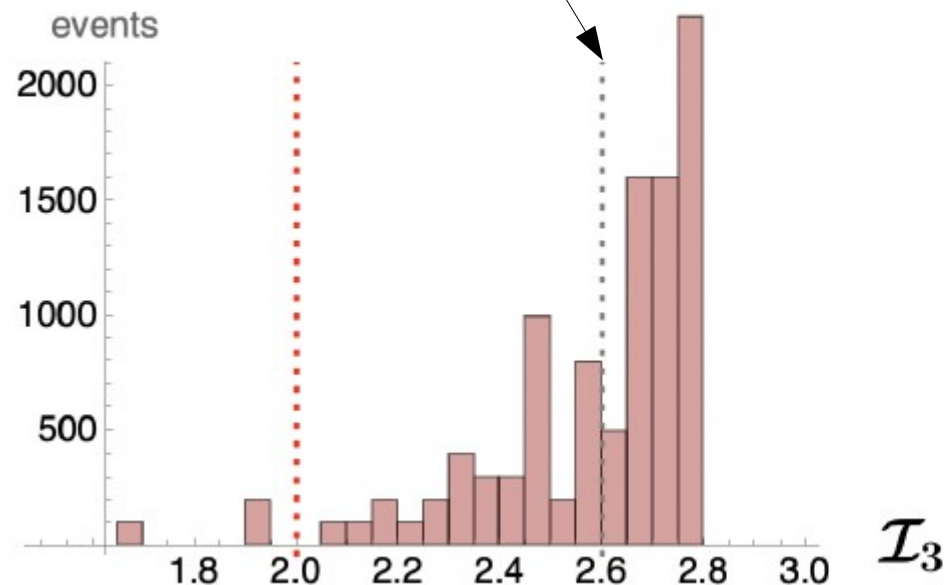
$$5.8 \times 10^4$$

$$m_{WW} > 200 \text{ GeV and } \cos \Theta < 0.25$$

Significance = 2 to reject the null hypothesis
both at muon collider and FCC

- Events computed with MADGRAPH@LO
- Efficiency 70% in each lepton identification
→ n. of events reduced accordingly
- systematic error due to missing neutrino
in WW mass reconstruction
largely dominates against statistical one
- no background considered

mean value = 2.6



mean value = 2.6

