



NA62 Physics Handbook

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Light-Mesons Radiative Decays & Photon Polarization Asymmetries

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Outlook

- Light mesons radiative polarized decays

$$\pi^+ \rightarrow \nu \ell^+ \gamma$$

$$K^+ \rightarrow \nu \ell^+ \gamma$$

- photon polarization asymmetries
- anomalous helicity-flip of final charged lepton
- conclusions

- Radiative decays of mesons and leptons widely studied both experimentally and theoretically
- more detailed information with respect to the unpolarized measurements are available
- excellent sources for precision tests of SM
- polarized decays analyzed only more recently

Pion (Choi et.al. ('89), Trentadue-Verbeni ('00), EG-Trentadue ('08))

Muon (Fischer et. al.('03) , Sehgal-Schulz ('03), EG-Trentadue ('08))

- non vanishing rate for helicity-flip of electron when $m(e) \rightarrow 0$: manifestation of **axial anomaly**
(Dolgov-Zakharov ('71))
- finiteness of polarized processes against infrared and collinear singularities take place with different mechanisms distinguish between **right- and left-handed** final leptons.
- Polarized processes are more sensitive to **New Physics** effects beyond the Standard Model with respect to the unpolarized ones
- in particular, to new chirality structures beyond **V-A theory** in the low-energy effective Hamiltonians

- we consider the possibility of "measuring" the **photon polarization** in radiative light-mesons & muon decays

- **photon polarization:**

- selects particular combinations of Form Factors
- (V-A) theory + angular-momentum conservation distinguish among L- and R-handed polarizations
- a sensitive probe of New Physics

- **a new observable can be defined:**

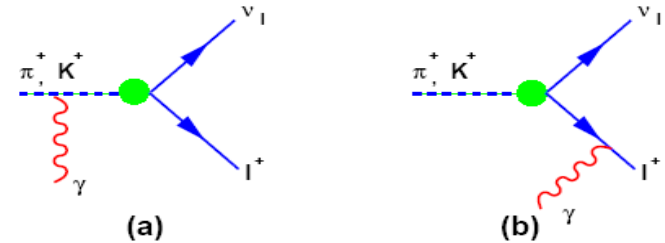
- photon polarization asymmetry (L-R):**

- free from infrared singularities
- gives a direct measure of parity violation
- quite sensitive to Form Factors (e-channel only)

$$M^+(p) \rightarrow \nu(p_\nu) + \ell^+(p_\ell, \lambda_\ell) + \gamma(k, \lambda_\gamma)$$

$$M = (K, \pi)$$

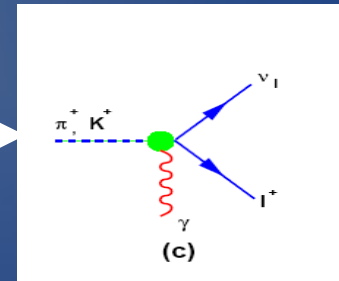
Inner Bremsstrahlung $\rightarrow$
 $f_M \rightarrow$ meson decay constant



$$M_{IB}^{(\lambda_l, \lambda_\gamma)} = \frac{ieG_F}{\sqrt{2}} m_l f_M V_{uq} \epsilon_\mu^*(k, \lambda_\gamma) \left[\bar{u}(p_\nu) \left(\frac{p^\mu}{(p \cdot k)} - \frac{k^\mu \gamma^\mu + 2p_l^\mu}{2(p_l \cdot k)} \right) (1 + \gamma_5) v(p_l, \lambda_l) \right]$$

Structure Dependent (SD) $\rightarrow$

V and A Form Factors



Gauge Invariant

$$M_{SD}^{(\lambda_l, \lambda_\gamma)} = -\frac{ieG_F}{\sqrt{2}} V_{uq} \epsilon_\mu^*(k, \lambda_\gamma) \left\{ (p \cdot k) \frac{A}{m_M} \left(-\eta^{\mu\nu} + \frac{p^\mu k^\nu}{(p \cdot k)} \right) + i\epsilon^{\mu\nu\alpha\beta} \frac{V}{m_M} k_\alpha p_\beta \right\} \\ \times \left[\bar{u}(p_\nu) \gamma_\nu (1 - \gamma_5) v(p_l, \lambda_l) \right],$$

Polarized amplitude

Amplitude is a mixture of both **L-** and **R-handed** photon polarizations

Measuring photon polarizations select specific combination of Form Factors

$V - A \rightarrow$ **L-photon**

$V + A \rightarrow$ **R-photon**

Matrix density for polarized processes

$$x \equiv \frac{2p \cdot k}{m_M^2}, \quad y \equiv \frac{2p \cdot p_l}{m_M^2}, \quad z \equiv \frac{2p_l \cdot k}{m_M^2}$$

$$\lambda \equiv z/x$$

In Meson rest-frame

$$\frac{d^2\Gamma^{(\lambda_\gamma)}}{dx d\lambda} = \frac{m_M}{256\pi^3} \sum_{\lambda_l=\pm 1} |M^{(\lambda_l, \lambda_\gamma)}|^2 = \rho^{(\lambda_\gamma)}(x, \lambda)$$

$$\lambda = E_e/m_M(1 - \cos\theta_{\gamma e})$$

$$x = 2E_\gamma/m_M$$

L- $\rho^{(-1)}(x, \lambda) = A_{IB} f_{IB}^L(x, \lambda) + A_{SD} \frac{1}{2}(V - A)^2 f_{SD}^L(x, \lambda) + A_{INT}(V - A)f_{INT}^L(x, \lambda)$

R- $\rho^{(+1)}(x, \lambda) = A_{IB} f_{IB}^R(x, \lambda) + A_{SD} \frac{1}{2}(V + A)^2 f_{SD}^R(x, \lambda) + A_{INT}(V + A)f_{INT}^R(x, \lambda)$

$$A_{IB} = 2r_l \left(\frac{f_M}{m_M} \right)^2 A_{SD}, \quad A_{INT} = 2r_l \frac{f_M}{m_M} A_{SD}$$

$$A_{SD} = \frac{\alpha}{32\pi^2} G_F^2 m_M^5 |V_{uq}|^2.$$

$$f_{\text{IB}}^{\text{L}}(x, \lambda) = \frac{1 - \lambda}{x\lambda} \left(1 + r_l(x - 1) - \frac{r_l}{\lambda}(1 + x - r_l) \right)$$

$$f_{\text{IB}}^{\text{R}}(x, \lambda) = \frac{1 - \lambda}{x\lambda} \left(x - 1 + \frac{r_l}{\lambda} \right) (x - 1 + r_l),$$

$$f_{\text{SD}}^{\text{R}}(x, \lambda) = x^2 \lambda \left((1 - x)(x\lambda + r_l) - r_l \right),$$

$$f_{\text{SD}}^{\text{L}}(x, \lambda) = x^2 (1 - \lambda) \left((x - 1)(r_l + x(\lambda - 1)) + r_l \right)$$

- IB dominates in the soft-photon region ($x \sim 0$)
- $x \rightarrow 0$: **L-handed** and **R-handed** distributions tend to same value: **photon-spin decoupling**
- hard photons ($x \sim 1$) accompanied by soft or collinear positrons ($\lambda \ll 1$) tend to be **Left-handed**
- **a consequence of V-A and A.M. conservation**

Photon Polarization Asymmetries

$$\frac{dA_\gamma}{d\xi} \equiv \frac{d_\xi(\text{BR}_L) - d_\xi(\text{BR}_R)}{d_\xi(\text{BR}_L) + d_\xi(\text{BR}_R)}$$

$$d_\xi(\text{BR}_{L,R}) \equiv \frac{d\text{BR}_{L,R}}{d\xi}$$

- **Free from infrared singularities at tree-level**

$$\lim_{x \rightarrow 0} \{ \rho_L(x, y) - \rho_R(x, y) \} \rightarrow \mathcal{O}(x)$$

$$\lim_{x \rightarrow 0} \{ \rho_L(x, y) + \rho_R(x, y) \} \rightarrow \log(x)$$

- **quite sensitive to hadronic form factors V and A**
- **provide a direct measure of parity violation**

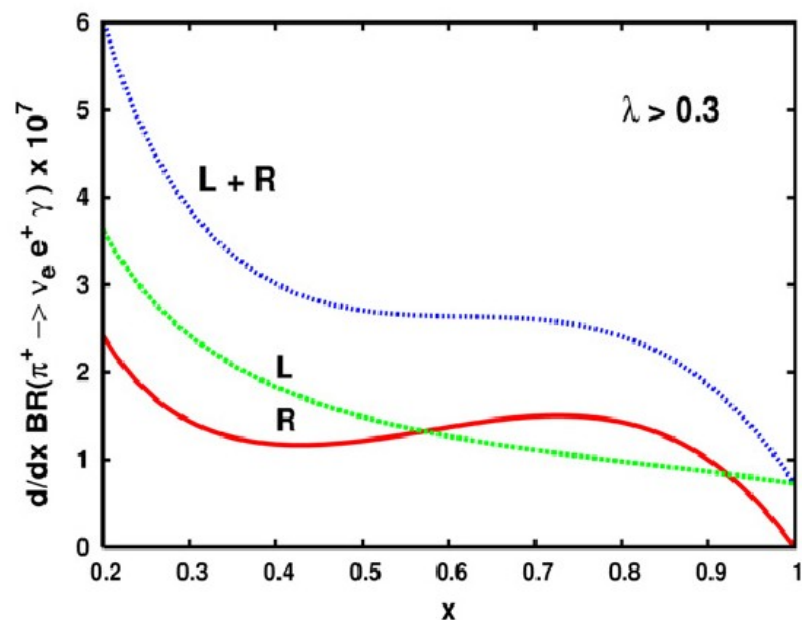
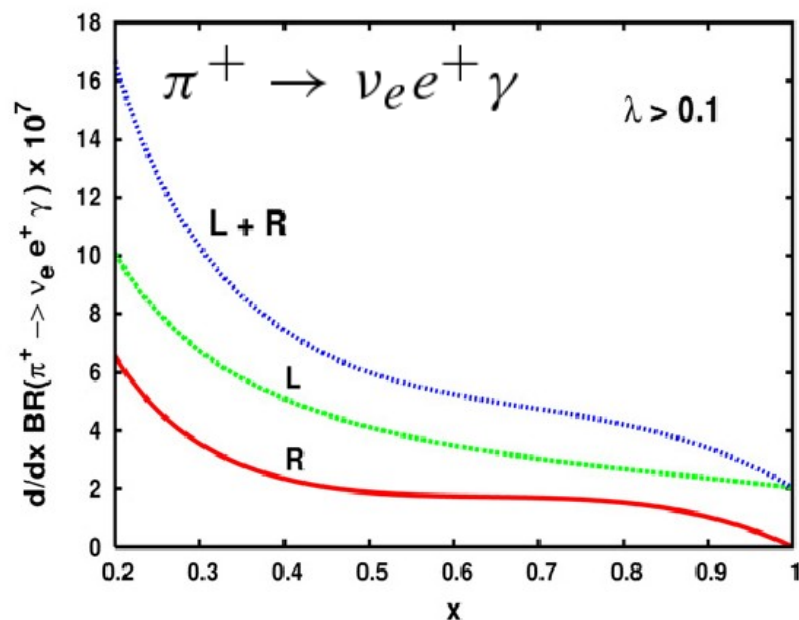
Radiative Pion decays

- form factors V and A contain the information about hadronic structure
- in general are not constant over all phase space
- ChPt at NLO (p^6) shows a mild dependence on momenta (Bijnens and Talavera (1997))
- V can be extracted from $\Pi \rightarrow \gamma \gamma$ by using PCAC

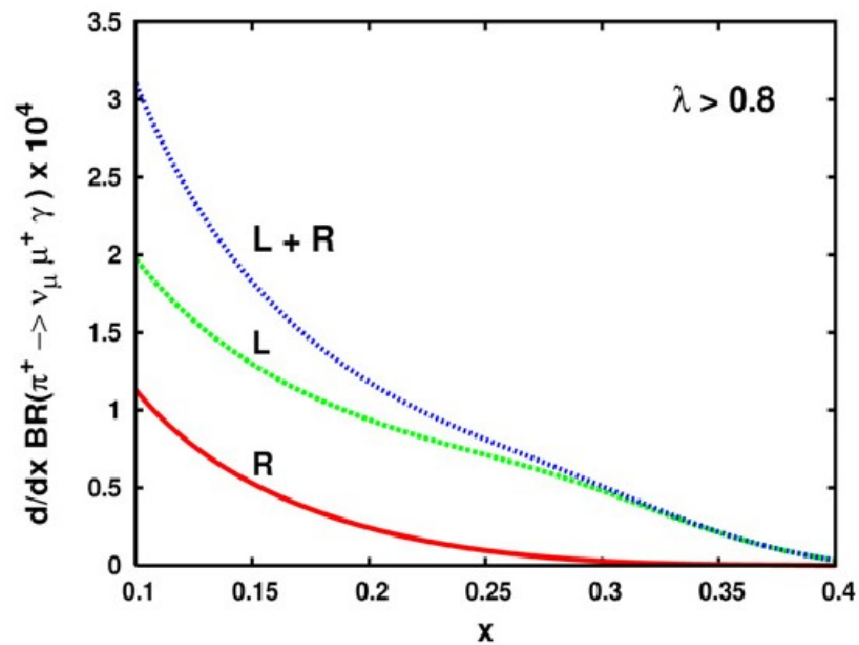
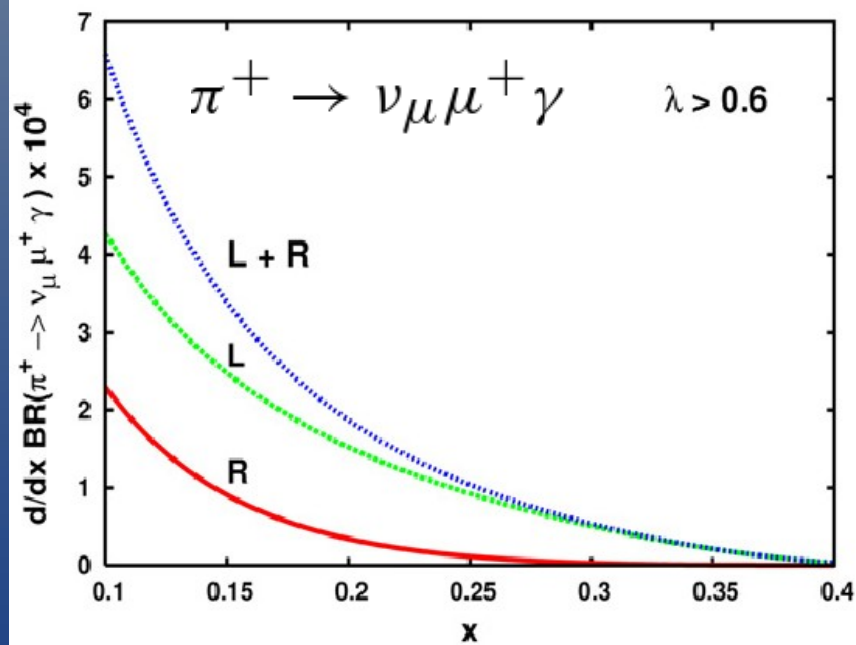
$$|V| = \frac{1}{\alpha} \sqrt{\frac{2\Gamma(\pi^0 \rightarrow \gamma\gamma)}{\pi m_{\pi^0}}} = 0.0259 \pm 0.0005$$

SD contributions are quite small in the case of μ since IB is not chiral suppressed

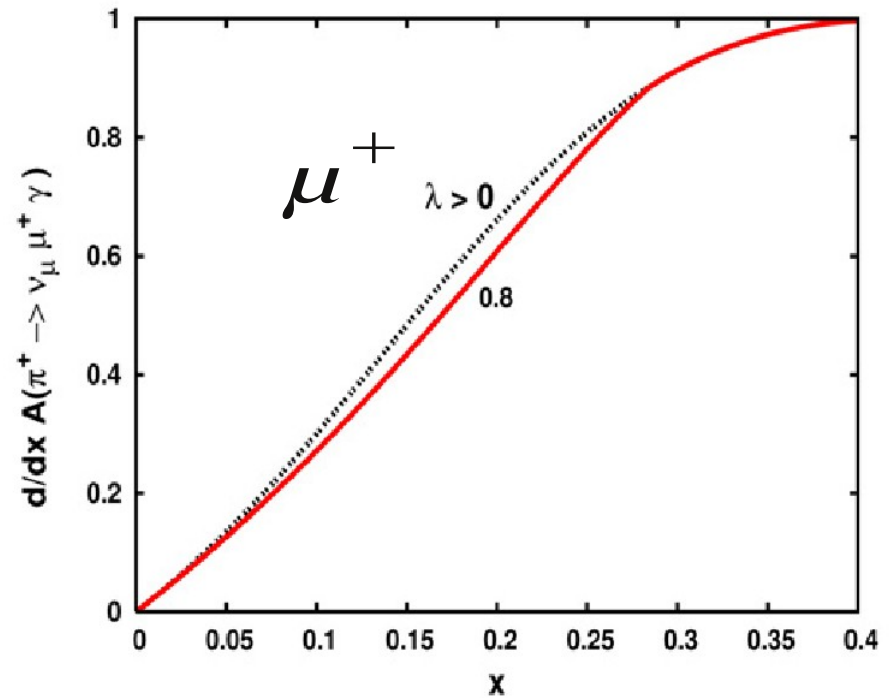
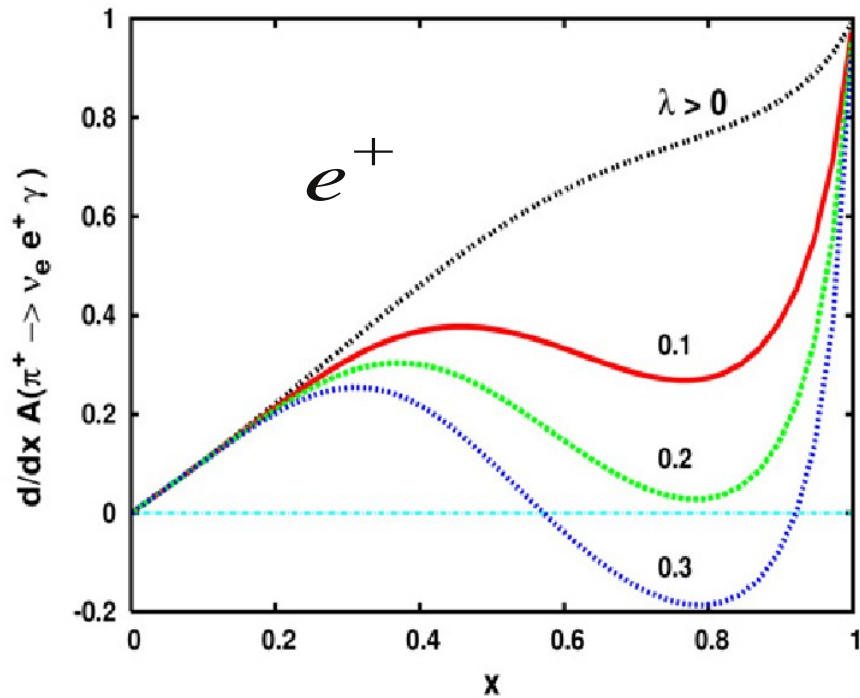
- Axial form factor A can be measured from the ratio $\gamma = V/A$ (extracted from e-channel)
- the most recent measurements of γ has been performed by **PIBETA collaboration** at PSI (2003)
- central value of best CVC fit consistent also with ChPt $\gamma = 0.443 \pm 0.015$
- We use A extracted from $\gamma = 0.443$ and $V = 0.0259$ and predict the distributions for polarized processes
- summing over all positron polarizations but fixing the photon polarization



Photon-energy distributions



PIONs: Photon Polarization Asymmetries



e-channel:

- **PPA** very sensitive to λ -cuts in hard-photons region
- **becomes negative at large x and λ**
- positions of zeros, quite sensitive to Form Factors

mu-channel:

- **PPA** always positive
- photons mainly L-handed polarized (due to IB)

Radiative Kaon decays

Form Factors

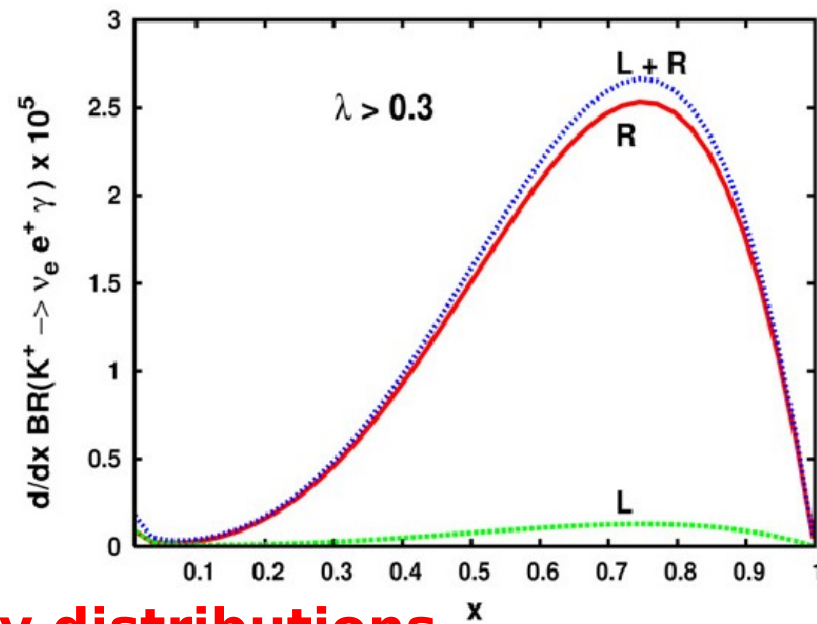
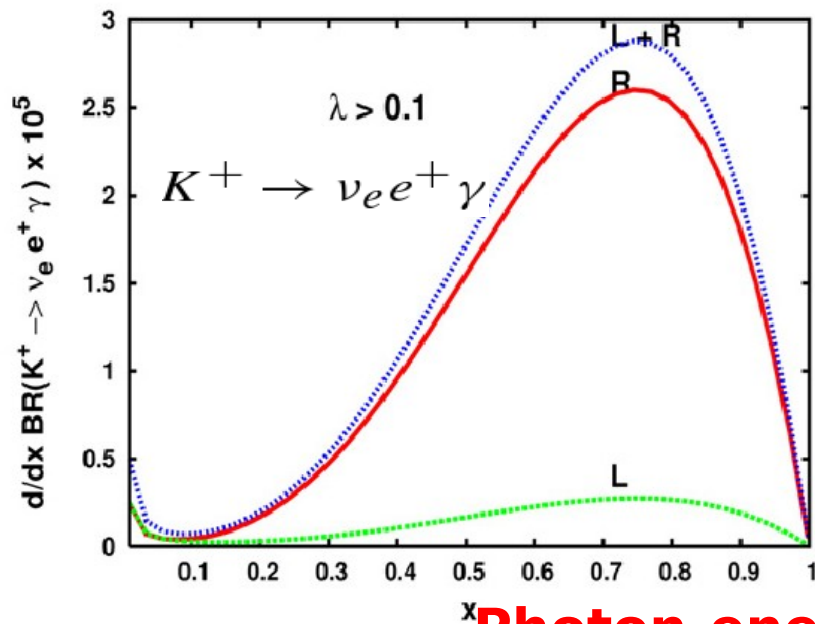
larger than in pion

$$|V + A| = 0.165 \pm 0.007 \pm 0.011$$
$$-0.24 < V - A < 0.04 \text{ at } 90\% C.L.$$

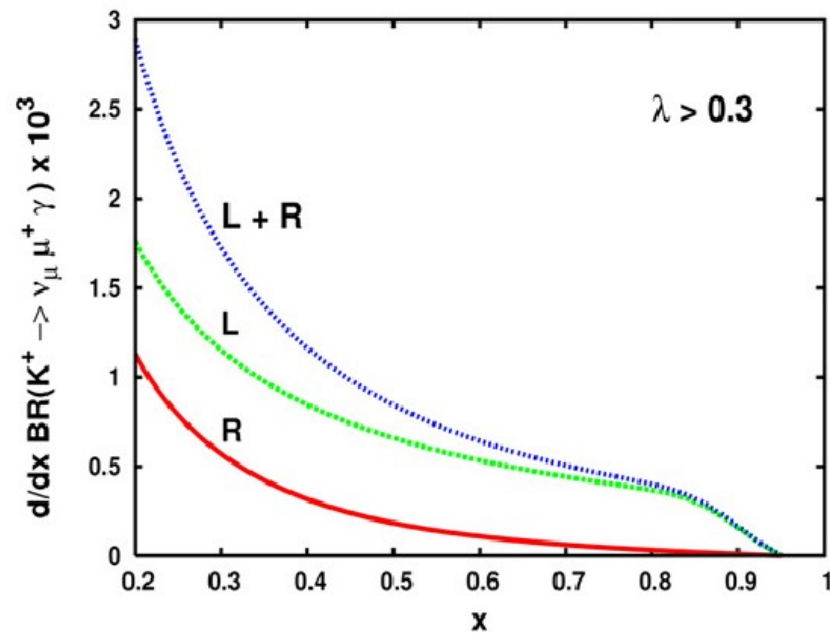
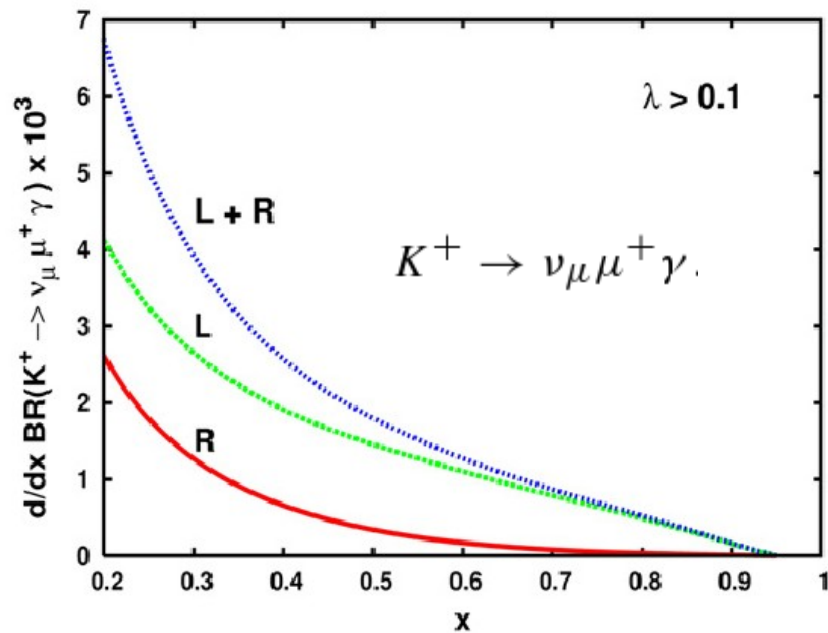
(E787 coll. (2001))

- ~ 2 -sigma deviation with respect to ChPt at LO
- **momentum dependence starts at NLO (p^6)**
 - expected to be sizeable (10-20%) (Geng, Ho, Wu (2004))
 - more sizeable in V than in A
- we use V and A inputs from ChPt predictions at 1-loop

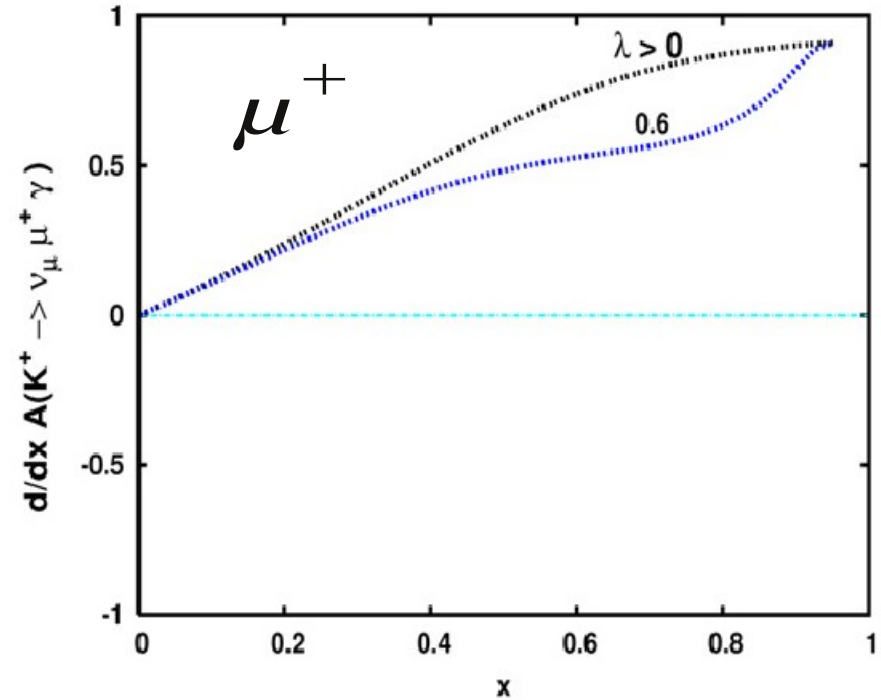
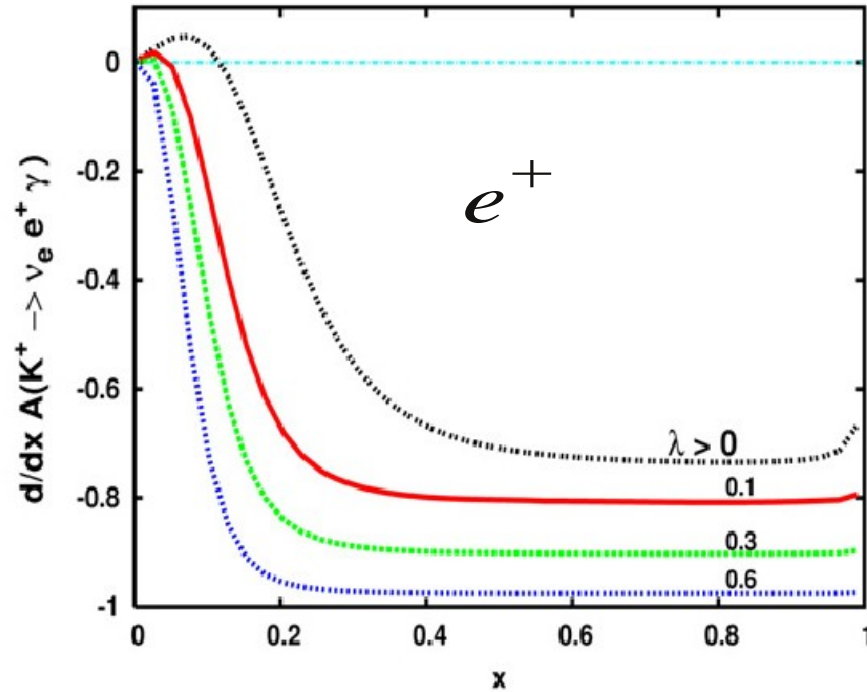
$$V + A = -0.137, \quad V - A = -0.052$$



Photon-energy distributions



KAONs: Photon Polarization Asymmetries



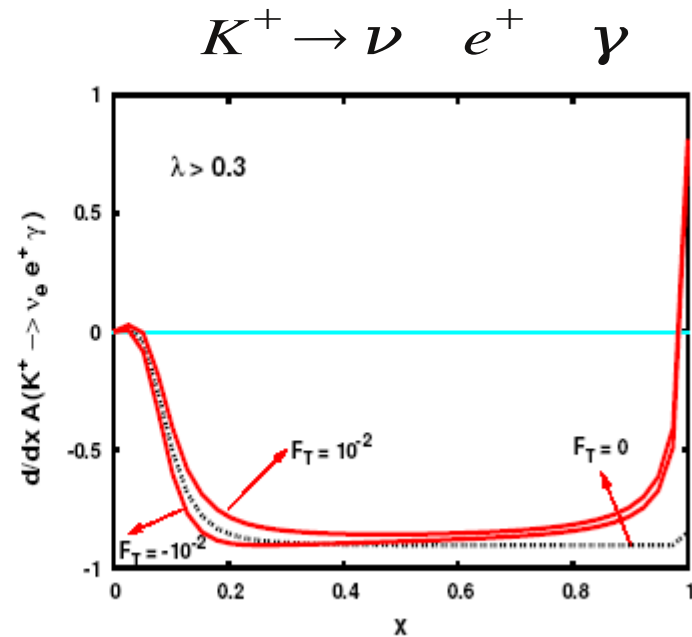
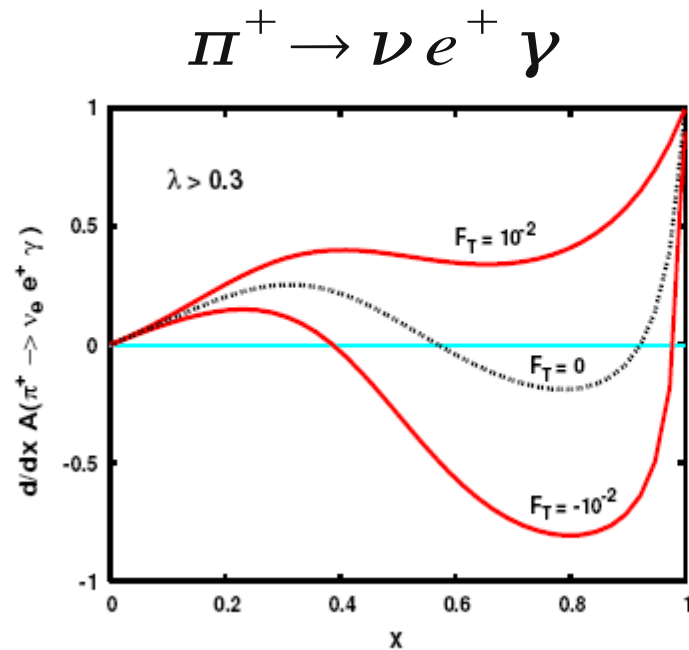
e-channel:

- for $x > 0.2$, PPA < 0 (very different from Pion case)
- hard photons mainly R-handed $\Rightarrow$ due to SD terms

mu-channel:

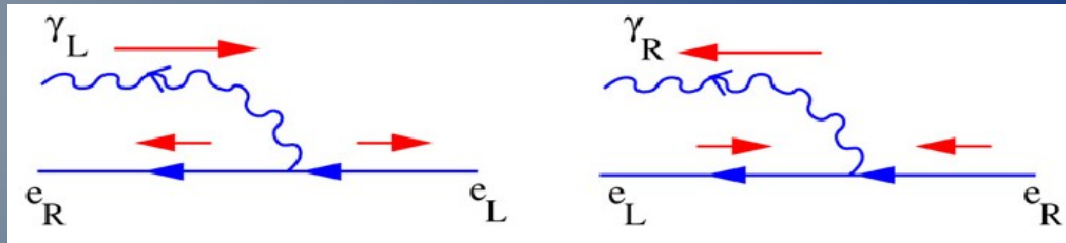
- PPA always positive (as in Pion case)
- photons mainly L-handed polarized (due to IB)

Tensorial contributions to PPA



PPA for Pion is quite sensitive to tensorial couplings of centiweak strenght in G-Fermi unity

Anomalous electron helicity-flip



- In massless QED, helicity-flip contribution vanishes at any order in perturbation theory
- **Lee and Nauenberg ('64)** pointed out that there will be a non-zero helicity-flip contribution from collinear photon emissions surviving the limit

$$m_e \rightarrow 0$$

- this phenomena happens also in meson and muon radiative decays (**Trentadue-Verbeni ('03)**, **Fischer et al. ('02)** , **Sehgal - Schulz ('03)**, **Gabrielli-Trentadue ('08)**)

Helicity-flip as a mass-discontinuity

$$\frac{d\sigma}{d\theta^2} \sim \frac{\left(\frac{m_e}{E_e}\right)^2}{\left(\left(\frac{m_e}{E_e}\right)^2 + \theta^2\right)^2}$$

In radiative meson decays

- **massless QED:** positron emitted **Left-handed** in meson decay (in IB only)
- **Right-handed positron gives a finite contribution in the limit $m(e) \rightarrow 0$**
- **due to Axial anomaly:** anomalous helicity-flip contribution can be related to the absorptive part of VVA triangle diagram in massless QED
(Dolgov-Zakharov ('71))

Right-handed positron contribution (soft-photon region, $x \sim 0$ or $y \sim 1$)

photon-polarization  positron-polarization 

$$\lim_{r_l \rightarrow 0} \frac{1}{\Gamma_0} \frac{d\Gamma_{\text{IB}}^{(\text{L},\text{R})}}{dy} = \frac{\alpha}{2\pi} (1 - y).$$

$$\lim_{r_l \rightarrow 0} \frac{1}{\Gamma_0} \frac{d\Gamma_{\text{IB}}^{(\text{R},\text{R})}}{dy} = 0$$

- **R-handed photon** contribution vanishes
- **L-handed photon** contribution is finite from infrared ($y \rightarrow 1$) and collinear divergencies ($r \rightarrow 0$)

- Analogous behavior for the SD contribution
(**soft-photon region near $y \sim 1$**)

- $|\text{SD}|^2$ contribution

$$\lim_{r_l \rightarrow 0} \frac{1}{\Gamma_0} \frac{d\Gamma_{\text{SD}}^{(\text{L},\text{L})}}{dy} = 0,$$

e(R)

$$\lim_{r_l \rightarrow 0} \frac{1}{\Gamma_0} \frac{d\Gamma_{\text{SD}}^{(\text{L},\text{R})}}{dy} = \frac{\alpha}{2\pi} \frac{m_M^2}{f_M^2} \frac{(V - A)^2}{8r_l} y^2 (1 - y)^2,$$

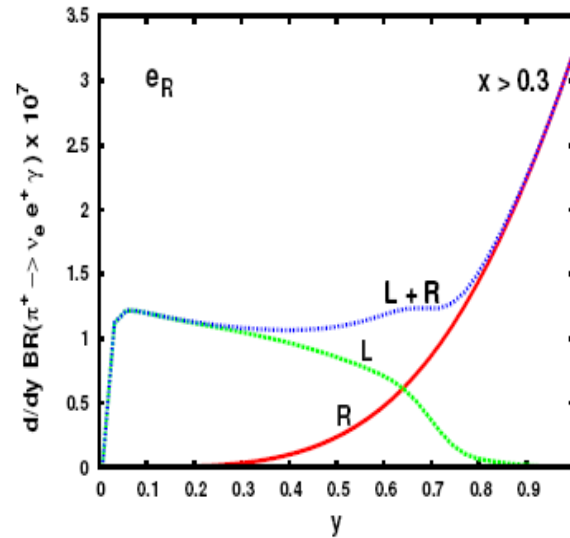
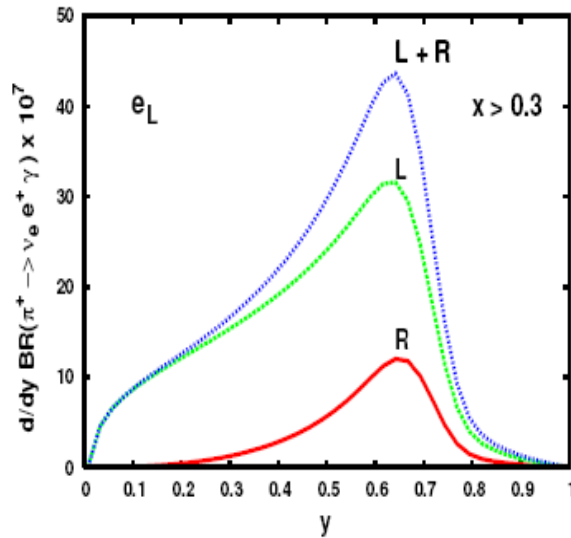
$$\lim_{r_l \rightarrow 0} \frac{1}{\Gamma_0} \frac{d\Gamma_{\text{SD}}^{(\text{R},\text{L})}}{dy} = 0,$$

e(R)

$$\lim_{r_l \rightarrow 0} \frac{1}{\Gamma_0} \frac{d\Gamma_{\text{SD}}^{(\text{R},\text{R})}}{dy} = \frac{\alpha}{2\pi} \frac{m_M^2}{f_M^2} \frac{(V + A)^2}{48r_l} y^4;$$

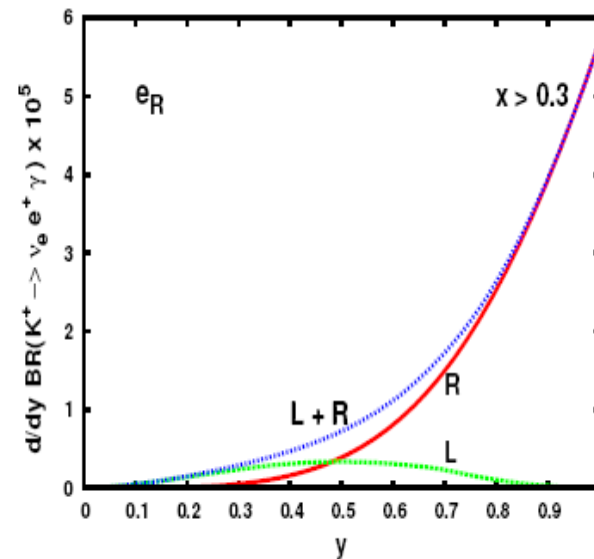
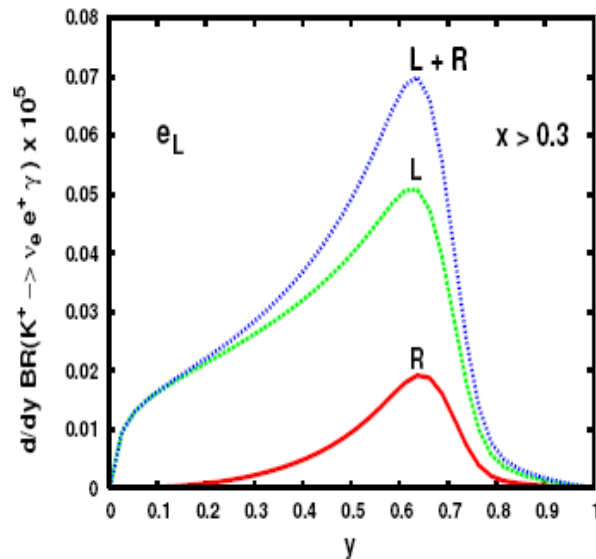
Pion

$e(L) \gg e(R)$



Kaon

$e(R) \gg e(L)$



In K, SD gives main contribution to $e(R)$ in large y -regions

Conclusions

- We considered the possibility of measuring the photon polarization in radiative meson and muon decays
- **Photon-Polarization-Asymmetries** allow a new approach to investigate interaction dynamics via finite and universal quantities:
 - **directly connected to parity violation**
 - **quite sensitive to Form Factors in meson decays**
 - **probe of New Physics**
- Mechanisms of cancellation of mass singularities have been analyzed for polarized radiative decays (included muon)
- Right-handed charged lepton contributions are finite from IR and collinear divergencies
- while Left-handed ones are only finite for inclusive processes (real+ virtual)

Concluding remark:

“Standard Model physics and as well as signal of New Physics could be put under scrutiny and more closely investigated by using tests involving **polarized quantities**”